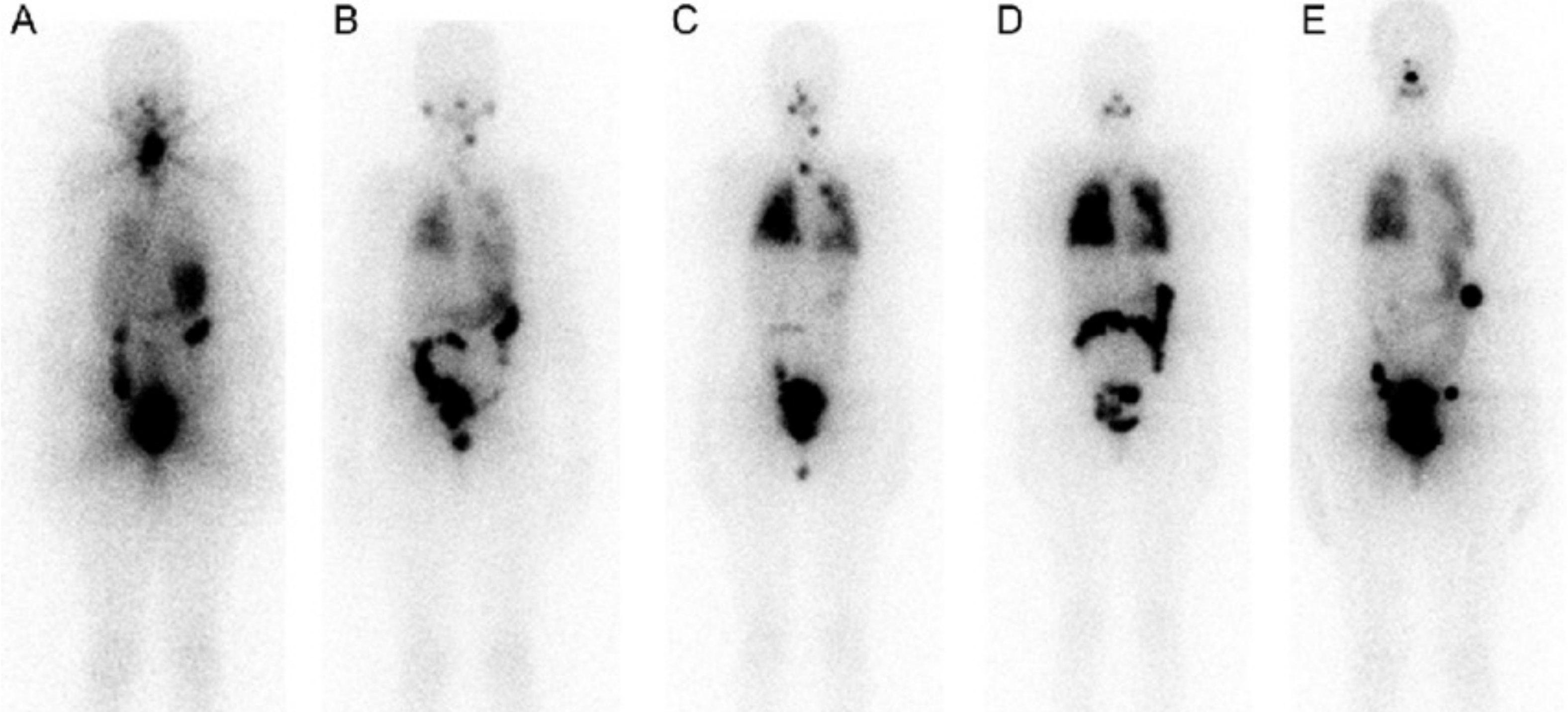
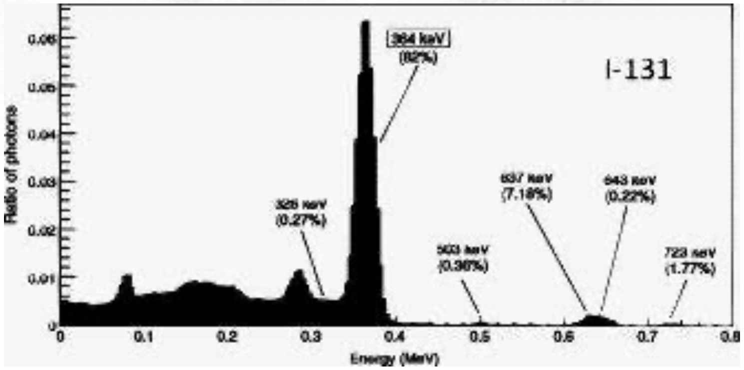
Accumulated Dose from Internally Deposited Radioactive Sources

Case Report: “Management of metastatic thyroid cancer in pregnancy: risk and uncertainty”, by Christopher W Rowe et al., Endocrinology, Diabetes and Metabolism, 2016



Serial post-I 131 therapy scans (anterior whole body views). (A) Age 10, 3.01 GBq; (B) age 11, 2.95 GBq; (C) age 13, 4.3 GBq; (D) age 14, 5.1 GBq; (E) age 15, 9.9 GBq, 7 months before conception. Neck disease present at age 13 (C) was treated surgically. The final study (E) showed the presence of radioiodine avid bilateral pulmonary metastases (<5 mm maximum diameter on computed tomography) and very small, low-grade lower neck disease. The focal uptake in the left upper abdomen is colonic and is likely physiological in nature.



^{225}Ac-PSMA-617 for PSMA-Targeted α-Radiation Therapy of Metastatic Castration-Resistant Prostate Cancer

Clemens Kratochwil*[1], Frank Bruchertseifer*[2], Frederik L. Giesel[1], Mirjam Weis[2], Frederik A. Verburg[3], Felix Mottaghy[3], Klaus Kopka[4], Christos Apostolidis[2], Uwe Haberkorn[1], and Alfred Morgenstern[2]

[1]Department of Nuclear Medicine, University Hospital Heidelberg, Heidelberg, Germany; [2]European Commission, Joint Research Centre, Institute for Transuranium Elements, Karlsruhe, Germany; [3]Department of Nuclear Medicine, RWTH University Hospital Aachen, Aachen, Germany; and [4]Division of Radiopharmaceutical Chemistry, German Cancer Research Center, Heidelberg, Germany

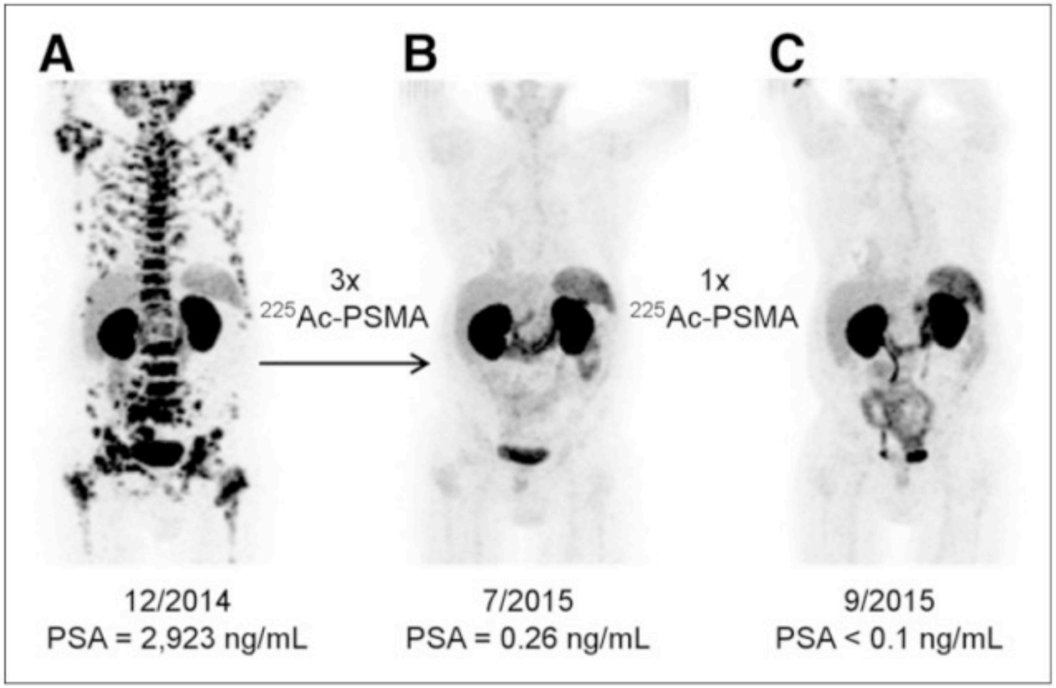


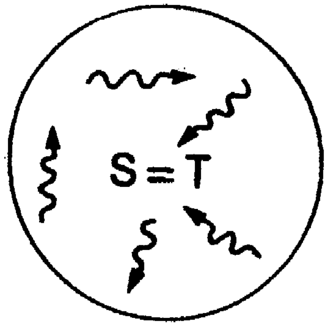
FIGURE 1. ^{68}Ga-PSMA-11 PET/CT scans of patient A. Pretherapeutic tumor spread (A), restaging 2 mo after third cycle of ^{225}Ac-PSMA-617 (B), and restaging 2 mo after one additional consolidation therapy (C).

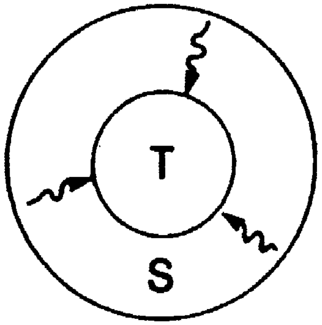
Partial Absorption of Gamma-Ray Energy – MIRD Method

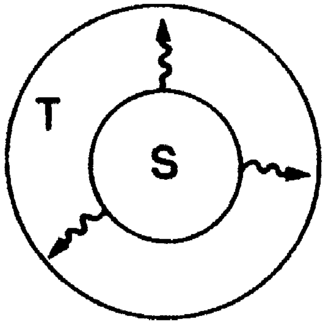
- ☞ To account for the partial absorption of gamma-ray energy in organs and tissues, the Medical Internal Radiation Dose (MIRD) Committee of the Society of Nuclear Medicine (SNM) has developed a formal system for calculating the dose to a target organ or tissue from a source organ containing a uniformly distributed radioisotope.
- ☞ The absorption fraction – the fraction of the energy radiated by the source organ, which is absorbed by the target organ.

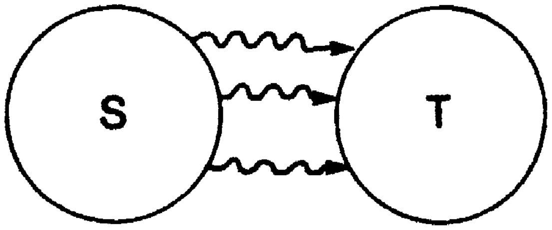
Partial Absorption of Gamma-Ray Energy – MIRD Method

- The absorption fraction – the fraction of the energy radiated by the source organ and absorbed by the target organ.









Partial Absorption of Gamma Ray Energy – MIRD Method

- ☞ The absorbed fraction are calculated by the application of Monte Carlo methods.

$$\text{Absorbed fraction} = \varphi = \frac{\text{energy absorbed by target}}{\text{energy emitted by source}}$$

- ☞ Standard data on the absorbed dose for photons of various energies for point isotropic sources and for uniformly distributed sources are published by MIRD in several Supplements to the Journal of Nuclear Medicine

TABLE 6-8. Absorbed Fractions (and Coefficients of Variation), Gamma Emitter Uniformly Distributed Throughout the Body[a] (Continued)

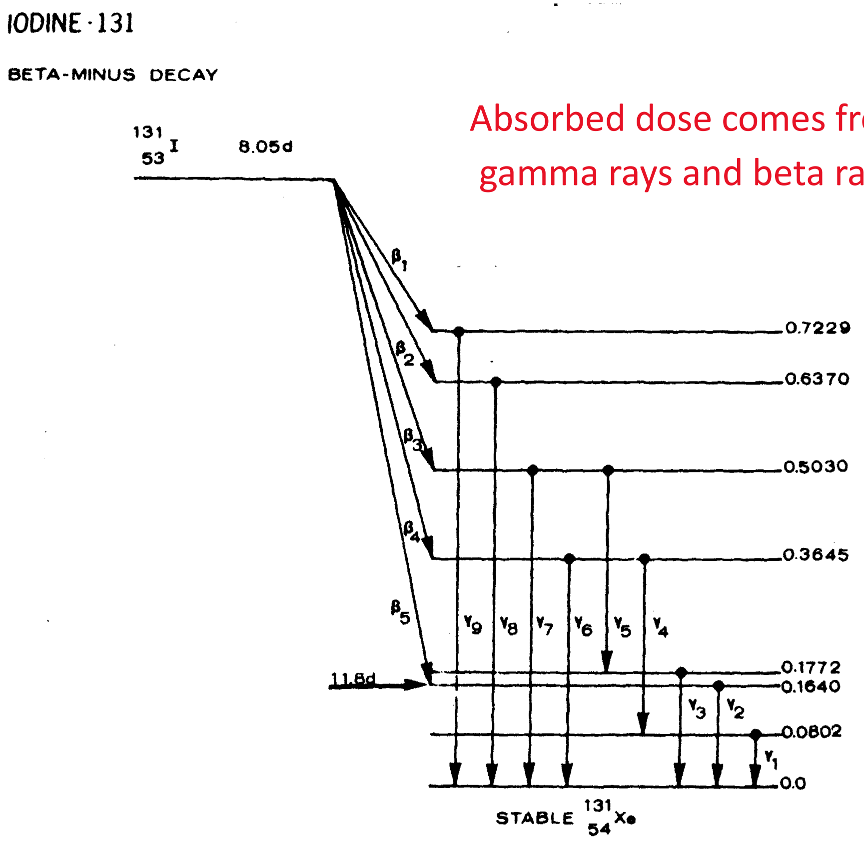
TARGET ORGAN	PHOTON ENERGY (MeV) 0.200 φ	$\frac{100\sigma_\varphi}{\varphi}$	0.500 φ	$\frac{100\sigma_\varphi}{\varphi}$	1.000 φ	$\frac{100\sigma_\varphi}{\varphi}$	1.500 φ	$\frac{100\sigma_\varphi}{\varphi}$	2.000 φ	$\frac{100\sigma_\varphi}{\varphi}$	4.000 φ	$\frac{100\sigma_\varphi}{\varphi}$	Target Organ
Adrenals	0.352E−04	36.	0.138E−03	35.	0.100E−03	42.	0.107E−03	43.	0.114E−03	43.			Adrenals
Bladder	0.327E−02	5.0	0.341E−02	6.6	0.274E−02	8.3	0.291E−02	8.4	0.231E−02	9.6	0.147E−02	12.	Bladder
GI (stom)	0.218E−02	7.0	0.258E−02	7.7	0.181E−02	9.8	0.199E−02	10.	0.212E−02	10.	0.119E−02	14.	GI (stom)
GI (SI)	0.106−01	3.4	0.114E−01	3.8	0.109E−01	4.2	0.915E−02	4.8	0.820E−02	5.2	0.409E−02	7.3	GI (SI)
GI (ULI)	0.256E−02	6.3	0.306−02	7.0	0.228E−02	8.9	0.209E−02	9.4	0.197E−02	10.	0.160E−02	12.	GI (ULI)
GI (LLI)	0.151E−02	7.6	0.184E−02	8.8	0.178E−02	9.7	0.181E−02	11.	0.157E−02	12.	0.673E−03	18.	GI (LLI)
Heart	0.337E−02	5.8	0.372E−02	6.6	0.301E−02	8.1	0.345E−02	7.8	0.312E−02	8.3	0.145−02	13.	Heart
Kidneys	0.171E−02	7.4	0.142E−02	9.7	0.161E−02	10.	0.152E−0.2	11.	0.154E−02	12.	0.904E−03	16.	Kidneys
Liver	0.111−01	3.4	0.101E−01	4.1	0.896E−02	4.7	0.912E−02	4.9	0.847E−02	5.1	0.560E−02	6.4	Liver
Lungs	0.507E−02	4.3	0.496E−02	5.2	0.466E-02	6.1	0.466E−02	6.5	0.427E−02	6.9	0.568E−02	6.4	Lungs
Marrow	0.221E−01	1.5	0.194E−01	1.0	0.182E-01	2.0	0.164E−01	2.2	0.156E−01	2.3	0.969E−02	3.0	Marrow
Pancreas	0.444E−03	14.	0.382E−03	17.	0.534E−03	19.	0.348E−03	22.	0.358E−03	24.	0.142−03	39.	Pancreas
Sk. (rib)	0.505E−02	4.1	0.435E−02	5.6	0.421E−02	6.3	0.405E−02	7.0	0.350E−02	7.7	0.338E−02	8.0	Sk. (rib)
Sk. (pelvis)	0.668E−02	3.9	0.569E−02	5.0	0.562E−02	5.7	0.511E−02	6.3	0.422E−02	7.0	0.256E−02	9.3	Sk. (pelvis)
Sk. (spine)	0.910E−02	3.6	0.763E−02	4.5	0.751E−02	5.1	0.610E−02	5.7	0.606E−02	5.9	0.341E−02	8.1	Sk. (spine)
Sk. (skull)	0.277E−02	6.3	0.304E−02	7.2	0.280E−02	8.0	0.254E−02	9.0	0.292E−02	8.8	0.224E−02	10.	Sk. (skull)
Skeleton (total)	0.550E−01	1.4	0.488E−01	1.7	0.456E−01	2.0	0.413E−01	2.2	0.396E−01	2.3	0.252E−01	3.0	Skeleton (total)
Skin	0.677E−02	3.5	0.757E−02	4.2	0.745E−02	4.8	0.759E−02	5.0	0.664E−02	5.5	0.123E−01	4.3	Skin
Spleen	0.798E−03	11.	0.116E−02	11.	0.914E−03	14.	0.903E−03	16.	0.740E−03	17.	0.368E−03	24.	Spleen
Thyroid	0.418E−04	42.							0.810E−04	46.			Thyroid
Uterus	0.408E−03	15.	0.473E−03	16.	0.517E−03	18.	0.323E−03	23.	0.364E−03	25.	0.238−03	33.	Uterus
Trunk	0.223	0.81	0.225	0.84	0.210	0.92	0.198	0.99	0.186	1.0	0.156	1.2	Trunk
Legs	0.102	1.3	0.101	1.4	0.965E−01	1.5	0.917E−01	1.6	0.846E−01	1.6	0.710E−01	1.8	Legs
Head	0.134E−01	3.2	0.147E−01	3.5	0.145E−01	3.8	0.130E−01	4.1	0.139E−01	4.1	0.127E−01	4.4	Head
Total body	0.338	0.57	0.340	0.60	0.321	0.67	0.302	0.73	0.284	0.77	0.240	0.90	Total body

[a] The digits following the symbol E indicate the powers of 10 by which each number is to be multiplied; A blank in the table indicates that the coefficient of variation was greater than 50%; Total body = head + trunk + legs.

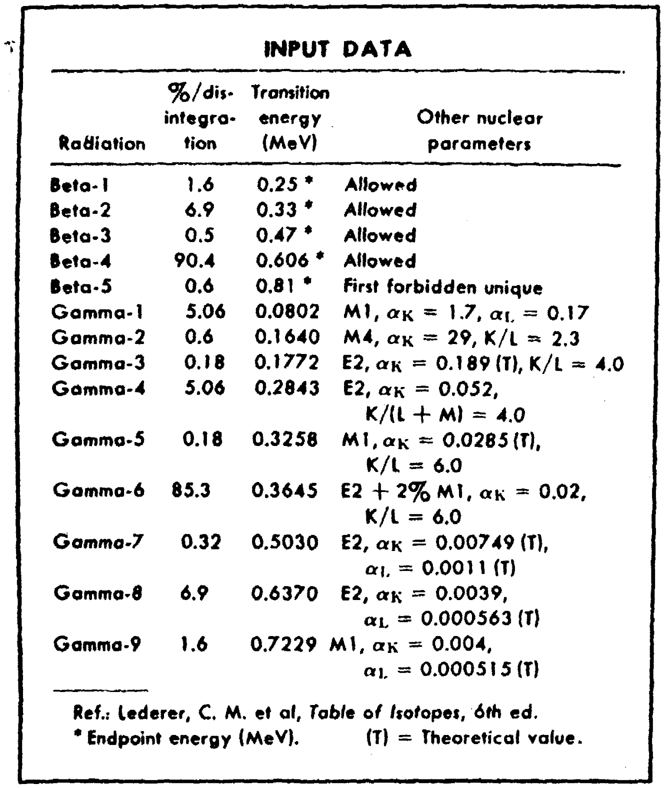
Reprinted by permission of the Society of Nuclear Medicine from Snyder WS, Ford MR, Warner GG, Fisher HL Jr. MIRD Pamphlet No. 5: Estimates of Absorbed Fractions for Monoenergetic Photon Sources Uniformly Distributed in Various Organs of a Heterogeneous Phantom. *J Nucl Med.* August 1969; 10(3 Suppl).

MIRD Method – An Example (Cember, p203-206)

Evaluate the dose rate to a 0.6kg sphere made of tissue-equivalent material in which 1MBq of I-131 is uniformly distributed.



Absorbed dose comes from gamma rays and beta rays



INPUT DATA

Radiation	%/disintegration	Transition energy (MeV)	Other nuclear parameters
Beta-1	1.6	0.25 *	Allowed
Beta-2	6.9	0.33 *	Allowed
Beta-3	0.5	0.47 *	Allowed
Beta-4	90.4	0.606 *	Allowed
Beta-5	0.6	0.81 *	First forbidden unique
Gamma-1	5.06	0.0802	M1, α_K = 1.7, α_L = 0.17
Gamma-2	0.6	0.1640	M4, α_K = 29, K/L = 2.3
Gamma-3	0.18	0.1772	E2, α_K = 0.189 (T), K/L = 4.0
Gamma-4	5.06	0.2843	E2, α_K = 0.052, K/(L + M) = 4.0
Gamma-5	0.18	0.3258	M1, α_K = 0.0285 (T), K/L = 6.0
Gamma-6	85.3	0.3645	E2 + 2% M1, α_K = 0.02, K/L = 6.0
Gamma-7	0.32	0.5030	E2, α_K = 0.00749 (T), α_L = 0.0011 (T)
Gamma-8	6.9	0.6370	E2, α_K = 0.0039, α_L = 0.000563 (T)
Gamma-9	1.6	0.7229	M1, α_K = 0.004, α_L = 0.000515 (T)

Ref.: Lederer, C. M. et al, *Table of Isotopes*, 6th ed.
* Endpoint energy (MeV). (T) = Theoretical value.

FIGURE 6.11. Transformation scheme and input and output data for ^{131}I dosimetry. (From L.T. Dillman: Radionuclide Decay Schemes and Nuclear Parameters for Use in Radiation Dose Estimation. *J. Nuclear Medicine*, Vol. 10, Supplement No. 2, MIRD Pamphlet No. 4, 1969. By permission.)

MIRD Method – An Example

- The absorbed dose is the sum of beta dose and gamma ray dose.
- The absorbed gamma-ray energy per I-131 transformation is given by

$$E_e(\gamma) = \sum_i E_{\gamma i} \times n_i \times \varphi_i ,$$

what is absorption fraction?

where $E_e(\gamma)$ = absorbed gamma ray energy, MeV/transformation,
$E_{\gamma i}$ = energy of the ith gamma photon, MeV,
n_i = number of photons of ith energy per transformation,
φ_i = absorbed fraction of the ith photon's energy.

Photon energy, $E_{\gamma i}$, MeV	×	Photons per transformation, n_i	×	Absorbed fraction, φ	=	Absorbed energy, MeV/t
0.723		0.016		0.123		0.0014
0.637		0.069		0.124		0.0055
0.503		0.003		0.123		0.0002
0.326		0.002		0.120		0.0001
0.177		0.002		0.112		0.0000
0.365		0.853		0.122		0.0380
0.284		0.051		0.118		0.0017
0.080		0.051		0.111		0.0005
0.164		0.006		0.111		0.0001

$E_e(\gamma) = 0.0474$ MeV/t

MIRD Method – An Example

- Since the absorption fraction is 1 for internally distributed beta sources, the absorbed beta energy per I-131 transformation is given by

$$
\begin{aligned}
E_e(\beta) &= \sum \overline{E}_{\beta i} \times n_{\beta i} \\
&= (0.0701 \times 0.016) + (0.0955 \times 0.069) + (0.1428 \times 0.005) \\
&\quad + (0.1917 \times 0.904) + (0.285 \times 0.006) = 0.183 \text{ MeV/t}.
\end{aligned}
$$

- So the total energy absorbed by the 0.6kg tissue-equivalent sphere per I-131 transformation is

$$
\begin{aligned}
E_e &= E_e(\gamma) + E_e(\beta) \\
&= 0.047 + 0.183 = 0.230 \text{ MeV/t}.
\end{aligned}
$$

MIRD Method – An Example

☞ Therefore the daily dose rate to the 0.6kg tissue-equivalent mass is

$$\dot{D} = \frac{q \text{ Bq} \times 1 \text{ tps/Bq} \times E_e \text{ MeV/t} \times 1.6 \times 10^{-13} \text{ J/MeV} \times 8.64 \times 10^4 \text{ s/day}}{m \text{ kg} \times 1 \frac{\text{J}}{\text{kg}} \Big/ \text{Gy}}.$$

If we substitute $q = 1 \times 10^6$ Bq,
$E_e = 0.230$ MeV/t,
$m = 0.6$ kg

we find the dose rate to be

$$\dot{D} = 5.30 \times 10^{-3} \text{ Gy/day}$$

Internally Deposited Radioisotope (II)
Effective Half-Life

- The total dose absorbed by an organ during any given time interval after the deposition of the isotope in the organ may be calculated by integrating the dose rate over the required time interval. For this purpose, two factors must be considered:

  In situ radioactive decay of the isotope → exponential decay

  Biological elimination of the isotope → follows the first-order kinetics → exponential decay

- The equation for the quantity of radioisotope within an organ at any given time after the deposition of a quantity Q_0 is given by

$$Q = (Q_0 e^{-\lambda_R t})(e^{-\lambda_B t})$$

where λ_R is the radioactive decay constant and λ_B is the biological elimination constant.

Internally Deposited Radioisotope (II)
Effective Half-Life

- ☞ One can define an effective elimination constant $\lambda_E = \lambda_R + \lambda_B$ that represents the combined effects of these two decay processes,

$$Q = Q_0 e^{-\lambda_E t}$$

and

$$T_E = \frac{0.693}{\lambda_E}$$

is called the effective half-life.

Internally Deposited Radioisotope (III)
Accumulated Dose and Dose Commitment

- ☞ Given the initial dose rate $\dot{D}_0$, the accumulated dose received during a time interval t after the deposition of the isotope is

$$D = \dot{D}_0 \int_0^t e^{-\lambda_E t} dt = \frac{\dot{D}_0}{\lambda_E}(1 - e^{-\lambda_E t})$$

For an infinitely long time—that is, when the isotope is completely gone-

$$D = \frac{\dot{D}_0}{\lambda_E}$$

- ☞ For practical purpose, an infinitely long time corresponding to about 6 half-lives. The total dose received from complete decay is called the dose commitment.

Internally Deposited Radioisotope (III)
Total Dose: Dose Commitment

Generally, if there is more than one compartment, the body burden at any time t after deposition of q_0 units of a radionuclide is given by

$$q(t) = f_1 q_0 \mathrm{e}^{-\lambda_1 t} + f_2 q_0 \mathrm{e}^{-\lambda_2 t} + \cdots + f_n q_0 \mathrm{e}^{-\lambda_n t}, \tag{6.60}$$

where $f_1, f_2, \ldots, f_n$ = fraction of the total activity deposited in compartments 1, 2, . . . , n, and $\lambda_1, \lambda_2, \ldots, \lambda_n$ = effective clearance rates for compartments 1, 2, . . . , n.

Since the activity in each compartment contributes to the dose to that organ or tissue, Eq. (6.57) becomes, for the multicompartment case,

$$D = \frac{\dot{D}_{10}}{\lambda_{1\mathrm{E}}}\left(1 - e^{-\lambda_{1\mathrm{E}} t}\right) + \frac{\dot{D}_{20}}{\lambda_{2\mathrm{E}}}\left(1 - e^{-\lambda_{2\mathrm{E}} t}\right) + \cdots + \frac{\dot{D}_{n0}}{\lambda_{n\mathrm{E}}}\left(1 - e^{-\lambda_{n\mathrm{E}} t}\right), \tag{6.61}$$

and when the radionuclide has completely been eliminated, Eq. (6.61) reduces to

$$D(t) = \frac{\dot{D}_{10}}{\lambda_{1\mathrm{E}}} + \frac{\dot{D}_{20}}{\lambda_{2\mathrm{E}}} + \cdots + \frac{\dot{D}_{n0}}{\lambda_{n\mathrm{E}}}. \tag{6.62}$$

Overall dose commitment

Internally Deposited Radioisotope (III)
Total Dose: Dose Commitment

first-order kinetics and is emptied at its own clearance rate. Thus, for example, cesium is found to be uniformly distributed throughout the body, although the body behaves as if the cesium were stored in two compartments. One compartment contains 10% of the total body burden and has a retention half-time of 2 days, while the second compartment contains the other 90% of the body's cesium content and has a clearance half-time of 110 days. The retention curve for cesium, therefore, is given by the equation

$$q(\mathrm{t}) = 0.1\, q_0\, \mathrm{e}^{-(0.693\ \mathrm{t}/2\,\mathrm{days})} + 0.9\, q_0\, \mathrm{e}^{-(0.693\ \mathrm{t}/110\,\mathrm{days})}, \qquad \mathbf{(6.59)}$$

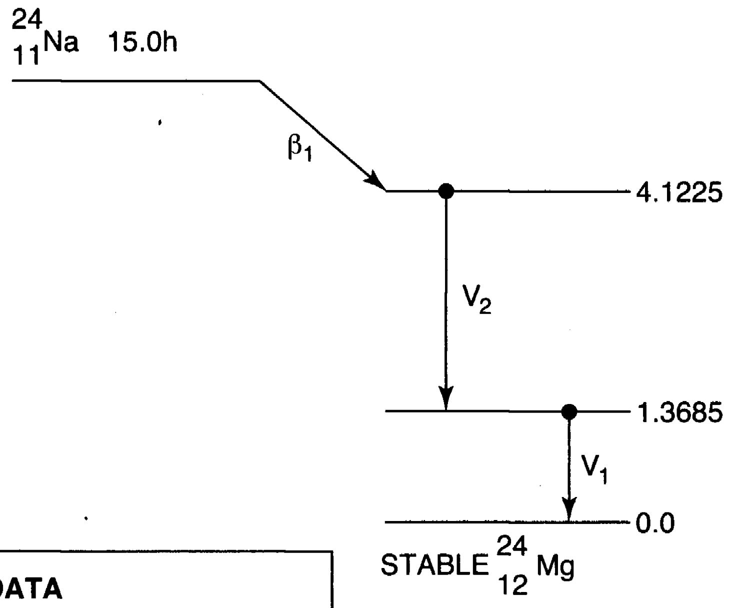
where $q(t)$ is the body burden at time t after deposition of q_0 amount of cesium in the body. Ten percent of the total is deposited in compartment 1 and 90% is deposited in compartment 2.

MIRD Method – General Treatment of Internal Dose

An Example

Calculate the total dose and initial dose rate to a 70-kg, 160-cm-tall reference man who is intraveneously injected with 1-MBq ^{24}NaCl. Assume the ^{24}NaCl to become uniformly distributed within a very short time and to have a biological half-life of 11 days (264 hours).

SODIUM-24 BETA-MINUS DECAY



INPUT DATA

Radiation	%/disintegration	Transition energy (MeV)	Other nuclear parameters
Beta-1	99.9	1.392*	Allowed
All other betas	<0.1	—	—
Gamma-1	100.	1.3685	E2, αx < 0.00001 (T)
Gamma-2	99.9	2.7539	E2, αx < 0.00001 (T)
All other gammas	<0.1	—	—

Ref. Lederer, C. M. et al, *Table of isotopes*, 6th ed.

*Endpoint energy (MeV). (T) = Theoretical value.

OUTPUT DATA

Radiation [1]	Mean number/disintegration (n_i)	Mean energy (MeV) [$\bar{E}_i$]	$\Delta i \left(\frac{\text{g-rad}}{\mu\text{Ci-h}}\right)$
Beta-1	0.999	0.5547	1.1803
Gamma-1	0.999	1.3685	2.9149
Gamma-2	0.999	2.7539	5.8599

MIRD Method – General Treatment of Internal Dose

An Example

absorption fraction

amount of E absorbed per decay

RADIATION	E_i (MeV)	φ_i	Δ_i, $\frac{\text{kg-fGy}}{\text{Bq-s}}$	$\varphi_i \Delta_i$
Beta 1	0.555	1.000	88.64	88.64
Gamma 1	1.369	0.31	218.91	67.86
Gamma 2	2.754	0.265	440.08	116.62
				$\sum = 273.12(\text{kg} \cdot \text{fGy})/(\text{Bq} \cdot \text{s})$

amount of E emitted per decay

The decay scheme and the accompanying table of input data show one beta (actually >0.999) particle whose maximum energy is 1.392 MeV, and one 1.3685-MeV gamma per decay. The output data list the integral dose in an infinite medium, per unit of cumulated activity, in units of $\frac{\text{g} \cdot \text{rads}}{\mu\text{Ci} \cdot \text{h}}$ for each radiation. To convert from the old system of units found in the MIRD publications to the SI system, that is, to go from $\frac{\text{g} \cdot \text{rads}}{\mu\text{Ci} \cdot \text{h}}$ to $\frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}}$, we use the following relation:

$$\frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}} = \frac{\text{g} \cdot \text{rad}}{\mu\text{Ci} \cdot \text{h}} \times \frac{10^{-3}\ \frac{\text{kg}}{\text{g}} \times 10^{-2}\ \frac{\text{Gy}}{\text{rad}}}{3.7 \times 10^{4}\ \frac{\text{Bq}}{\mu\text{Ci}} \times 3.6 \times 10^{3}\ \frac{\text{s}}{\text{h}}}$$

$$\frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}} = \frac{\text{g} \cdot \text{rad}}{\mu\text{Ci} \cdot \text{h}} \times 75.1 \times 10^{-14}. \qquad \textbf{(6.87)}$$

To convert from SI units to traditional units:

$$\frac{\text{g} \cdot \text{rad}}{\mu\text{Ci} \cdot \text{h}} = \frac{1}{7.51 \times 10^{-14}} \times \frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}} = 1.33 \times 10^{13} \times \frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}}. \qquad \textbf{(6.88)}$$

Now let us return to the problem. Since the ^{24}Na is cleared exponentially at an effective rate λ_E, the amount of activity in the source organ is given by

$$A_s(t) = A_s(0) \times e^{-\lambda_E t}, \tag{6.89}$$

where $A_s(0)$ is the initial activity in the source.

$$\tilde{A} = \int_0^\infty A_s(t)\,dt = A_s(0) \int_0^\infty e^{-\lambda_E t} dt = \frac{A_s(0)}{\lambda_E}. \tag{6.90}$$

$\tilde{A}$, organ burden: total number of decays that would happen in the organ.

Since

$$\lambda_E = \frac{0.693}{T_E} = \frac{0.693}{(T_R \times T_B)/(T_R + T_B)}$$

the biological half-life T_B is found in International Commission on Radiological Protection (ICRP) Publication 2 to be 264 hours, and the radioactive half-life T_R is 15 hours, therefore

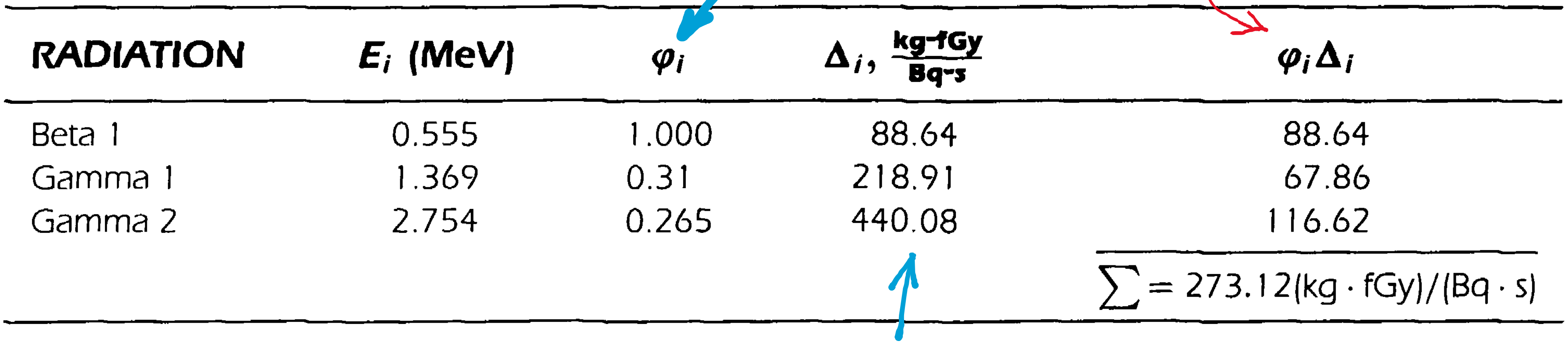
$$\tilde{A} = \frac{10^6\ \text{Bq}}{1.36 \times 10^{-5}\ \text{s}^{-1}} = 7.35 \times 10^{10}\ \text{Bq} \cdot \text{s}.$$

MIRD Method – General Treatment of Internal Dose

An Example

absorption fraction

amount of E absorbed per decay



RADIATION	E_i (MeV)	φ_i	Δ_i, $\frac{kg\cdot fGy}{Bq\cdot s}$	$\varphi_i \Delta_i$
Beta 1	0.555	1.000	88.64	88.64
Gamma 1	1.369	0.31	218.91	67.86
Gamma 2	2.754	0.265	440.08	116.62
				$\sum = 273.12(kg \cdot fGy)/(Bq \cdot s)$

amount of E emitted per decay

TABLE 6-8. Absorbed Fractions (and Coefficients of Variation), Gamma Emitter Uniformly Distributed Throughout the Body[a]

	PHOTON ENERGY (MeV)												
	0.010		**0.015**		**0.020**		**0.030**		**0.050**		**0.100**		
TARGET ORGAN	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	**Target Organ**
Adrenals	0.270E−03	35.	0.228E−03	34.	0.175E−03	37.	0.209E−03	28.	0.131E−03	23.	0.101E−03	26.	Adrenals
Bladder	0.757E−02	6.6	0.762E−02	6.5	0.683E−02	6.6	0.625E−02	6.1	0.445E−02	5.6	0.352E-02	5.2	Bladder
GI (stom)	0.570E−02	7.6	0.507E−02	8.0	0.573E−02	7.1	0.560E−02	6.4	0.391E−02	5.8	0.273E−02	5.9	GI (stom)
GI (SI)	0.254E−01	3.6	0.236E−01	3.7	0.234E−01	3.6	0.209E−01	3.4	0.163E−01	3.1	0.120E−01	3.2	GI (SI)
GI (ULI)	0.541E−02	7.8	0.561E−02	7.5	0.647E−02	6.6	0.533E−02	5.9	0.374E−02	5.4	0.262E−02	5.7	GI (ULI)
GI (LLI)	0.350E−02	9.7	0.441E−02	8.5	0.457E−02	7.7	0.285E−02	7.9	0.256E−02	6.2	0.187E−02	6.3	GI (LLI)
Heart	0.756E−02	6.6	0.804E−02	6.3	0.769E-02	6.2	0.635E−02	6.0	0.469E−02	5.4	0.420E−02	5.0	Heart
Kidneys	0.410E−02	9.0	0.446E−02	8.5	0.412E-02	8.3	0.338E−02	7.4	0.233E−02	6.4	0.183E−02	6.6	Kidneys
Liver	0.260E−01	3.5	0.244E−01	3.6	0.249E-01	3.5	0.221E−01	3.3	0.154E−01	3.2	0.120E-01	3.2	Liver
Lungs	0.127E−01	5.1	0.142E−01	4.7	0.138E−01	4.4	0.122E−01	3.8	0.808E−02	3.4	0.551E−02	3.6	Lungs
Marrow	0.560E−01	1.4	0.594E−01	1.4	0.655E−01	1.3	0.740E−01	1.1	0.613E−01	1.1	0.329E−01	1.3	Marrow
Pancreas	0.134E−02	16.	0.103E−02	18.	0.828E−03	17.	0.780E−03	14.	0.567E−03	12.	0.449E−03	12.	Pancreas
Sk. (rib)	0.168E−01	4.4	0.206E−01	3.9	0.247E−01	3.4	0.263E−01	2.9	0.176E−01	2.9	0.764E−02	3.3	Sk. (rib)
Sk. (pelvis)	0.147E−01	4.7	0.160E−01	4.51	0.163E−01	4.3	0.224E−01	3.4	0.199E−01	3.0	0.103E−01	3.3	Sk. (pelvis)
Sk. (spine)	0.186E−01	4.2	0.190E−01	4.1	0.234E−01	3.7	0.253E−01	3.3	0.229E−01	3.0	0.144E−01	3.2	Sk. (spine)
Sk. (skull)	0.103E−01	5.6	0.115E−01	5.3	0.123E−01	5.	0.128E−01	4.6	0.722E−02	5.1	0.313E−02	6.0	Sk. (skull)
Skeleton (total)	0.144	1.4	0.153	1.3	0.167	1.2	0.188	1.1	0.153	1.1	0.810E−01	1.3	Skeleton (total)
Skin	0.258E−01	3.5	0.227E−01	3.5	0.169E−01	3.7	0.116E−01	3.3	0.758E−02	2.9	0.585E−02	3.1	Skin
Spleen	0.260E−02	11.	0.237E−02	12.	0.242E−02	11.	0.223E−02	9.1	0.149E−02	8.5	0.111E−02	8.7	Spleen
Thyroid	0.265E−03	35.	0.263E−03	34.	0.602E−04	48.	0.111E−03	36.	0.114E−03	27.	0.873E−04	29.	Thyroid
Uterus	0.999E−03	18.	0.109E−02	17.	0.122E−02	15.	0.924E−03	13.	0.712E−03	12.	0.611E−03	11.	Uterus
Trunk	0.604	0.47	0.589	0.48	0.566	0.50	0.500	0.55	0.358	0.67	0.245	0.79	Trunk
Legs	0.309	0.86	0.299	0.88	0.285	0.90	0.242	0.97	0.171	1.1	0.113	1.3	Legs
Head	0.488E−01	2.5	0.474E−01	2.5	0.440E−01	2.6	0.342E−01	2.7	0.200E−01	3.1	0.127E−01	3.1	Head
Total body	0.959	0.11	0.933	0.15	0.892	0.19	0.774	0.27	0.548	0.43	0.370	0.56	Total body

(Continued)

TABLE 6-8. Absorbed Fractions (and Coefficients of Variation), Gamma Emitter Uniformly Distributed Throughout the Body[a] (Continued)

	PHOTON ENERGY (MeV)												
	0.200		0.500		1.000		1.500		2.000		4.000		
TARGET ORGAN	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	φ	$\frac{100\sigma_\varphi}{\varphi}$	**Target Organ**
Adrenals	0.352E−04	36.	0.138E−03	35.	0.100E−03	42.	0.107E−03	43.	0.114E−03	43.			Adrenals
Bladder	0.327E−02	5.0	0.341E−02	6.6	0.274E−02	8.3	0.291E−02	8.4	0.231E−02	9.6	0.147E−02	12.	Bladder
GI (stom)	0.218E−02	7.0	0.258E−02	7.7	0.181E−02	9.8	0.199E−02	10.	0.212E−02	10.	0.119E−02	14.	GI (stom)
GI (SI)	0.106−01	3.4	0.114E−01	3.8	0.109E−01	4.2	0.915E−02	4.8	0.820E−02	5.2	0.409E−02	7.3	GI (SI)
GI (ULI)	0.256E−02	6.3	0.306−02	7.0	0.228E−02	8.9	0.209E−02	9.4	0.197E−02	10.	0.160E−02	12.	GI (ULI)
GI (LLI)	0.151E−02	7.6	0.184E−02	8.8	0.178E−02	9.7	0.181E−02	11.	0.157E−02	12.	0.673E−03	18.	GI (LLI)
Heart	0.337E−02	5.8	0.372E−02	6.6	0.301E−02	8.1	0.345E−02	7.8	0.312E−02	8.3	0.145−02	13.	Heart
Kidneys	0.171E−02	7.4	0.142E−02	9.7	0.161E−02	10.	0.152E−0.2	11.	0.154E−02	12.	0.904E−03	16.	Kidneys
Liver	0.111−01	3.4	0.101E−01	4.1	0.896E−02	4.7	0.912E−02	4.9	0.847E−02	5.1	0.560E−02	6.4	Liver
Lungs	0.507E−02	4.3	0.496E−02	5.2	0.466E-02	6.1	0.466E−02	6.5	0.427E−02	6.9	0.568E−02	6.4	Lungs
Marrow	0.221E−01	1.5	0.194E−01	1.0	0.182E-01	2.0	0.164E−01	2.2	0.156E−01	2.3	0.969E−02	3.0	Marrow
Pancreas	0.444E−03	14.	0.382E−03	17.	0.534E−03	19.	0.348E−03	22.	0.358E−03	24.	0.142−03	39.	Pancreas
Sk. (rib)	0.505E−02	4.1	0.435E−02	5.6	0.421E−02	6.3	0.405E−02	7.0	0.350E−02	7.7	0.338E−02	8.0	Sk. (rib)
Sk. (pelvis)	0.668E−02	3.9	0.569E−02	5.0	0.562E−02	5.7	0.511E−02	6.3	0.422E−02	7.0	0.256E−02	9.3	Sk. (pelvis)
Sk. (spine)	0.910E−02	3.6	0.763E−02	4.5	0.751E−02	5.1	0.610E−02	5.7	0.606E−02	5.9	0.341E−02	8.1	Sk. (spine)
Sk. (skull)	0.277E−02	6.3	0.304E−02	7.2	0.280E−02	8.0	0.254E−02	9.0	0.292E−02	8.8	0.224E−02	10.	Sk. (skull)
Skeleton (total)	0.550E−01	1.4	0.488E−01	1.7	0.456E−01	2.0	0.413E−01	2.2	0.396E−01	2.3	0.252E−01	3.0	Skeleton (total)
Skin	0.677E−02	3.5	0.757E−02	4.2	0.745E−02	4.8	0.759E−02	5.0	0.664E−02	5.5	0.123E−01	4.3	Skin
Spleen	0.798E−03	11.	0.116E−02	11.	0.914E−03	14.	0.903E−03	16.	0.740E−03	17.	0.368E−03	24.	Spleen
Thyroid	0.418E−04	42.							0.810E−04	46.			Thyroid
Uterus	0.408E−03	15.	0.473E−03	16.	0.517E−03	18.	0.323E−03	23.	0.364E−03	25.	0.238−03	33.	Uterus
Trunk	0.223	0.81	0.225	0.84	0.210	0.92	0.198	0.99	0.186	1.0	0.156	1.2	Trunk
Legs	0.102	1.3	0.101	1.4	0.965E−01	1.5	0.917E−01	1.6	0.846E−01	1.6	0.710E−01	1.8	Legs
Head	0.134E−01	3.2	0.147E−01	3.5	0.145E−01	3.8	0.130E−01	4.1	0.139E−01	4.1	0.127E−01	4.4	Head
Total body	0.338	0.57	0.340	0.60	0.321	0.67	0.302	0.73	0.284	0.77	0.240	0.90	Total body

[a] The digits following the symbol E indicate the powers of 10 by which each number is to be multiplied; A blank in the table indicates that the coefficient of variation was greater than 50%; Total body = head + trunk + legs.

Reprinted by permission of the Society of Nuclear Medicine from Snyder WS, Ford MR, Warner GG, Fisher HL Jr. MIRD Pamphlet No. 5: Estimates of Absorbed Fractions for Monoenergetic Photon Sources Uniformly Distributed in Various Organs of a Heterogeneous Phantom. *J Nucl Med.* August 1969; 10(3 Suppl).

Substituting the values (and 10^{15} femto Gy(fGy) per Gy):

$$\tilde{A} = 7.35 \times 10^{10}\ \mathrm{Bq \cdot s}, \quad \sum \varphi_i \Delta_i = 273.12 \frac{\mathrm{kg \cdot fGy}}{\mathrm{Bq \cdot s}}, \quad \text{and } m = 70\ \mathrm{kg}$$

into Eq. (6.86) yields

$$\boxed{D = \frac{7.35 \times 10^{10}\ \mathrm{Bq \cdot s}}{70\ \mathrm{kg}} \times 273.12 \frac{\mathrm{kg \cdot fGy}}{\mathrm{Bq \cdot s}}}$$

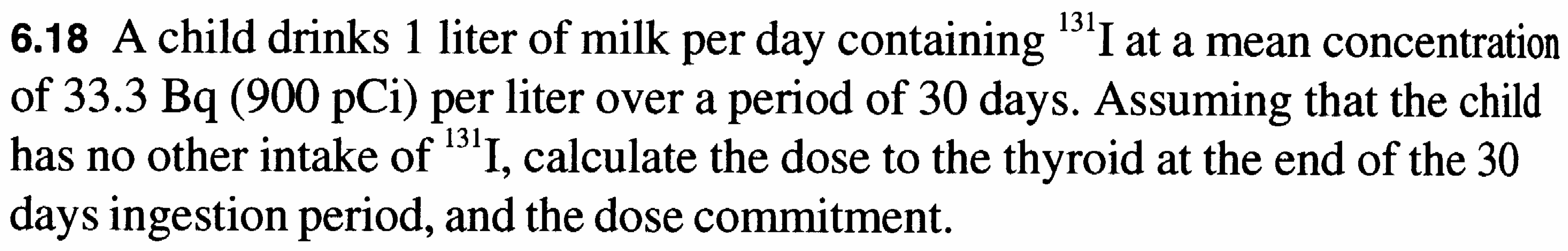
$$= 2.868 \times 10^{11}\ \mathrm{fGy} \qquad (29\ \mathrm{mrads}).$$

The total dose that the organ will be receiving from the internally administrated radioactivity

The initial dose rate may be found by substituting 10^6 Bq for A_s in Eq. (6.83):

$$\boxed{\dot{D} = \frac{10^6\ \mathrm{Bq}}{70\ \mathrm{kg}} \times 273.12 \frac{\mathrm{kg \cdot fGy}}{\mathrm{Bq \cdot s}}}$$

$$= 3.9 \times 10^6\ \mathrm{fGy/s} \quad (1.4\ \mathrm{mrads/h}).$$



6.18 A child drinks 1 liter of milk per day containing ^{131}I at a mean concentration of 33.3 Bq (900 pCi) per liter over a period of 30 days. Assuming that the child has no other intake of ^{131}I, calculate the dose to the thyroid at the end of the 30 days ingestion period, and the dose commitment.

MIRD Method – Another Example, Cember, 6.18

Step 1: Derive the effective half-live of I-131

First calculate the effective half life of the I–131 in the body, use equation 6.54:

$T_R = 8.05$ d
$T_B = 138$ d (ICRP 28)

$$T_E = \frac{T_R \times T_B}{T_R + T_B} = \frac{8.05\text{ d} \times 138\text{ d}}{8.05\text{ d} + 138\text{ d}} = 7.6\text{ d effective half life of }^{131}\text{I in body.}$$

Converting to effective elimination constant, using equation 6.52:

$$\lambda = \frac{0.693}{T} = \frac{0.693}{7.6\text{ d}} = 0.091\text{ d}^{-1}$$

Note that we will use λ to symbolize the effective decay constant in the following derivations.

Step 2: Derive the absorbed dose to the thyroid per I-131 decay

The average energy of each ^{131}I beta particle is found (Figure 6.11), and the yield from each decay is also tabulated:

Energy, MeV/t	Yield, *f*
0.0701	0.016
0.0955	0.069
0.1428	0.005
0.1917	0.904
0.2856	0.006

The mean β energy/transformation is:

$$\bar{E}_e(\beta) = \Sigma \bar{E} \times f_{\beta i} = (0.0701 \times 0.016) + (0.0955 \times 0.069) + (0.1428 \times 0.005) +$$

$$0.1917 \times 0.904) + (0.2856 \times 0.006)$$

$$\bar{E}_e(\beta) = 0.184 \text{ MeV/t}$$

Beta energy absorbed in the thyroid per I-131 decay

Absorption fraction

MeV	*f*	Spec. Abs	MeV/t
0.723	0.016	0.00166	1.92E-05
0.637	0.069	0.00166	7.3E-05
0.503	0.003	0.00166	2.5E-06
0.326	0.002	0.00155	1.01E-06
0.177	0.002	0.00155	5.49E-07
0.365	0.853	0.00155	0.000483
0.284	0.051	0.00155	2.25E-05
0.08	0.051	0.0429	0.000175
0.164	0.006	0.00155	1.53E-06
		Sum	0.000778

Beta energy absorbed in the thyroid per I-131 decay

alculating the contribution due to the γ, the specific absorbed fraction is found in ppendix 4 for an adult. Assume all the ^{131}I is deposited in the thyroid. So for this ase, the contribution from the γ is not significant and can be ignored, especially nce the child's thyroid is small (~2-5g, ICRP 53).

Step 3: derive the I-131 activity in the thyroid as a function of t

The intake is $q = 33.3$ Bq/day, however, only one–third of the iodine is directly deposited in the thyroid (ICRP 30).

$$K = 33.3\frac{\text{Bq}}{\text{d}} \times \frac{1\text{ deposited}}{3\text{ ingested}} = 11.1\frac{\text{Bq}}{\text{d}}$$

← rate of intake

Some of the iodine is eliminated daily, so find the concentration in the thyroid at any time:

$$\frac{dq}{dt} = \text{deposition} - \text{disappearance}$$

$$\frac{dq}{dt} = K - \lambda \quad q$$

Separating the variables, we have

$$\int_0^q \frac{dq}{\left(K - \lambda \quad q\right)} = \int_0^t dt$$

Note that

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln(ax+b) + C,$$

and consider the following conditions:

$$\begin{cases} a = -\lambda_{eff} \\ b = K \\ q(t=0) = 0 \end{cases}$$

After integration, and solving for q as a function of t;

$$q(t) = \frac{K}{\lambda}\left(1 - e^{-\lambda t}\right) \quad \Rightarrow$$

If the uptake of I-131 continues, the dose rate as a function of time is

$$\begin{aligned}\dot{D}(t) &= \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t}\right) \\ &= \dot{D}_\infty \cdot \left(1 - \mathrm{e}^{-\lambda t}\right)\end{aligned}$$

As $t \to \infty$, q approaches

$$q_\infty = \frac{K}{\lambda} = \frac{11.1\,\frac{\text{Bq}}{\text{day}}}{0.091\ \text{day}} = 122\ \text{Bq} \quad \Rightarrow$$

If the uptake of I-131 continues, the saturation dose rate is given by

$$\dot{D}_\infty = \frac{\bar{E}}{m} \cdot \frac{K}{\lambda}$$

$m = 20$ g (for adult, from appendix C), for a child, assume 10% of adult mass (10 CFR 20), 2 g.

Step 4: Derive the accumulated dose received within the first 30 days.

If the uptake continues, the accumulated dose received by a given time t is

$$D = \int_{0}^{t} \dot{D}(t') \cdot dt'$$

$$\int_0^t (1 - e^{-\lambda t}) dt$$
$$= \left[t - \left(-\frac{1}{\lambda}\right) e^{-\lambda t}\right]\Big|_0^t$$
$$= \left[t + \frac{1}{\lambda} e^{-\lambda t}\right] - \left[0 + \frac{1}{\lambda}\right]$$
$$= \left[t + \frac{1}{\lambda}(e^{-\lambda t} - 1)\right]$$

where

$$\dot{D}(t) = \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t}\right),$$

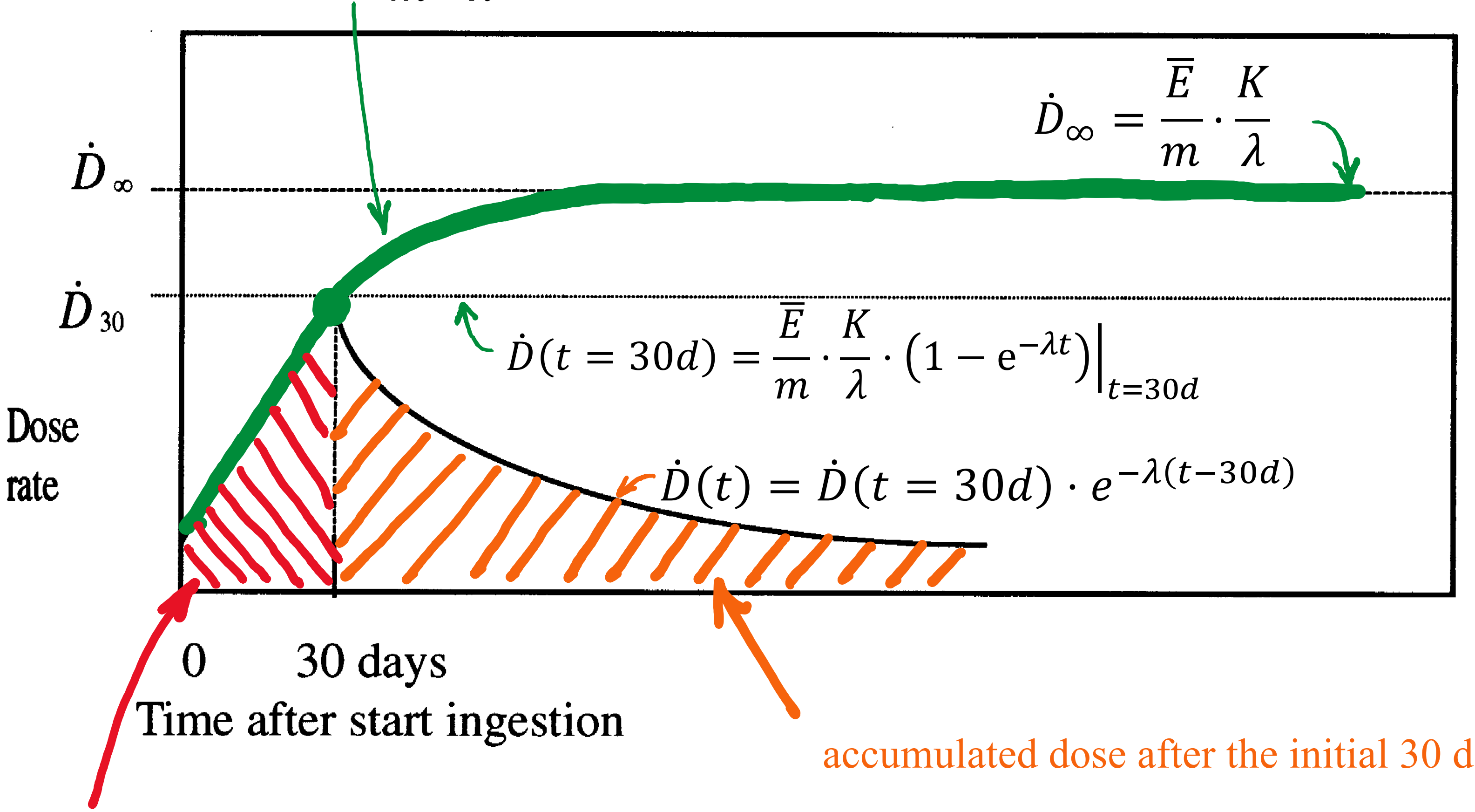
and $\bar{E}$ is the mean absorbed energy in the organ per decay of I-131 in the thyroid.

So the accumulated dose is given by

$$D = \int_{0}^{t} \dot{D}(t') \cdot dt' = \int_{0}^{t} \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t'}\right) \cdot dt'$$

$$= \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left[t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right)\right]$$

$$\dot{D}(t) = \frac{\bar{E}}{m}\cdot\frac{K}{\lambda}\cdot\left(1-e^{-\lambda t}\right)$$



$$\dot{D}_\infty = \frac{\bar{E}}{m}\cdot\frac{K}{\lambda}$$

$$\dot{D}(t=30d) = \frac{\bar{E}}{m}\cdot\frac{K}{\lambda}\cdot\left(1-\mathrm{e}^{-\lambda t}\right)\Big|_{t=30d}$$

$$\dot{D}(t) = \dot{D}(t=30d)\cdot e^{-\lambda(t-30d)}$$

accumulated dose after the initial 30 days.

accumulated dose within the first 30 days

Step 4: Derive the accumulated dose received within the first 30 days (continued)

Using the following equation for accumulated dose till time t,

$$D = \dot{D}_\infty \left[t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right) \right] \quad and \quad \dot{D}_\infty = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda},$$

the accumulated dose at 30 days is:

$$\dot{D}_\infty = 0.155 \text{ mGy/day}$$
$$\lambda = 0.091 \text{ d}^{-1}$$
$$t = 30 \text{ d}$$

$$D = 0.155 \frac{\text{mGy}}{\text{d}} \times \left[30 \text{ d} + \frac{1}{0.091 \text{d}^{-1}} \times \left(e^{-0.091 \times (30)} - 1\right) \right] = 3 \text{ mGy}$$

3 mGy is the accumulated dose at the end of the 30 day period.

Internally Deposited Radioisotope: Total Dose or Dose Commitment (Revisited)

- The total dose received during a time interval t after the deposition of the isotope is

$$D = \dot{D}_0 \int_0^t e^{-\lambda t} \mathrm{d}t = \frac{\dot{D}_0}{\lambda_E}(1 - e^{-\lambda t})$$

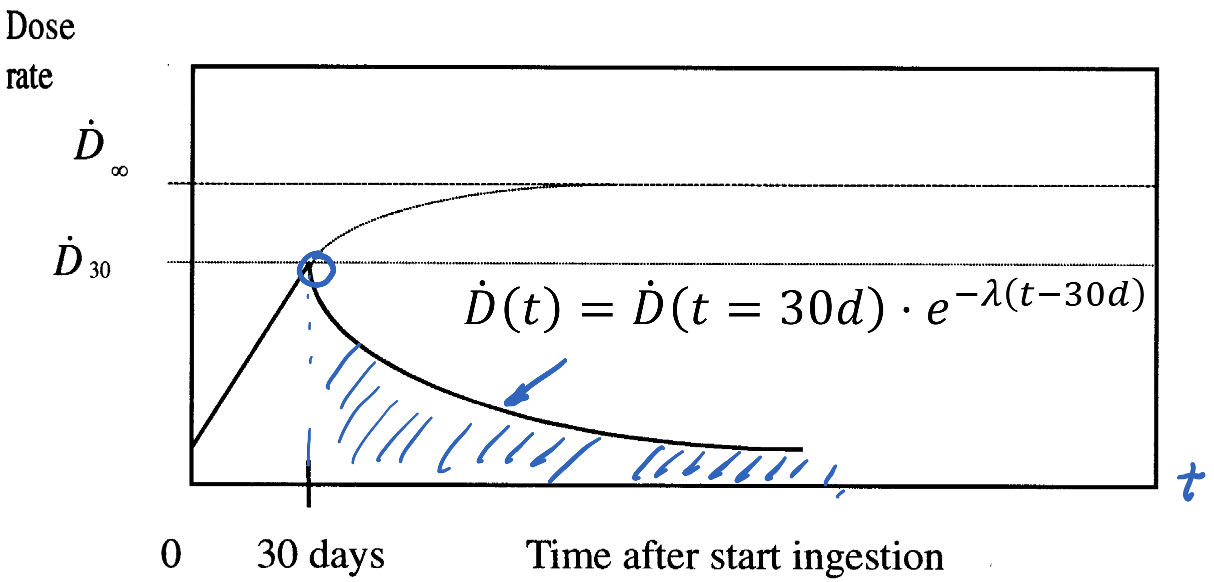
For an infinitely long time—that is, when the isotope is completely gone-

$$D = \frac{\dot{D}_0}{\lambda}$$

- For practical purpose, an infinitely long time corresponding to about 6 half-lives. The total dose received from complete decay is called the dose commitment.

Step 5: Derive the total dose (dose commitment) received after t=30d.

The dose commitment is the sum of the dose accumulated during intake, and then during elimination (washout).



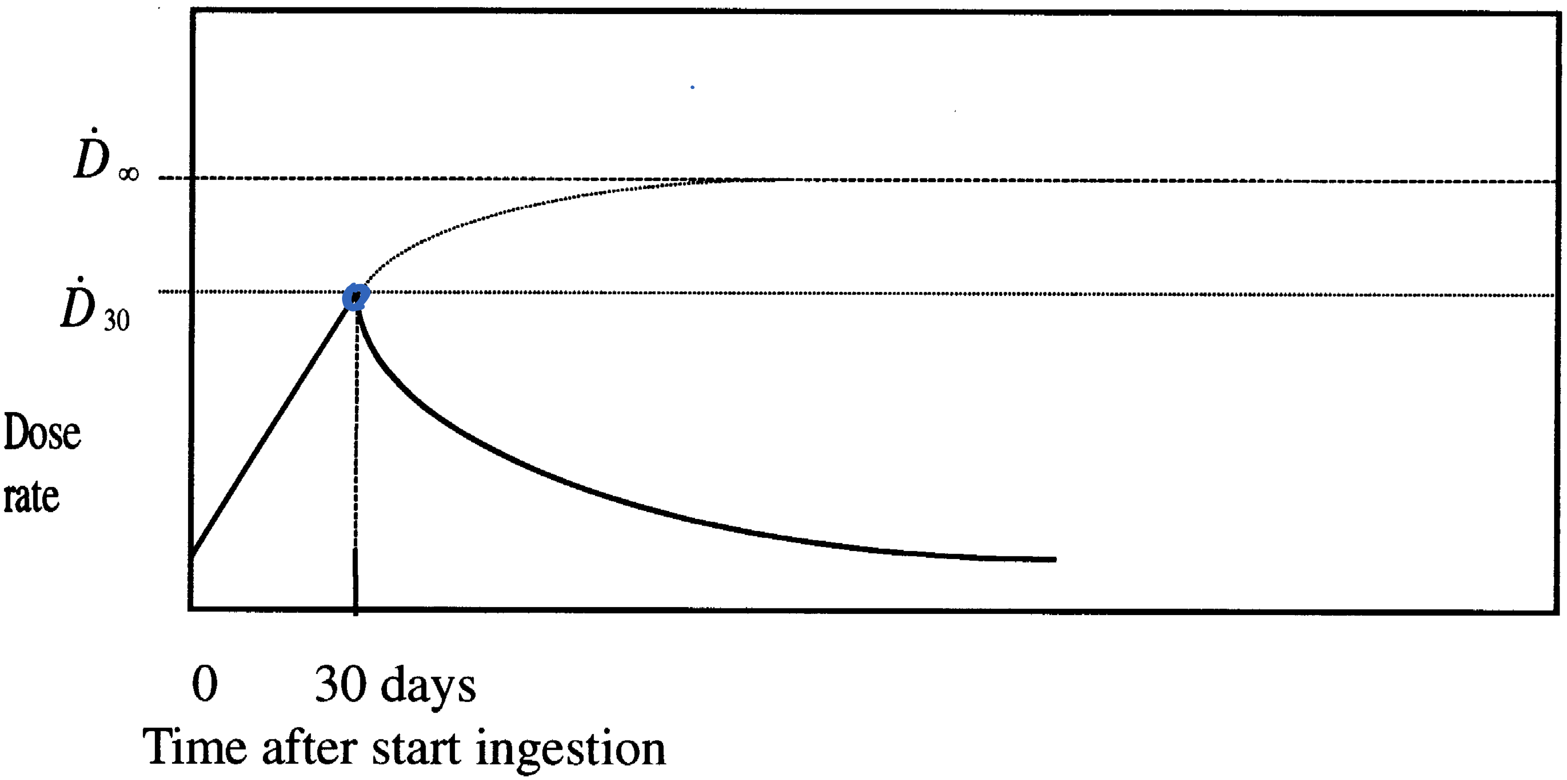
$$\dot{D}(t) = \dot{D}(t = 30d) \cdot e^{-\lambda(t-30d)}$$

Find the dose rate at $t = 30$ days

$$\dot{D}_{30} = \dot{D}_{\infty}(1 - e^{-\lambda t}) = 0.155\frac{\text{mGy}}{\text{d}} \times \left(1 - e^{-0.091\times(30)}\right) = 0.145\ \frac{\text{mGy}}{\text{day}}$$

$D = \frac{\dot{D}_{30}}{\lambda} = \frac{0.145\frac{\text{mGy}}{\text{d}}}{0.091\frac{1}{\text{d}}} = 1.59$ mGy is the dose after ingestion stops.

(Final) Step 6: Derive the total dose (dose commitment) from the initial intake of I-131



The total dose, from the time intake started to the end of the first 30 days, plus the dose after the intake stopped will be;

3 mGy + 1.6 mGy = 4.6 mGy total dose to the childs thyroid.