Chapter 5: Radiation Dosimetry

- Units of exposure and dose
- Equipment and approaches for measuring exposure

Units for Absorbed Dose

- Radiation damage depends on the energy absorption from the radiation and is approximately proportional to the concentration of absorbed energy in tissue.
- The basic unit of radiation dose is expressed in terms of absorbed energy per unit mass of tissue, which is called Gary (Gy)

$$1Gy = 1J / Kg = 100rad$$

where Rad stands for Radiation Absorbed Dose, which is a non - SI unit.

- The Gary is universally applicable to all types of ionizing radiation dosimetry – irradiation due to external field of gamma rays, neutrons or charged particles as well as that due to internally deposited radioisotopes.

The SI Unit for Exposure

- ☞ For external radiation of any given energy flux, the absorbed dose to any point within an organism depends on the type and energy of radiation, the depth within the organism of the point at which the absorbed dose is required and the elemental composition of the absorbing medium at that point.
- ☞ The x-ray fields to which an organism may be exposed is normally specified in exposure unit.

$$1\,X\ unit = 1C/Kg\ \text{air}$$

$$1\ \text{X unit} = 1\frac{\text{C}}{\text{kg air}} \times \frac{1\ \text{ion}}{1.6 \times 10^{-19}\ \text{C}} \times 34\frac{\text{eV}}{\text{ion}} \times 1.6 \times 10^{-19}\frac{\text{J}}{\text{eV}} \times 1\frac{\text{Gy}}{\text{J/kg}}$$

$$= 34\ \text{Gy (in air)}.$$

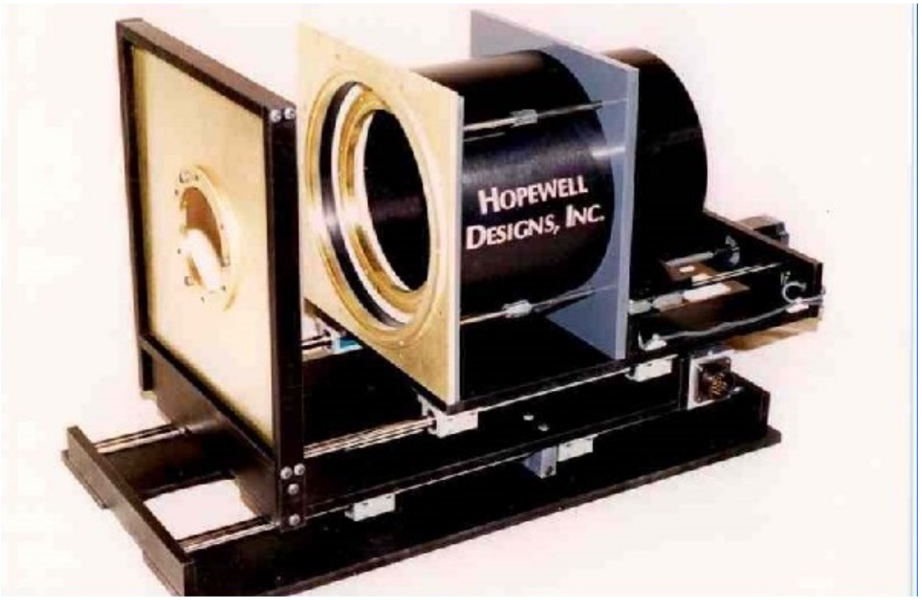
- ☞ Alternatively, exposure is also measured with the unit Roentgen (R),

$$1\ \text{R} = 2.58 \times 10^{-4}\ \text{C} \cdot \text{kg}^{-1}$$

The Free Air Chamber (FAC)

Typically used for

- **Primary Standards:** They are used by national laboratories to define the Roentgen or air Kerma for X-ray dosimetry.
- **Low/Medium Energy Focus:** They are ideal for low- or medium-energy photon-beam calibration.
- **Reference Instrument:** They provide the foundational accuracy for calibrating secondary instruments.
- **Modern Advancements:** While the principle is old, modern FAC designs (like the Attix chamber) and Monte Carlo computer simulations are used to improve accuracy and calculate correction factors.



https://slidetodoc.com/hopewell-designs-inc-calibration-of-radiation-instruments-overview/

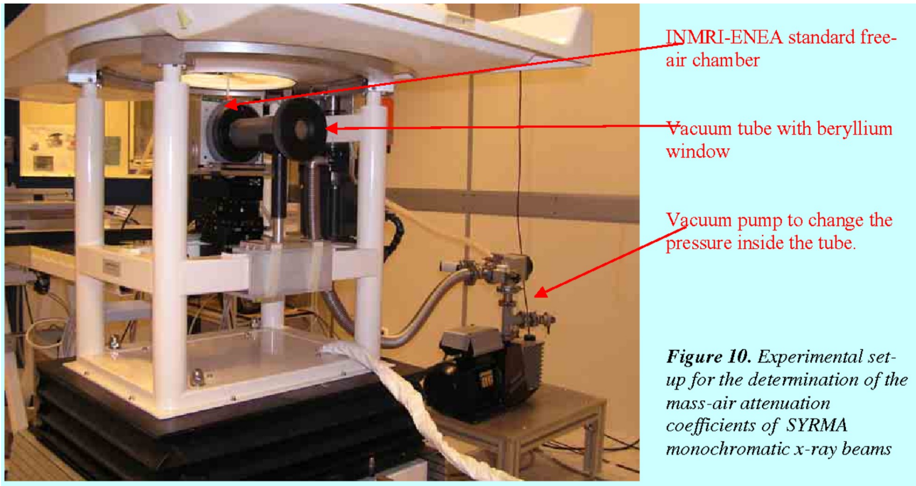


Figure 10. *Experimental set-up for the determination of the mass-air attenuation coefficients of SYRMA monochromatic x-ray beams*

Exposure Measurement: The Free Air Chamber

Electronic Equilibrium, or Charged-Particle Equilibrium (CPE)

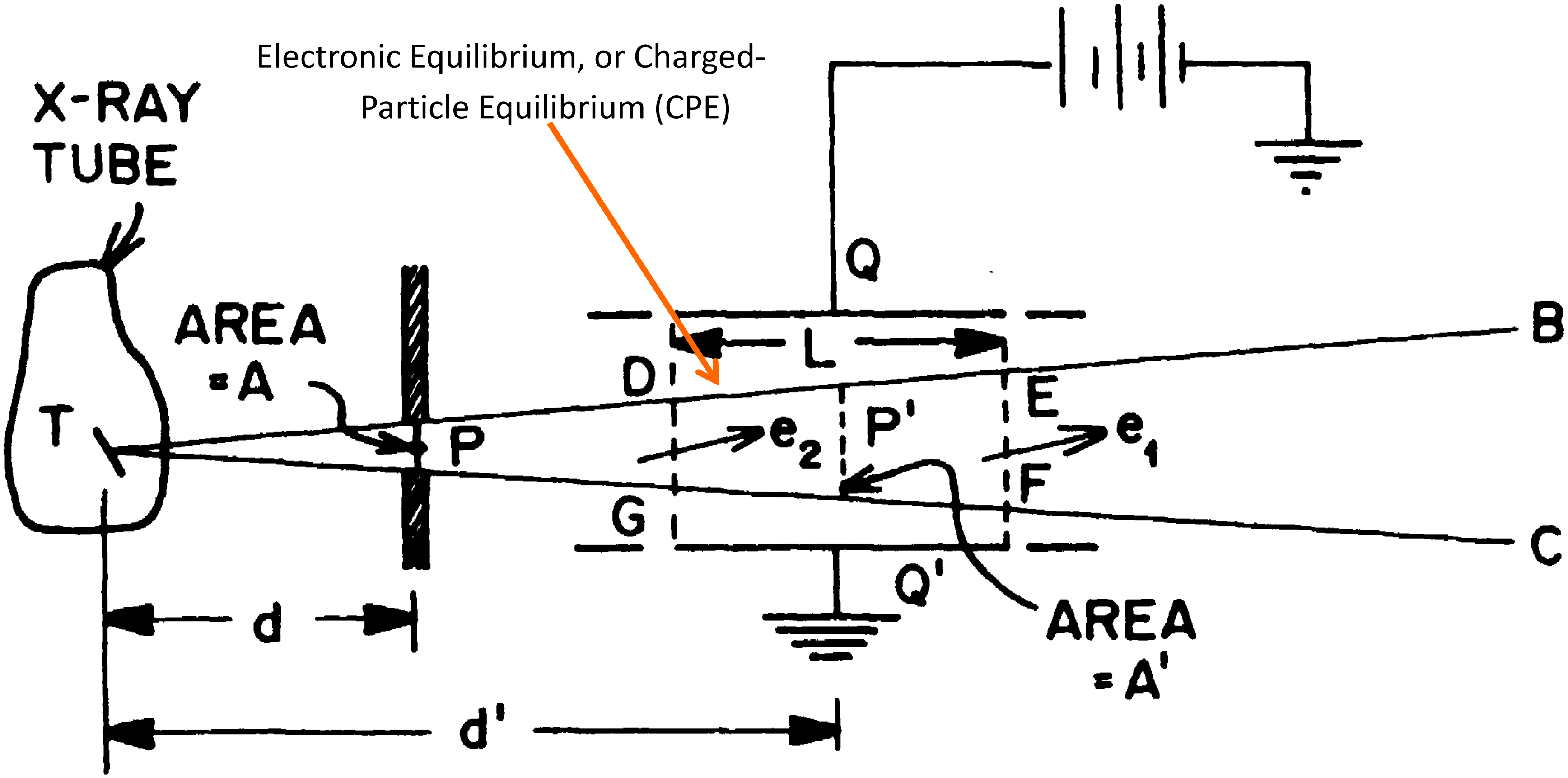


Fig. 12.1 Schematic diagram of the “free-air” or “standard” ionization chamber.

Turner, pp. 365.

Exposure Measurement: The Free Air Chamber

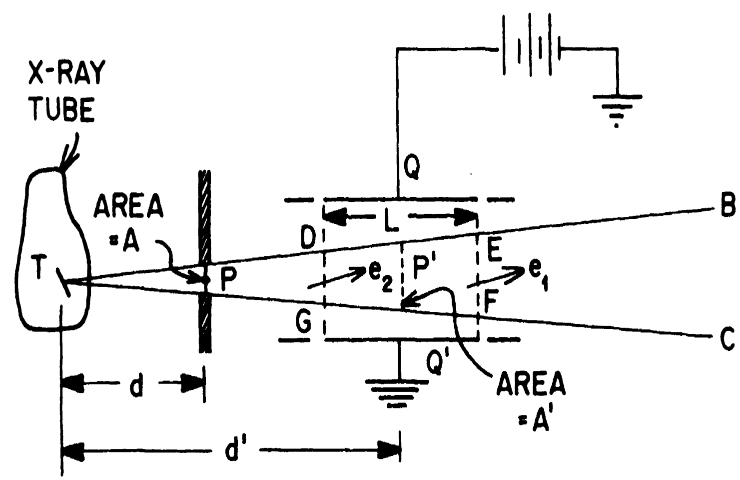
Example

The entrance port of a free-air ionization chamber has a diameter of 0.25 cm and the length of the collecting plates is 6 cm. Exposure to an X-ray beam produces a steady current of 2.6×10^{-10} A for 30 s. The temperature is 26°C and the pressure is 750 torr. Calculate the exposure rate and the exposure.

Solution

We can apply Eq. (12.7) to exposure rates as well as to exposure. The rate of charge collection is $\dot{q} = 2.6 \times 10^{-10}\ \text{A} = 2.6 \times 10^{-10}\ \text{C s}^{-1}$. The density of the air under the stated conditions is $\rho = (0.00129)(273/299)(750/760) = 1.16 \times 10^{-3}\ \text{g cm}^{-3}$. The entrance-port area is $A = \pi(0.125)^2 = 4.91 \times 10^{-2}\ \text{cm}^2$ and $L = 6$ cm. Equation (12.7) implies, for the exposure rate,

$$\dot{E}_P = \frac{\dot{q}}{\rho AL} = \frac{2.6 \times 10^{-10}\ \text{C s}^{-1}}{1.16 \times 10^{-3} \times 4.91 \times 10^{-2} \times 6\ \text{g}} \times \frac{1\ \text{R}}{2.58 \times 10^{-7}\ \text{C g}^{-1}} = 2.95\ \text{R s}^{-1}.$$



The total exposure is 88.5 R.

Exposure Measurement: The Air Wall Chamber

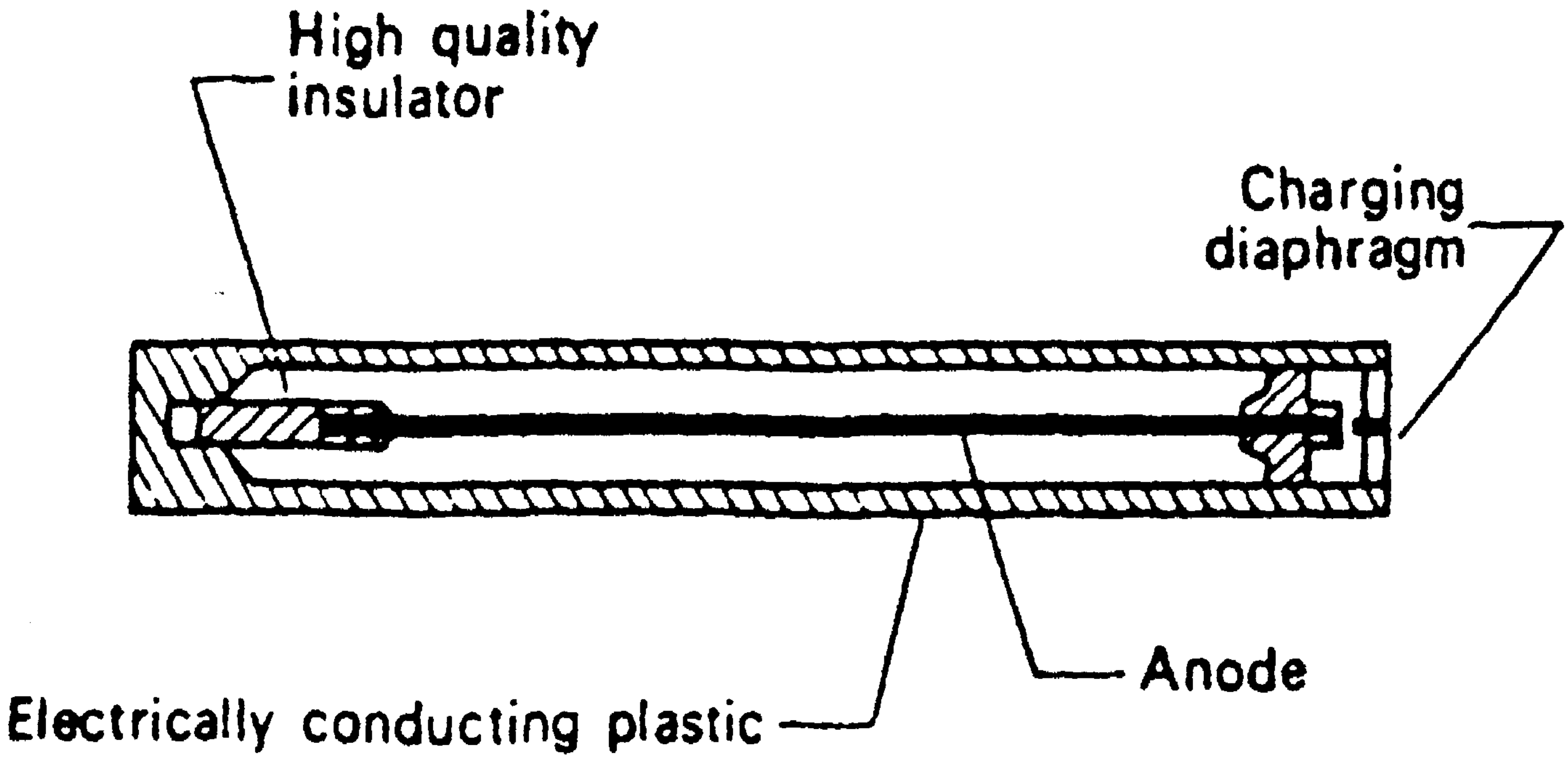


FIGURE 6.2. Non-self-reading condenser-type pocket ionization chamber.

Exposure Measurement: The Air Wall Chamber

Example 6.3

Chamber volume = 2 cm^3.

Chamber filled with air at STP.

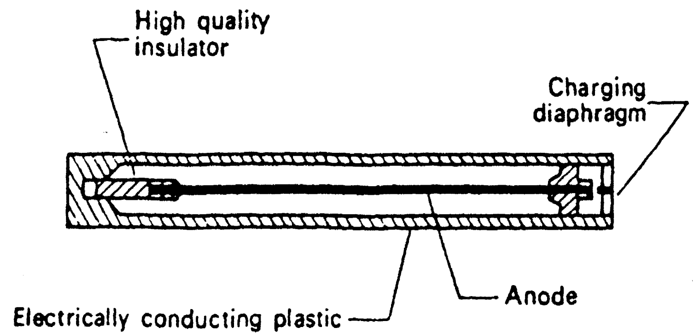
Electrical capacity = 5 $\mu\mu F$.

Voltage across chamber before exposure to radiation = 180 V.

Voltage across chamber after exposure to radiation = 160 V.

Exposure time = $\frac{1}{2}$ h.

Calculate the radiation exposure and the exposure rate.



Solution

The exposure is calculated as follows:

$$C \times \Delta V = \Delta Q \tag{6.8}$$

$$5 \times 10^{-12} \text{ farads} \times (180 - 160) \text{ volts} = 1 \times 10^{-10} \text{ coulombs.}$$

Since one exposure unit is equal to 1 C/kg, the exposure measured by this chamber is

$$\frac{1 \times 10^{-10}\ \text{C}}{2\ \text{cm}^3 \times 1.293 \times 10^{-6}\ \text{kg/cm}^3} = 3.867 \times 10^{-5}\ \text{C/kg},$$

which corresponds to

$$3.867 \times 10^{-5}\ \text{C/kg} \times 3881 \frac{\text{R}}{\text{C/kg}} = 0.150\ \text{R},$$

Dose-Exposure Relationship

Does-Exposure Relationship

- ☞ Air wall chamber measures exposure in air, which should, in some instances, be converted to the energy absorption in tissue.

- ☞ Since energy absorption is approximately proportional to the electron density of the absorber in the energy region where exposure units are valid, the tissue dose is NOT necessarily equal to the air dose for any given exposure.

Does-Exposure Relationship

☞ The relationship between exposure and dose is obtained from the ratio of the absorbed dose rate and the exposure rate,

$$\frac{\dot{D}}{\dot{X}} = \frac{(\phi \times E \times 1.6 \times 10^{-13} \times \mu_m)/\rho_m}{(\phi \times E \times 1.6 \times 10^{-13} \times \mu_a)/(\rho_a \times 34)}$$

$$\dot{D} = 34 \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a} \times \dot{X}\ \text{Gy/s}.$$

μ_{m} is the linear energy absorption coefficient of the medium and
ρ_{m} is the density of the medium.

μ_a is the linear energy absorption coefficient for air for the photon energy and
ρ_a is the density of air.

Does-Exposure Relationship

An example:

What is the radiation absorbed dose corresponding to an exposure of 25.8 μC/kg (100 mR) from 300-keV photons?

Solution

When the value for the energy absorption coefficient for muscle tissue for 0.3-MeV photons, $\mu_{\text{medium}} = 0.0317\ \text{cm}^2/\text{g}$ and $\mu_{\text{air}} = 0.0288\ \text{cm}^2/\text{g}$, from Table 5-4, are substituted into Eq. (6.12a), we have

$$\text{Dose} = 34\frac{\text{Gy}}{\text{C/kg}} \times \frac{0.0317\ \text{cm}^2/\text{g}}{0.0288\ \text{cm}^2/\text{g}} \times 25.8 \times 10^{-6}\ \frac{\text{C}}{\text{kg}} = 9.7 \times 10^{-4}\ \text{Gy}$$

$$\frac{\dot{D}}{\dot{X}} = \frac{(\phi \times E \times 1.6 \times 10^{-13} \times \mu_m)/\rho_m}{(\phi \times E \times 1.6 \times 10^{-13} \times \mu_a)/(\rho_a \times 34)}$$

$$\dot{D} = 34 \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a} \times \dot{X}\ \text{Gy/s}.$$

Does-Exposure Relationship

A few remarks:

- ☞ In the case of tissue, the ratio of dose to exposure remains approximately constant over the quantum energy range of about 0.1 to 10MeV. (Why?)
- ☞ At lower energy, material with greater atomic number absorbs much more energy.

$$\frac{\dot{D}}{\dot{X}} = \frac{(\phi \times E \times 1.6 \times 10^{-13} \times \mu_m)/\rho_m}{(\phi \times E \times 1.6 \times 10^{-13} \times \mu_a)/(\rho_a \times 34)}$$

$$\dot{D} = 34 \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a} \times \dot{X} \text{ Gy/s}.$$

μ_m is the linear energy absorption coefficient of the medium and
ρ_m is the density of the medium.

μ_a is the linear energy absorption coefficient for air for the photon energy and
ρ_a is the density of air.

Does-Exposure Relationship

$$\dot{D} = 34 \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a} \times \dot{X} \text{ Gy/s} .$$

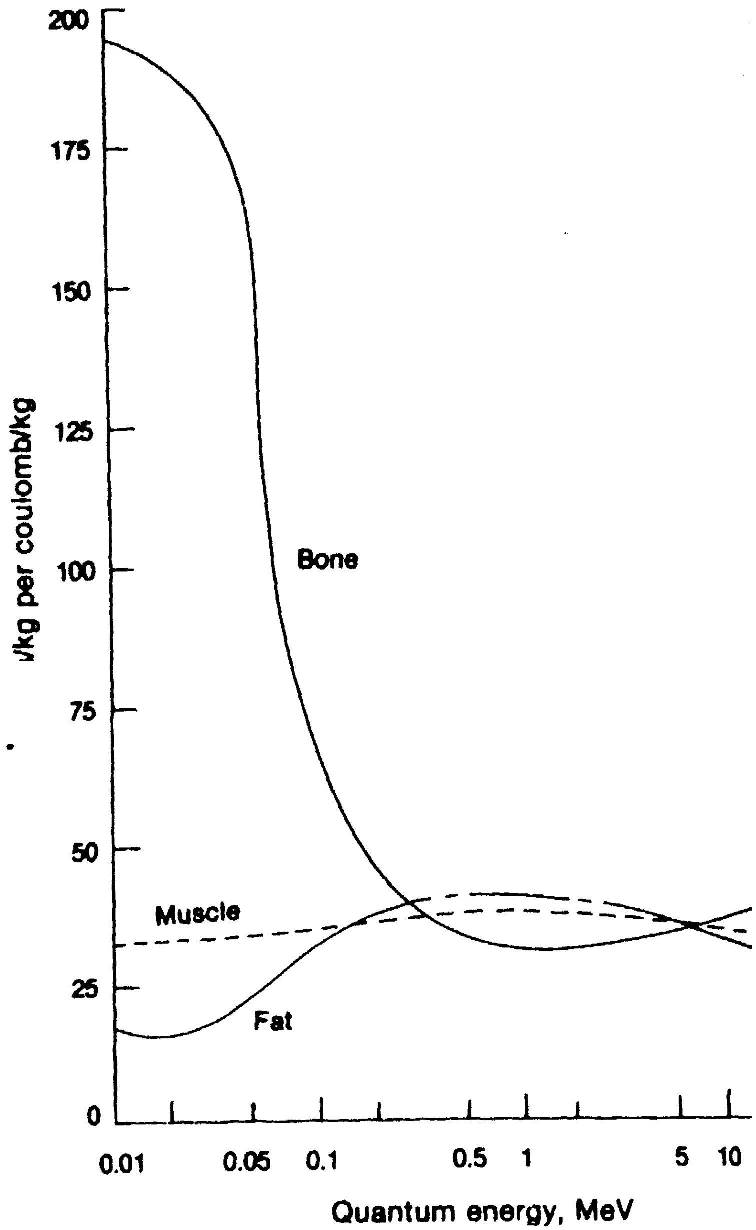
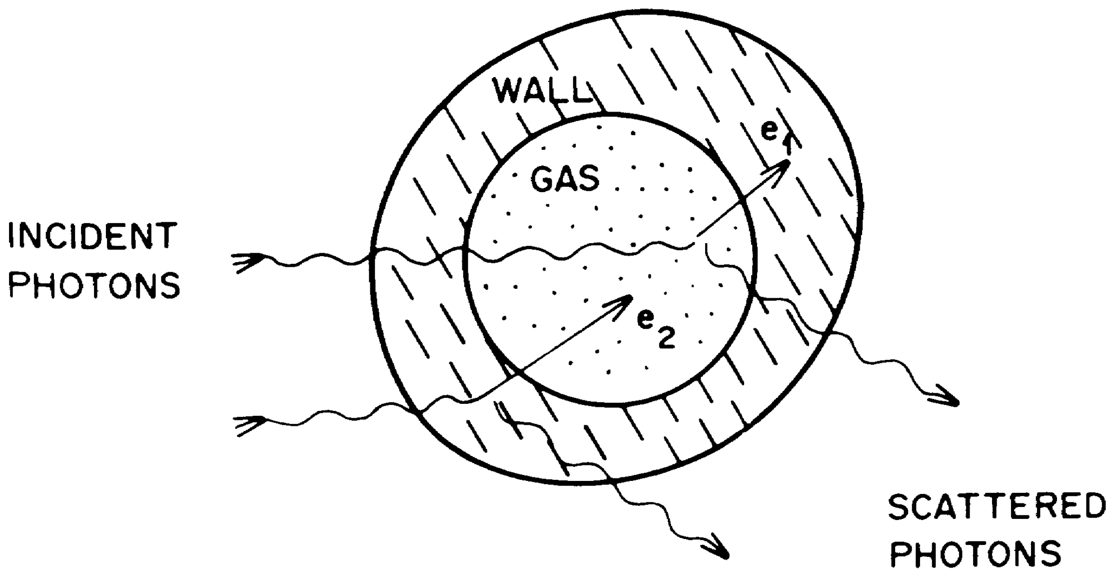


FIGURE 6.6. Energy absorption per X unit (coulomb per kilogram) exposure for several tissues. (Adapted from O. Glasser, *Medical Physics*, Vol. II. Yearbook Publishers, Chicago, IL, 1950. By permission.)

The Bragg-Gray Principle

Absorbed Dose Measurement: Bragg-Gray Principle



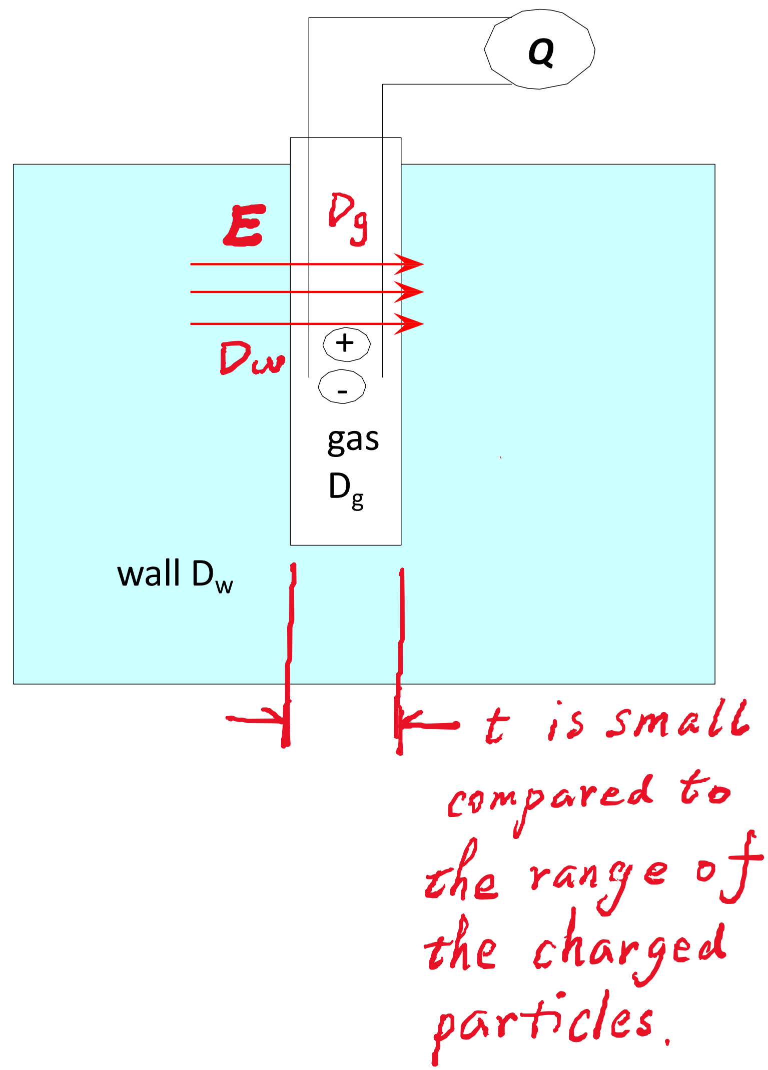
Conditions for Reaching the Electronic Equilibrium

- ☞ Dimension of the gas volume is small compared to the range of the secondary charged particles.
- ☞ Wall thickness > maximum range of secondary charged particles.
- ☞ Wall thickness is not great enough to significantly attenuate the incident radiation.
- ☞ Wall and gas have similar atomic compositions.

Bragg-Gray Principle: Problem Statement

- Homogeneous medium, wall (w)
- Probe - cavity - thin layer of gas (g)
- Charged particles crossing w-g interface
- Objective: find a relation between the dose

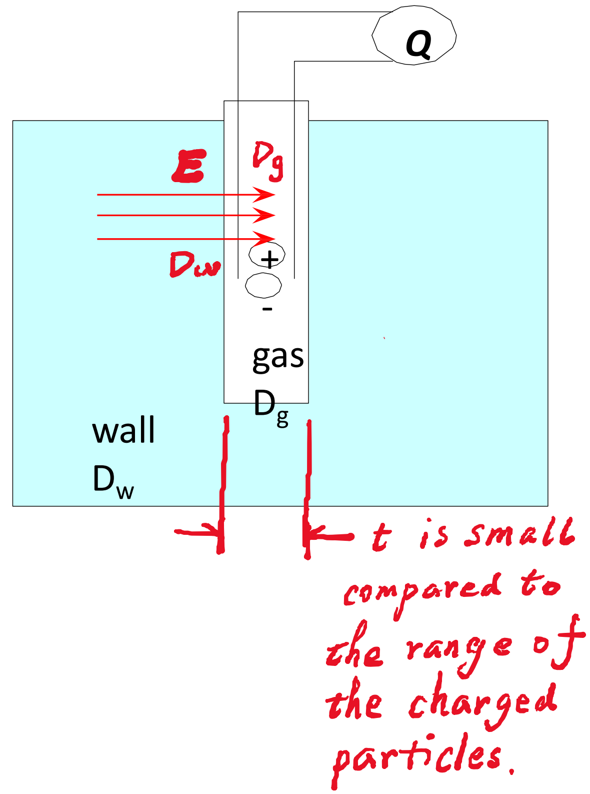
  in a probe to that in the medium
- Basis for dosimetry



F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Bragg-Gray Principle

If we look at the very thin layers of wall media immediately adjacent to the interface, then the flux of the charged particles is almost unchanged across the boundary. The dose rate to the wall is given by



$$\dot{D}_w\left(\frac{J}{g\cdot s}\right)=\phi\left(\frac{particles}{s\cdot cm^2}\right)\cdot E\left(\frac{J}{particle}\right)\cdot\mu_w(cm^{-1})/\rho_w\left(\frac{g}{cm^3}\right),$$

where $\boldsymbol{\mu}_w$ is the linear energy absorption coefficient.

Then the ratio of dose (rate) in the wall and in the gas is

$$\frac{\dot{D}_w}{\dot{D}_g}=\frac{D_w}{D_g}=\frac{\mu_w/\rho_w}{\mu_g/\rho_g}$$

For charged particles, the linear energy absorption coefficient μ is roughly the same as the linear stopping power, $s=\frac{\mathrm{d}E}{\mathrm{d}x}$, therefore

$$\frac{\dot{D}_w}{\dot{D}_g}=\frac{D_w}{D_g}=\frac{\mu_w/\rho_w}{\mu_g/\rho_g}=\frac{s_w/\rho_w}{s_g/\rho_g}\ .$$

Absorbed Dose Measurement: Bragg-Gray Principle

- The Bragg-Gray principle provides a means of relating ionization measurements in a gas volume to the absorbed dose in some convenient materials (such tissue equivalent materials) from which a dosimeter can be fabricated.
- If the gas cavity is surrounded by a wall medium of proper thickness to establish electronic equilibrium, then the energy absorbed per unit mass of the wall, dE_m/dM_m, is related to the energy absorbed per unit mass of gas, dE_g/dM_g, by

$$\frac{dE_m}{dM_m} = \frac{S_m}{S_g} \times \frac{dE_g}{dM_g}$$

where S_m is the main mass stopping power of the wall medium and S_g is the mass stopping power of the gas to the secondary electrons.

Absorbed Dose Measurement: Bragg-Gray Principle

TABLE 6.2. Mean Mass Stopping Power Ratios, S_m/S_{air} for Equilibrium Electron Spectra Generated by ^{198}Au, ^{137}Cs, and ^{60}Co, on the Assumption That the Electrons Slow Down in a Continuous Manner

	Medium		
Energy, MeV	Graphite	Water	Tissue
0.411 (^{198}Au)	1.032		
0.670 (^{137}Cs)	1.027	1.162	1.145
1.25 (^{60}Co)	1.017	1.155	1.137

Source: N.B.S. Handbook 78, *Report of the International Commission on Radiological Units and Measurements,* National Bureau of Standards, U.S. Govt. Printing Office, Washington, D.C., 1959.

$$\frac{dE_m}{dM_m} = \frac{S_m}{S_g} \times \frac{dE_g}{dM_g}$$

Measurement of X- and Gamma Ray Dose

☞ For gamma rays with different energies

$$\frac{dE_m}{dM_m} = \frac{S_m}{S_g} \times \frac{dE_g}{dM_g}$$

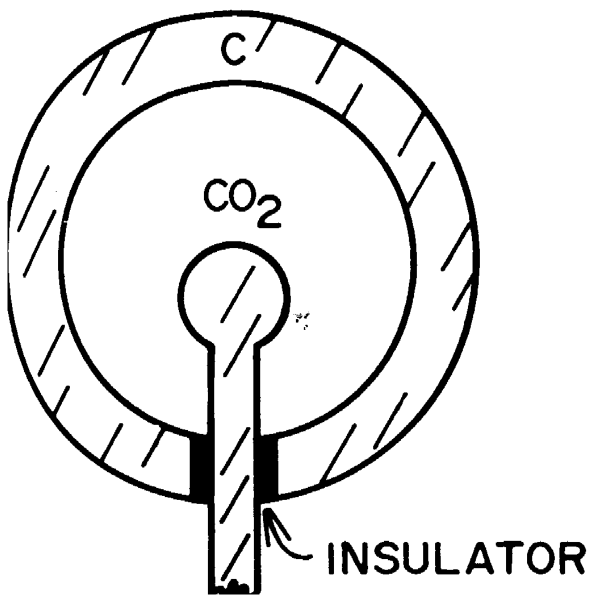


FIGURE 12.4. Cross section of graphite-walled CO_2 chamber for measuring photon dose.

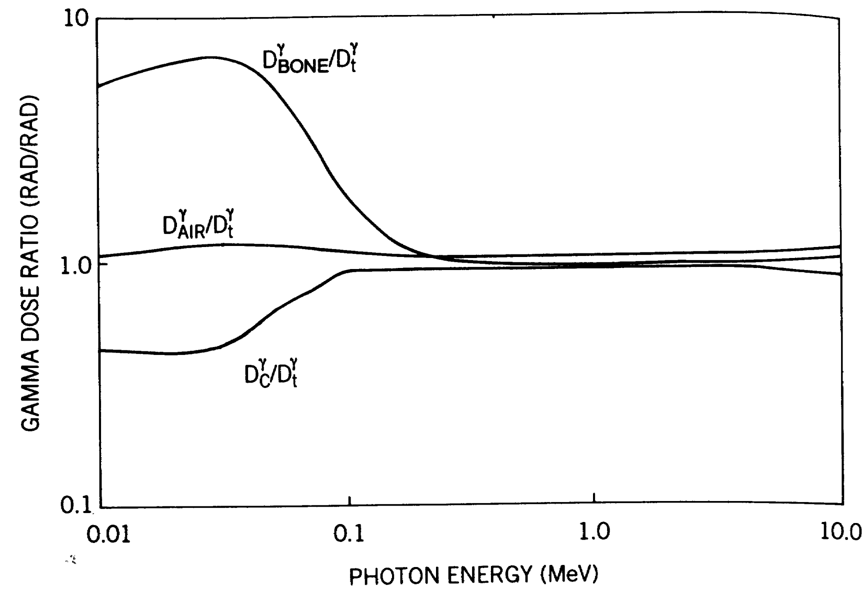


FIGURE 12.5. Ratio of absorbed doses in bone, air, and carbon to that in soft tissue, D_t.

Kinetic Energy Released per Unit Mass (Kerma)

Kinetic Energy Released per Unit Mass (Kerma)

- Kerma: Initial kinetic energy of the "primary" ionizing particles (including the photoelectrons, positron-electron pairs, recoil electrons and the scattered nuclei in case of fast neutrons) produced by the interaction of incident radiation per unit mass of the interacting medium.
- Measured in Gy (Joules per kilogram).
- An example (Cember, p183)

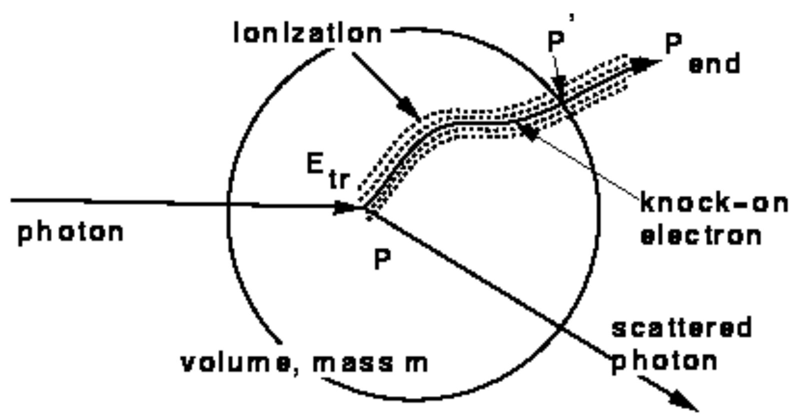


Figure 1: The exposure, air kerma and absorbed dose for a single photon which Compton scatters and transfers an energy $\mathbf{E_{tr}}$ to an electron at point **P**. The volume of interest is shown as a circle and the mass of this volume is m. The energetic electron set in motion at **P** slows down and stops at $\mathbf{P_{end}}$. As it slows down it loses energy which results in 30 ion pairs being created near the track, per keV of energy lost.

Kinetic Energy Released per Unit Mass (Kerma)

Example 6.6

A 10-MeV photon penetrates into a 100-g mass, and a pair-production interaction leads to a positron and an electron of 4.5 MeV each. Both charged particles dissipate all their kinetic energy within the mass through ionization and bremsstrahlung production. Three bremsstrahlung photons of 1.6, 1.4, and 2 MeV each are produced and escape from the mass before they interact. The positron, after expending all its kinetic energy, interacts with an ambient electron as they mutually annihilate one another to produce two photons of 0.51 MeV each. Calculate

(a) The kerma
(b) The absorbed dose.

Kinetic Energy Released per Unit Mass (Kerma)

(a) Kerma is defined as the *sum of the initial kinetic energies per unit mass of all charged particles produced by the radiation*. In this case, a positron-negatron pair of 4.5 MeV each (2 × 4.5 MeV) represents all the initial kinetic energy:

$$K = \frac{2 \times 4.5\ \text{MeV} \times 1.6 \times 10^{-13}\ \dfrac{\text{J}}{\text{MeV}}}{0.1\ \text{kg} \times 1\dfrac{\text{J}}{\text{kg}} \Big/ \text{Gy}} = 1.44 \times 10^{-11}\ \text{Gy}.$$

(b) Dose is defined as the *energy absorbed per unit mass.* Here we have th MeV of initial kinetic energy, of which (1.6 + 1.4 + 2) MeV was converted i bremsstrahlung and escaped from the mass. The absorbed dose, therefore is

$$D = \frac{(4.5 + 4.5)\ \text{MeV} - (1.6 + 1.4 + 2)\ \text{MeV}}{0.1\ \text{kg} \times 1\dfrac{\text{J}}{\text{kg}} \Big/ \text{Gy}} \times 1.6 \times 10^{-13}\ \frac{\text{J}}{\text{MeV}} = 6.4 \times 10^{-12}\ \text{Gy}.$$

Radiation Dose Induced by Gamma Radiation

Please consider the following questions:

- What is the so-called specific gamma ray constant?
- How to evaluate the specific gamma ray constant for a given isotopic source (such as In-111, Cs-137)?
- For a radiation source with a known specific gamma ray constant and a given volumetric distribution, how can we evaluate the exposure or dose delivered to a given target point?

Radiation Dose from a Gamma-Ray Point Source

Considering an I-131 point-source of 1 MBq, how do we evaluate the exposure it delivers at a distance of 1 m?

☞ The decay of ^{131}I produces gamma rays of various energies as shown below,

Quantum Energy, MeV	Photons per Transformation	Energy Absorption Coefficient for Air, m^{-1}
0.723	0.016	3.8×10^{-3}
0.637	0.069	3.9×10^{-3}
0.503	0.003	3.8×10^{-3}
0.326	0.002	3.8×10^{-3}
0.177	0.002	3.4×10^{-3}
0.365	0.853	3.8×10^{-3}
0.284	0.051	3.7×10^{-3}
0.080	0.051	3.2×10^{-3}
0.164	0.006	3.3×10^{-3}

Radiation Dose from a Gamma-Ray Point Source

For the 0.080-MeV gamma ray, we have

$$\dot{X} = \frac{5.1 \times 10^{-2} \times 8 \times 10^{-2} \times 1.6 \times 10^{-13} \times 1 \times 10^{6} \times 3.6 \times 10^{3} \times 3.2 \times 10^{-3}}{4\pi (1)^2 \times 1.293 \times 34}$$

$$= 1.36 \times 10^{-11} \frac{\text{C/(kg} \cdot \text{h)}}{\text{MBq}}.$$

☞ Repeating the calculation, we get the following results:

Quantun Energy, MeV	(C/kg)/h at 1 m
0.723	4.583×10^{-11}
0.637	17.787×10^{-11}
0.503	0.598×10^{-11}
0.326	0.258×10^{-11}
0.177	0.126×10^{-11}
0.365	123.400×10^{-11}
0.284	5.569×10^{-11}
0.080	1.361×10^{-11}
0.164	0.339×10^{-11}

$$\text{Total} = 1.540 \times 10^{-9} \frac{\text{(C/kg)/h}}{\text{MBq}} \text{ at 1 m}$$

Specific Gamma-Ray Constant

☞ In a more generic case, considering a gamma-ray point-source of activity *A* (*MBq*), the exposure rate it will deliver to a distance *d* (m) away in air is given by

$$\dot{X}\left(\frac{C/kg}{h}\right) = \sum_{i=1}^{I}\left[\frac{A\times 10^6 (t/_s)\times 3600(s/_h)\cdot f_i(p/_t)\cdot E_i(J/_p)}{4\pi[d\ (m)]^2}\cdot \mu_i(m^{-1})/\rho_a\ (kg/m^3)\right]\cdot \frac{1}{34\frac{J/kg}{C/kg}}$$

where

$\dot{X}$: *exposure rate*, $\left(\frac{C/kg}{h}\right)$
f_i: *fraction of transformation that result in a photon of the i'th energy*
E_i: *energy carried by each photon of the i'th group*, (J)
μ_i: *linear energy absorption coefficient for photons of the i'th group*, (m^{-1})
ρ_a: *the density of air*, (kg/m^3)

☞ The specific gamma-ray constant, Γ, , for this gamma-ray source, is given by

$$\Gamma\left(\frac{C/kg\cdot m^2}{h\cdot MBq}\right) = \frac{\dot{X}\left(\frac{C/kg}{h}\right)\cdot [d(m)]^2}{A(MBq)}$$

☞ The specific gamma-ray constant numerically equal to the exposure rate that a source of a unit activity delivers to a unit distance away in air.

Specific Gamma-Ray Constant

☞ Substitute all known constants into the generic equation, the specific Gamma-Ray Constant can be given by

$$\Gamma = 1.043 \times 10^{-6} \sum_i f_i \times E_i \times \mu_i \frac{(\mathrm{C/kg})\ \mathrm{m}^2}{\mathrm{MBq \cdot h}},$$

where f_i is the fraction of the transformations that yield a photon whose energy is E_i and μ_i is the linear energy absorption coefficient in air of the ith photon.

☞ For many practical situations, when photon energy is ranging from 60keV to 2MeV, the linear absorption coefficient varies little with energy, over this energy range, μ is about 3.5×10^{-3} per meter. Therefore, we can simplify the above equation as

$$\Gamma = 3.65 \times 10^{-9} \sum_i f_i \times E_i \frac{(\mathrm{C/kg})\ \mathrm{m}^2}{\mathrm{MBq \cdot h}}$$

Specific Gamma-Ray Constant

☞ Specific Gamma Ray Constant (Γ): The exposure rate from a gamma ray point source of unit activity and positioned at a unit distance. It is given in the unit of coulombs per kilogram per hour at a distance of 1 m from a 1 MBq point source, or (coulombs/kg/h/MBq at 1m).

TABLE 6.3. Specific Gamma-ray Constant of Some Radioisotopes

Isotope	Γ $\frac{R \cdot m^2}{Ci \cdot h}$ [a]	Γ $\frac{X \cdot m^2}{MBq \cdot h}$ [b]
Antimony 122	0.24	1.67E—09
Cesium 137	0.33	2.30E—09
Chromium 51	0.016	1.11E—10
Cobalt 60	1.32	9.19E—09
Gold 198	0.23	1.60E—09
Iodine 125	0.07	4.87E—10
Iodine 131	0.22	1.53E—09
Iridium 192	0.48	3.34E—09
Mercury 203	0.13	9.05E—10
Potassium 42	0.14	1.39E—09
Radium 226	0.825	5.75E—09
Sodium 22	1.20	8.36E—09
Sodium 24	1.84	12.80E—09
Zinc 65	0.27	1.88E—09

[a]From *Radiological Health Handbook*, rev. ed., U.S. Public Health Service, Bureau of Radiological Health, Rockville, MD, 1970.
[b]1 X unit = 1 C/kg.

Cember, pp. 226.

Specific Gamma-Ray Constant

Given the specific gamma ray constant, Γ, for an isotope, the exposure rate at a location at a distance, *r*, is simply

$$\dot{X} = \Gamma \frac{A}{r^2}$$

activity (A)

distance (r)

Example

(a) Estimate the specific gamma-ray constant for ^{137}Cs. (b) Estimate the exposure rate at a distance of 1.7 m from a 100-mCi point source of ^{137}Cs.

Solution

(a) The isotope emits only a 0.662-MeV gamma ray in 85% of its transformations (Appendix D). The average energy per disintegration released as gamma radiation is therefore $0.85 \times 0.662 = 0.563$ MeV. The estimated specific gamma-ray constant for ^{137}Cs is therefore $\Gamma =$ 0.28 R m^2 Ci^{-1} h^{-1}.

(b) From Eq. (12.28), the exposure rate at a distance $r = 1.7$ m from a point source of activity $C = 100$ mCi $= 0.1$ Ci is

$$\dot{X} = 0.28 \frac{\text{R m}^2}{\text{Ci h}} \times \frac{0.1 \text{ Ci}}{(1.7 \text{ m})^2} = 9.7 \times 10^{-3} \text{ R h}^{-1} = 9.7 \text{ mR h}^{-1}. \qquad (12.29)$$

Turner, pp. 382.

Internally Deposited Radioisotope (IV) Gamma Ray Emitters

☞ For a uniformly distributed gamma-ray-emitting isotope, the dose rate from the isotope in an infinitesimal volume *dV* to a point *p* at a distance *r* away is

Mass energy absorption coefficient of the dose-receiving media

$$d\dot{D} = C(MBq/m^3) \cdot \Gamma(\frac{C/kg \cdot m^2}{MBq \cdot hr}) \cdot \frac{e^{-\mu(m^{-1})r(m)}}{r^2(m^2)} \cdot dV(m^3) \cdot (34\,\frac{J/kg}{C/kg}) \cdot (\frac{\mu_m/\rho_m}{\mu_a/\rho_a})$$

Mass energy absorption coefficient of the air

where C is the concentration of the isotope, Γ is the specific gamma-ray emission, and μ is the linear energy absorption coefficient.

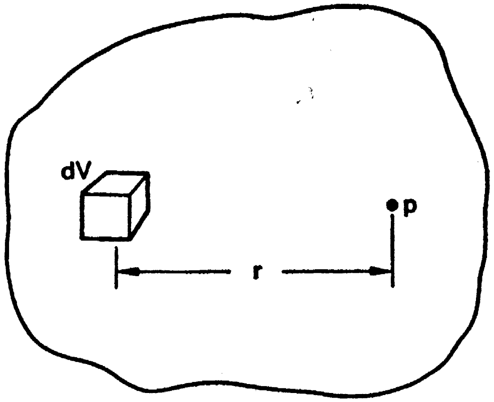


FIGURE 6.8. Diagram for calculating dose at point p from the gamma rays emitted from the volume element dV in a tissue mass containing a uniformly distributed isotope.

Internally Deposited Radioisotope (IV)
Gamma Ray Emitters

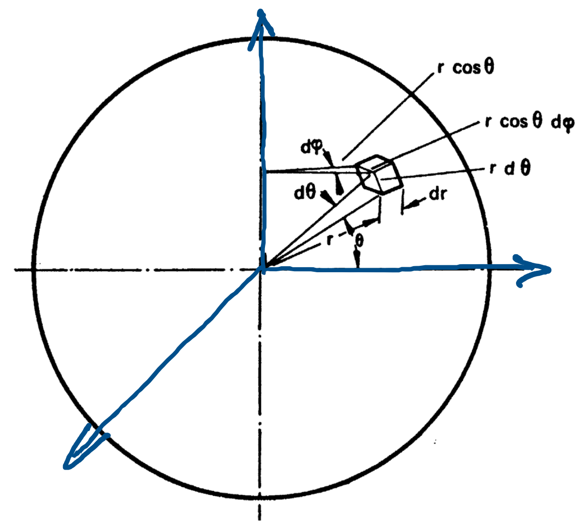
☞ For a uniform spherical source, the dose rate at the center is given by

$$\dot{D} = 4C\Gamma \int_{r=0}^{r=R} \int_{\theta=0}^{\theta=\pi/2} \int_{\varphi=0}^{\varphi=\pi} \frac{e^{-\mu r}}{r^2} \cdot r\, d\theta \cdot r\cos\theta\, d\varphi \cdot dr \cdot \left(34\, \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

$$= C\Gamma \cdot \frac{4\pi}{\mu}(1 - e^{-\mu R}) \cdot \left(34\, \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

☞ And the dose rate at the surface of the spherical source volume is given by

$$\dot{D}_{surface} = 0.5\, \dot{D}_{center}$$



$$dV = rd\theta \cdot rcos\theta d\varphi \cdot dr = r^2 cos\theta d\varphi d\theta dr$$

Example 6.12

A spherical tank, capacity 1 m^3 and radius 0.62 m, is filled with aqueous ^{137}Cs waste containing a total activity of 37,000 MBq (1 Ci). What is the dose rate at the tank surface if we neglect absorption by the tank wall?

Example 6.12

A spherical tank, capacity 1 m^3 and radius 0.62 m, is filled with aqueous ^{137}Cs waste containing a total activity of 37,000 MBq (1 Ci). What is the dose rate at the tank surface if we neglect absorption by the tank wall?

From Table 6.3 we find $\Gamma = 2.3 \times 10^{-9}$ X units/h/MBq at 1 m. Since water absorbs 38 Gy/X unit, the dose rate is 8.74×10^{-8} Gy/h/MBq at 1 m. The absorption coefficient of water for the 0.661 MeV gammas from ^{137}Cs is listed in Table 5.3 as 0.0327 cm^2/g. Since the density of water is 1 g/cm^3, the linear absorption coefficient is 0.0327/cm, or 3.27/m. The dose rate at the center of the sphere is found by substituting the respective values into Eq. (6.66):

$$\dot{D}_0 = C\Gamma \cdot \frac{4\pi}{\mu}(1 - e^{-\mu R}) \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

$$\dot{D}_0 = 3.7 \times 10^4 \text{ MBq/m}^3 \times 8.74 \times 10^{-8} \frac{\text{Gy} \cdot \text{m}^2}{\text{MBq} \cdot \text{h}} \times \frac{4\pi}{3.27 \text{ m}^{-1}}(1 - e^{-3.27 \times 0.62})$$

$$= 1.08 \times 10^{-2} \text{ Gy/h (1.08 rad/h).}$$

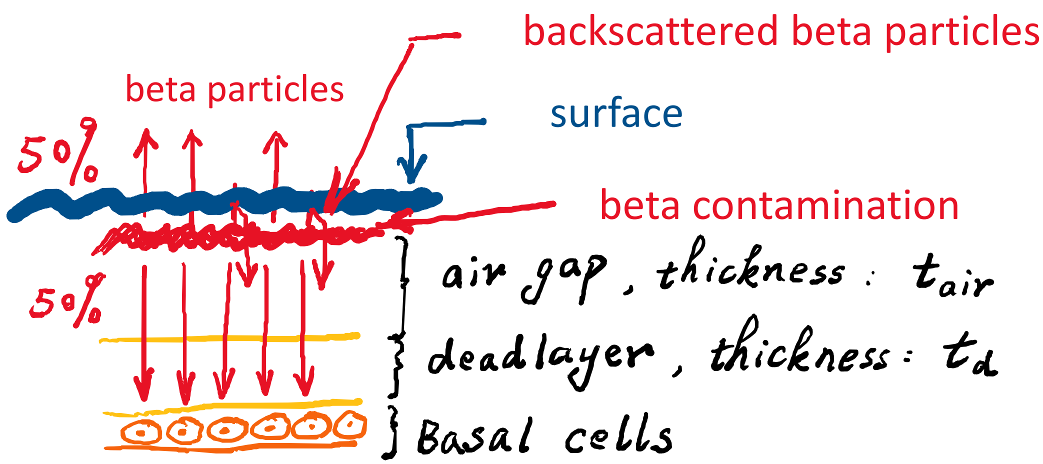
From Eq. (6.71), we see that the surface dose rate is $0.5 \times \dot{D}_0$. Therefore

$$\dot{D}_{\text{surface}} = 0.5 \times 1.08 \times 10^{-2} = 0.54 \times 10^{-2} \text{ Gy/h (0.54 rad/h).}$$

Radiation Dose Induced by Beta Radiation

Skin Dose from Surface Contamination

Skin Dose from Surface Contamination



For a planar beta emitting surface, the surface dose rate may be easily calculated. Suppose the surface concentration is C_a Bq/cm^2, the dose rate to the basal cell region is

Mass energy absorption coefficient of tissue (cm^2/g)

surface concentration is C_a (Bq/cm^2)

Average energy of beta particles

$$\dot{D} = C_a \cdot 0.5 \cdot f_b \cdot \bar{E} \cdot e^{-\mu_{air} \cdot t_{air}} \cdot e^{-\mu_d \cdot t_d} \cdot \mu_\beta$$

Backscattering correction, 1.25

Attenuation by air

Attenuation by dead skin layer

Beta Radiation – Dose from Surface Contamination
An Example (Cember, p. 190)

Example 6.7

A solution of ^{32}P is spilled, and contaminates a large surface to an areal concentration of 37 Bq/cm^2. What is the estimated beta-ray-contact dose rate to the skin and the dose rate at a height of 1 m above the contaminated area?

For ^{32}P:

$E_m = 1.71$ MeV $\qquad \bar{E} = 0.7$ MeV.

The beta absorption coefficients in air and in tissue are calculated by substituting 1.71 for the value of E_m in Eqs. (6.20) and (6.21):

$$\mu_{\beta,a} = 16(1.71 - 0.036)^{-1.4}\ \frac{cm^2}{g} = 7.78\,\frac{cm^2}{g}$$

$$\mu_{\beta,t} = 18.6(1.71 - 0.036)^{-1.37}\ \frac{cm^2}{g} = 9.18\,\frac{cm^2}{g},$$

Beta Radiation – Dose from Surface Contamination
An Example (Cember, p. 190)

and the dose rate to the skin in contact with the contaminated area is calculated with Eq. (6.26):

energy fluence rate

attenuation in the dead layer

$$\dot{D}_{\beta}, \frac{\text{Gy}}{\text{h}} = \frac{\left(3.6 \times 10^{-10} \times C_a \times \overline{E}\right) \frac{\text{J}}{\text{cm}^2} \Big/ \text{h} \times \mu_{\beta,t} \frac{\text{cm}^2}{\text{g}}}{0.001 \frac{\text{J}}{\text{g}} \Big/ \text{Gy}} \times e^{-.007 \frac{\text{g}}{\text{cm}^2} \times \mu_{\beta,t} \frac{\text{cm}^2}{\text{g}}}$$

$$= \frac{3.6 \times 10^{-10} \times 37 \times 0.7 \times 9.18}{0.001} \times e^{-0.007 \times 9.18} = 8 \times 10^{-5} \frac{\text{Gy}}{\text{h}} = 0.08 \frac{\text{mGy}}{\text{h}}.$$

The dose rate to the skin, at a height of 1 m above the contaminated surface, is calculated with Eq. (6.31):

extra attenuation by the air gap

$$\dot{D}_b = 3.6 \times 10^{-4} \times C_a \times \overline{E} \times e^{-\mu_{\beta,a} d} \times e^{-\mu_{\beta,t} \times 0.007} \times \mu_{\beta,t} \frac{\text{mGy}}{\text{h}}$$

$$= 3.6 \times 10^{-4} \times 37 \times 0.7 \times e^{-7.78 \times 0.129} \times e^{-9.18 \times 0.007} \times 9.18 = 2.9 \times 10^{-2} \frac{\text{mGy}}{\text{h}}.$$

Neutron Dose

Radiation Dose from Fast Neutrons

Example 6.16

What is the absorbed dose rate to soft tissue in a beam of 5-MeV neutrons whose intensity is 2000 neutrons per square centimeter per second?

Radiation Dose from Fast Neutrons

☞ Neutron dose is deposited through scattering and neutron induced nuclear reactions.

☞ In cases of elastic scattering, the scattered nuclei dissipate their energy in the immediate vicinity of the primary neutron interaction. The radiation dose absorbed locally in this way is called the first collision dose. The scattered neutron is not considered after this primary interaction.

☞ For fast neutrons, the first collision dose rate is given by

$$\dot{D}_n(E) = \frac{\phi(E)E\sum_i N_i\sigma_i f}{1\ \text{J/kg}\cdot\text{Gy}}, \tag{6.103}$$

where $\phi(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

Radiation Dose from Fast Neutrons

Example 6.16

What is the absorbed dose rate to soft tissue in a beam of 5-MeV neutrons whose intensity is 2000 neutrons per square centimeter per second?

Substituting the appropriate values into Eq. (6.103) yields

$$\dot{D}_n = \frac{2 \times 10^3\ n/\text{cm}^2 \cdot \text{s} \times 5\ \text{MeV}/n \times 1.6 \times 10^{-13}\ \text{J/MeV} \times 51.17\ \text{cm}^2/\text{kg}}{1\ \text{J/kg} \cdot \text{Gy}}$$

$$= 8.19 \times 10^{-8}\ \text{Gy/s} \left(8.19 \times 10^{-6}\ \text{rad/s}\right),$$

or

$$8.19 \times 10^{-8}\ \text{Gy/s} \times 10^6\ \mu\text{Gy/Gy} \times 3.6 \times 10^3\ \text{s/h}$$

$$= 295\ \mu\text{Gy/h}\ (29.5\ \text{mrad/h}).$$

The scattering cross sections of each of the tissue elements for 5-MeV neutrons are listed below:

ELEMENT	σ, x 10^{-24} cm^2	$N_i \sigma_i f_i$
O	1.55	4.628
C	1.65	1.502
H	1.50	4.485×10^1
N	1.00	1.848×10^{-1}
Na	2.3	7.231×10^{-3}
Cl	2.8	2.523×10^{-3}
		$\sum N_i \sigma_i f_i = 5.117 \times 10^1 \frac{\text{cm}^2}{\text{kg}}$

$$\dot{D}_n(E) = \frac{\phi(E) E \sum_i N_i \sigma_i f}{1 \text{ J/kg} \cdot \text{Gy}},$$

where $\phi\,(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

How do Thermal Neutrons Deposit Energy in Tissue?

Neutron Induced Reactions

$$ {}_{0}^{1}n + {}_{7}^{14}N \rightarrow {}_{6}^{14}C + {}_{1}^{1}p $$

- ☞ Cross section for thermal neutron is 1.70 barns.
- ☞ Q=0.626MeV.
- ☞ Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- ☞ Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Thermal Neutron Dose from the $^{14}N(n,p)^{14}C$ Reaction

☞ Two reactions are normally considered, namely $^{14}N(n,p)^{14}C$ and $^{1}H(n,r)^{2}H$ reactions.

☞ For the $^{14}N(n,p)^{14}C$ reaction, the dose is given by

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}\ \text{J/MeV}}{1\ \text{J/kg} \cdot \text{Gy}},$$

where
ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

Neutron Induced Reactions

$$^{1}_{0}n + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{0}\gamma$$

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22MeV released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Thermal Neutron Dose from the $^1H(n, \gamma)^2H$ Reaction

☞ For the $^1H(n, \gamma)^2H$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

$$A = \phi N_H \sigma_H \text{ "Bq"/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

☞ The resulting gamma ray dose is illustrated with the following example.

Example 6.17

What is the absorbed dose rate to a 70-kg person from a whole body exposure to a mean thermal flux of 10,000 neutrons per cm^2 per second?

The dose rate due to the n, p reaction is calculated from Eq. (6.105)

$$\dot{D}_{np} = 1 \times 10^4 \times 1.49 \times 10^{24} \times 1.75 \times 10^{-24} \times 0.63 \times 1.6 \times 10^{-13}$$

$$= 2.628 \times 10^{-9} \text{ Gy/s} \quad (2.628 \times 10^{-7} \text{ rad/s}),$$

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13} \text{ J/MeV}}{1 \text{ J/kg} \cdot \text{Gy}},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

or

$$\dot{D}_{np} = 9.461\ \mu\text{Gy/h} \quad (0.95 \text{ mrad/h}).$$

The autointegral gamma-ray dose rate is calculated with Eq. (6.82). The gamma-ray "activity," from Eq. (6.106) is

$$A = 10^4 \text{ cm}^2 \text{ s}^{-1} \times 5.98 \times 10^{25} \text{ atoms/kg} \times 3.3 \times 10^{-25} \text{ cm}^2/\text{atom}$$

$$= 1.973 \times 10^5 \text{ "Bq"/kg}.$$

$$A = \phi N_H \sigma_H \text{ "Bq"/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

The dose rate from this uniformly distributed gamma ray activity is calculated from Eq. (6.82):

$$\dot{D}_H = A \cdot E_r \cdot \varphi = 1.973\times10^5\, Bq/kg \cdot 2.23\text{MeV} \cdot 1.6\times10^{-16}\, J/MeV \cdot 0.278$$
$$= 1.19\times10^{-11}\, Gy/\text{sec} = 6.89\times10^{-2}\, \mu Gy/\text{h}$$

The absorbed fraction, φ, for the 2.23-MeV gamma ray is found, by interpolating in Table 6.8 between the 2.000- and 4.000-MeV values, to be 0.278

Accumulated Dose from Internally Deposited Radioactive Sources

- The MIRD method and absorbed fraction.
- The effective decay constant of a given form of radioactivity in the human body
- Please pay attention to the example on evaluating the dose delivered by internally deposited radioactivity.

^{225}Ac-PSMA-617 for PSMA-Targeted α-Radiation Therapy of Metastatic Castration-Resistant Prostate Cancer

Clemens Kratochwil*[1], Frank Bruchertseifer*[2], Frederik L. Giesel[1], Mirjam Weis[2], Frederik A. Verburg[3], Felix Mottaghy[3], Klaus Kopka[4], Christos Apostolidis[2], Uwe Haberkorn[1], and Alfred Morgenstern[2]

[1]Department of Nuclear Medicine, University Hospital Heidelberg, Heidelberg, Germany; [2]European Commission, Joint Research Centre, Institute for Transuranium Elements, Karlsruhe, Germany; [3]Department of Nuclear Medicine, RWTH University Hospital Aachen, Aachen, Germany; and [4]Division of Radiopharmaceutical Chemistry, German Cancer Research Center, Heidelberg, Germany

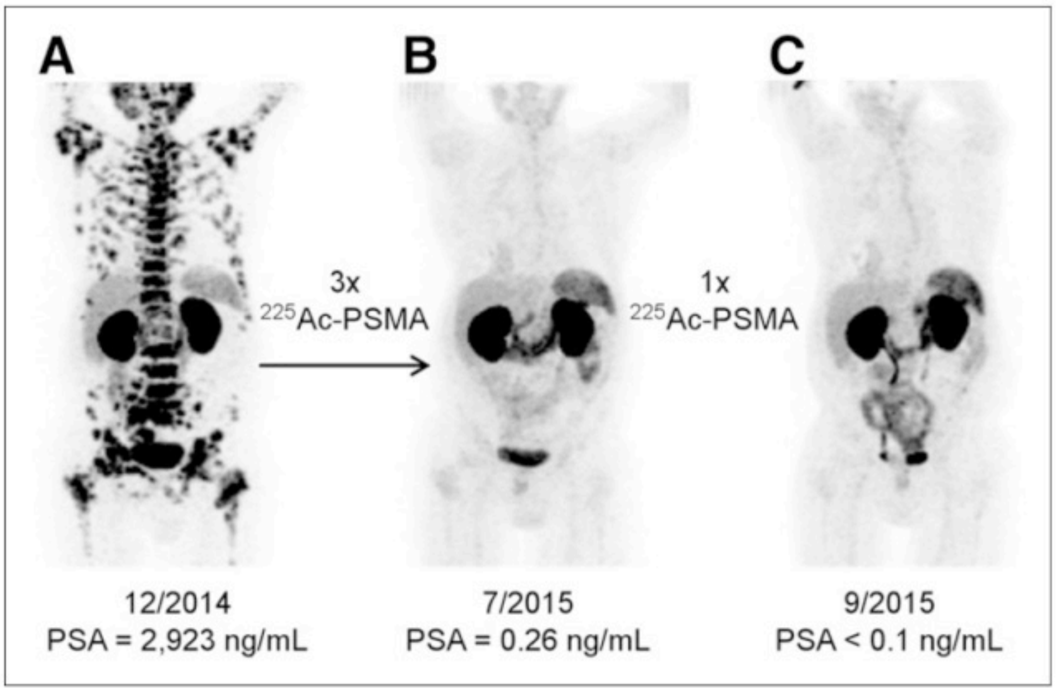


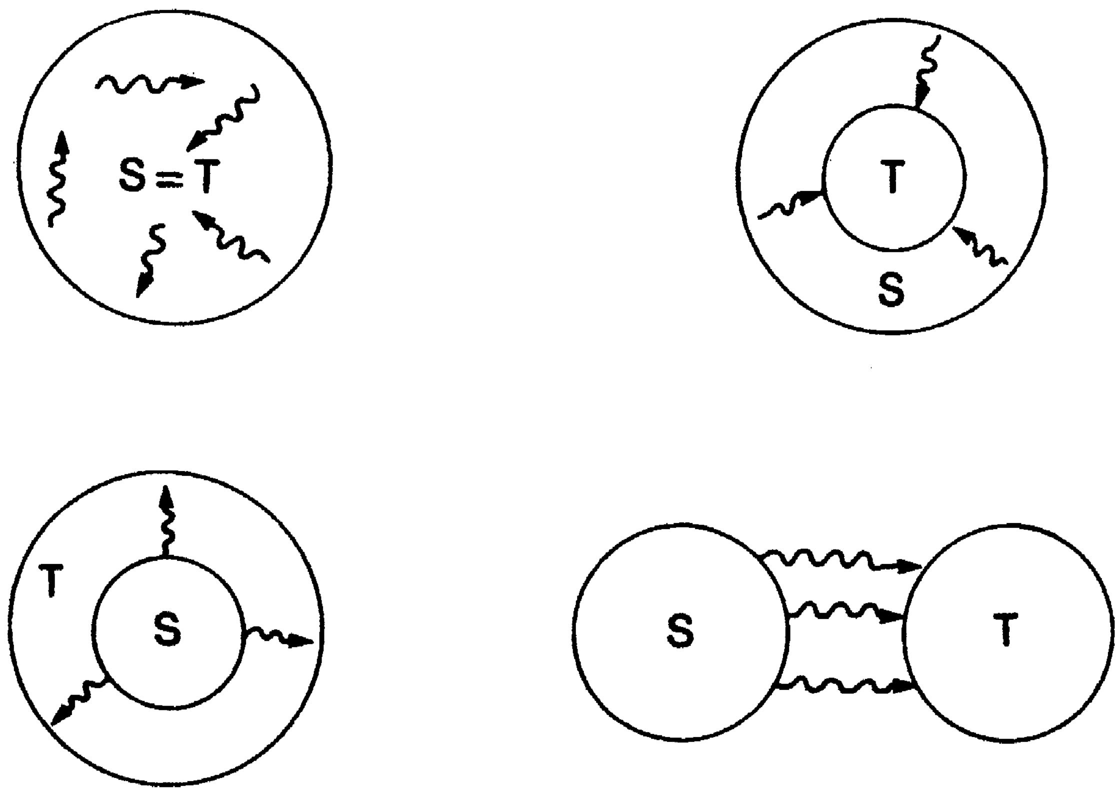
FIGURE 1. ^{68}Ga-PSMA-11 PET/CT scans of patient A. Pretherapeutic tumor spread (A), restaging 2 mo after third cycle of ^{225}Ac-PSMA-617 (B), and restaging 2 mo after one additional consolidation therapy (C).

Partial Absorption of Gamma-Ray Energy – MIRD Method

- To account for the partial absorption of gamma-ray energy in organs and tissues, the Medical Internal Radiation Dose (MIRD) Committee of the Society of Nuclear Medicine (SNM) has developed a formal system for calculating the dose to a target organ or tissue from a source organ containing a uniformly distributed radioisotope.
- The absorption fraction – the fraction of the energy radiated by the source organ, which is absorbed by the target organ.

Partial Absorption of Gamma-Ray Energy – MIRD Method

- The absorption fraction – the fraction of the energy radiated by the source organ and absorbed by the target organ.



Partial Absorption of Gamma Ray Energy – MIRD Method

☞ The absorbed fraction are calculated by the application of Monte Carlo methods.

$$\text{Absorbed fraction} = \varphi = \frac{\text{energy absorbed by target}}{\text{energy emitted by source}}$$

☞ Standard data on the absorbed dose for photons of various energies for point isotropic sources and for uniformly distributed sources are published by MIRD in several Supplements to the Journal of Nuclear Medicine

TABLE 6-8. Absorbed Fractions (and Coefficients of Variation), Gamma Emitter Uniformly Distributed Throughout the Body[a] (Continued)

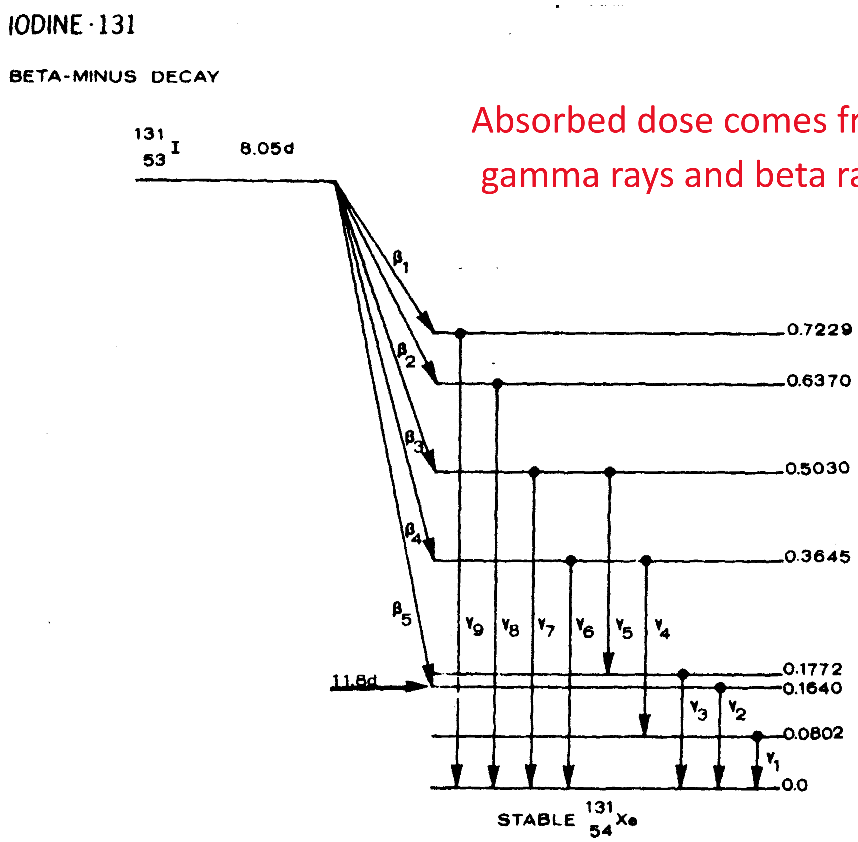
	PHOTON ENERGY (MeV)												
	0.200		0.500		1.000		1.500		2.000		4.000		
TARGET ORGAN	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	**Target Organ**
Adrenals	0.352E−04	36.	0.138E−03	35.	0.100E−03	42.	0.107E−03	43.	0.114E−03	43.			Adrenals
Bladder	0.327E−02	5.0	0.341E−02	6.6	0.274E−02	8.3	0.291E−02	8.4	0.231E−02	9.6	0.147E−02	12.	Bladder
GI (stom)	0.218E−02	7.0	0.258E−02	7.7	0.181E−02	9.8	0.199E−02	10.	0.212E−02	10.	0.119E−02	14.	GI (stom)
GI (SI)	0.106−01	3.4	0.114E−01	3.8	0.109E−01	4.2	0.915E−02	4.8	0.820E−02	5.2	0.409E−02	7.3	GI (SI)
GI (ULI)	0.256E−02	6.3	0.306−02	7.0	0.228E−02	8.9	0.209E−02	9.4	0.197E−02	10.	0.160E−02	12.	GI (ULI)
GI (LLI)	0.151E−02	7.6	0.184E−02	8.8	0.178E−02	9.7	0.181E−02	11.	0.157E−02	12.	0.673E−03	18.	GI (LLI)
Heart	0.337E−02	5.8	0.372E−02	6.6	0.301E−02	8.1	0.345E−02	7.8	0.312E−02	8.3	0.145−02	13.	Heart
Kidneys	0.171E−02	7.4	0.142E−02	9.7	0.161E−02	10.	0.152E−0.2	11.	0.154E−02	12.	0.904E−03	16.	Kidneys
Liver	0.111−01	3.4	0.101E−01	4.1	0.896E−02	4.7	0.912E−02	4.9	0.847E−02	5.1	0.560E−02	6.4	Liver
Lungs	0.507E−02	4.3	0.496E−02	5.2	0.466E-02	6.1	0.466E−02	6.5	0.427E−02	6.9	0.568E−02	6.4	Lungs
Marrow	0.221E−01	1.5	0.194E−01	1.0	0.182E-01	2.0	0.164E−01	2.2	0.156E−01	2.3	0.969E−02	3.0	Marrow
Pancreas	0.444E−03	14.	0.382E−03	17.	0.534E−03	19.	0.348E−03	22.	0.358E−03	24.	0.142−03	39.	Pancreas
Sk. (rib)	0.505E−02	4.1	0.435E−02	5.6	0.421E−02	6.3	0.405E−02	7.0	0.350E−02	7.7	0.338E−02	8.0	Sk. (rib)
Sk. (pelvis)	0.668E−02	3.9	0.569E−02	5.0	0.562E−02	5.7	0.511E−02	6.3	0.422E−02	7.0	0.256E−02	9.3	Sk. (pelvis)
Sk. (spine)	0.910E−02	3.6	0.763E−02	4.5	0.751E−02	5.1	0.610E−02	5.7	0.606E−02	5.9	0.341E−02	8.1	Sk. (spine)
Sk. (skull)	0.277E−02	6.3	0.304E−02	7.2	0.280E−02	8.0	0.254E−02	9.0	0.292E−02	8.8	0.224E−02	10.	Sk. (skull)
Skeleton (total)	0.550E−01	1.4	0.488E−01	1.7	0.456E−01	2.0	0.413E−01	2.2	0.396E−01	2.3	0.252E−01	3.0	Skeleton (total)
Skin	0.677E−02	3.5	0.757E−02	4.2	0.745E−02	4.8	0.759E−02	5.0	0.664E−02	5.5	0.123E−01	4.3	Skin
Spleen	0.798E−03	11.	0.116E−02	11.	0.914E−03	14.	0.903E−03	16.	0.740E−03	17.	0.368E−03	24.	Spleen
Thyroid	0.418E−04	42.							0.810E−04	46.			Thyroid
Uterus	0.408E−03	15.	0.473E−03	16.	0.517E−03	18.	0.323E−03	23.	0.364E−03	25.	0.238−03	33.	Uterus
Trunk	0.223	0.81	0.225	0.84	0.210	0.92	0.198	0.99	0.186	1.0	0.156	1.2	Trunk
Legs	0.102	1.3	0.101	1.4	0.965E−01	1.5	0.917E−01	1.6	0.846E−01	1.6	0.710E−01	1.8	Legs
Head	0.134E−01	3.2	0.147E−01	3.5	0.145E−01	3.8	0.130E−01	4.1	0.139E−01	4.1	0.127E−01	4.4	Head
Total body	0.338	0.57	0.340	0.60	0.321	0.67	0.302	0.73	0.284	0.77	0.240	0.90	Total body

[a]The digits following the symbol E indicate the powers of 10 by which each number is to be multiplied; A blank in the table indicates that the coefficient of variation was greater than 50%; Total body = head + trunk + legs.

MIRD Method – An Example (Cember, p203-206)

Evaluate the dose rate to a 0.6kg sphere made of tissue-equivalent material in which 1MBq of I-131 is uniformly distributed.

Absorbed dose comes from gamma rays and beta rays



INPUT DATA

Radiation	%/dis-integration	Transition energy (MeV)	Other nuclear parameters
Beta-1	1.6	0.25 *	Allowed
Beta-2	6.9	0.33 *	Allowed
Beta-3	0.5	0.47 *	Allowed
Beta-4	90.4	0.606 *	Allowed
Beta-5	0.6	0.81 *	First forbidden unique
Gamma-1	5.06	0.0802	M1, $\alpha_K = 1.7$, $\alpha_L = 0.17$
Gamma-2	0.6	0.1640	M4, $\alpha_K = 29$, K/L = 2.3
Gamma-3	0.18	0.1772	E2, $\alpha_K = 0.189$ (T), K/L = 4.0
Gamma-4	5.06	0.2843	E2, $\alpha_K = 0.052$, K/(L + M) = 4.0
Gamma-5	0.18	0.3258	M1, $\alpha_K = 0.0285$ (T), K/L = 6.0
Gamma-6	85.3	0.3645	E2 + 2% M1, $\alpha_K = 0.02$, K/L = 6.0
Gamma-7	0.32	0.5030	E2, $\alpha_K = 0.00749$ (T), $\alpha_L = 0.0011$ (T)
Gamma-8	6.9	0.6370	E2, $\alpha_K = 0.0039$, $\alpha_L = 0.000563$ (T)
Gamma-9	1.6	0.7229	M1, $\alpha_K = 0.004$, $\alpha_L = 0.000515$ (T)

Ref.: Lederer, C. M. et al, *Table of Isotopes*, 6th ed.
* Endpoint energy (MeV). (T) = Theoretical value.

FIGURE 6.11. Transformation scheme and input and output data for ^{131}I dosimetry. (From L. T. Dillman: Radionuclide Decay Schemes and Nuclear Parameters for Use in Radiation Dose Estimation. *J. Nuclear Medicine*, Vol. 10, Supplement No. 2, MIRD Pamphlet No. 4, 1969. By permission.)

MIRD Method – An Example

☞ The absorbed dose is the sum of beta dose and gamma ray dose.

☞ The absorbed gamma-ray energy per I-131 transformation is given by

$$E_e(\gamma) = \sum_i E_{\gamma i} \times n_i \times \varphi_i,$$

what is absorption fraction?

where $E_e(\gamma)$ = absorbed gamma ray energy, MeV/transformation,
$E_{\gamma i}$ = energy of the ith gamma photon, MeV,
n_i = number of photons of ith energy per transformation,
φ_i = absorbed fraction of the ith photon's energy.

Photon energy, $E_{\gamma i}$, MeV	×	Photons per transformation, n_i	×	Absorbed fraction, φ	=	Absorbed energy, MeV/t
0.723		0.016		0.123		0.0014
0.637		0.069		0.124		0.0055
0.503		0.003		0.123		0.0002
0.326		0.002		0.120		0.0001
0.177		0.002		0.112		0.0000
0.365		0.853		0.122		0.0380
0.284		0.051		0.118		0.0017
0.080		0.051		0.111		0.0005
0.164		0.006		0.111		0.0001
						$E_e(\gamma)$ = 0.0474 MeV/t

MIRD Method – An Example

- Since the absorption fraction is 1 for internally distributed beta sources, the absorbed beta energy per I-131 transformation is given by

$$E_e(\beta) = \sum \overline{E}_{\beta i} \times n_{\beta i}$$
$$= (0.0701 \times 0.016) + (0.0955 \times 0.069) + (0.1428 \times 0.005)$$
$$+ (0.1917 \times 0.904) + (0.285 \times 0.006) = 0.183\ \text{MeV/t}.$$

- So the total energy absorbed by the 0.6kg tissue-equivalent sphere per I-131 transformation is

$$E_e = E_e(\gamma) + E_e(\beta)$$
$$= 0.047 + 0.183 = 0.230\ \text{MeV/t}.$$

MIRD Method – An Example

☞ Therefore, the daily dose rate to the 0.6kg tissue-equivalent mass is

$$\dot{D} = \frac{q \text{ Bq} \times 1 \text{ tps/Bq} \times E_e \text{ MeV/t} \times 1.6 \times 10^{-13} \text{ J/MeV} \times 8.64 \times 10^4 \text{ s/day}}{m \text{ kg} \times 1 \frac{\text{J}}{\text{kg}} \Big/ \text{Gy}}.$$

If we substitute $q = 1 \times 10^6$ Bq,
$E_e = 0.230$ MeV/t,
$m = 0.6$ kg

we find the dose rate to be

$$\dot{D} = 5.30 \times 10^{-3} \text{ Gy/day}$$

Internally Deposited Radioisotope (II)
Effective Half-Life

☞ The total dose absorbed by an organ during any given time interval after the deposition of the isotope in the organ may be calculated by integrating the dose rate over the required time interval. For this purpose, two factors must be considered:

In situ radioactive decay of the isotope → exponential decay

Biological elimination of the isotope → follows the first-order kinetics → exponential decay

☞ The equation for the quantity of radioisotope within an organ at any given time after the deposition of a quantity Q_0 is given by

$$Q = (Q_0 e^{-\lambda_R t})(e^{-\lambda_B t})$$

where λ_R is the radioactive decay constant and λ_B is the biological elimination constant.

Internally Deposited Radioisotope (II)
Effective Half-Life

☞ One can define an effective elimination constant $\lambda_E = \lambda_R + \lambda_B$ that represents the combined effects of these two decay processes,

$$Q = Q_0 e^{-\lambda_E t}$$

and

$$T_E = \frac{0.693}{\lambda_E}$$

is called the effective half-life.

Internally Deposited Radioisotope (III)
Accumulated Dose and Dose Commitment

☞ Given the initial dose rate $\dot{D}_0$, the accumulated dose received during a time interval t after the deposition of the isotope is

$$D = \dot{D}_0 \int_0^t e^{-\lambda_E t} dt = \frac{\dot{D}_0}{\lambda_E}(1 - e^{-\lambda_E t})$$

For an infinitely long time—that is, when the isotope is completely gone-

$$D = \frac{\dot{D}_0}{\lambda_E}$$

☞ For practical purpose, an infinitely long time corresponding to about 6 half-lives. The total dose received from complete decay is called the dose commitment.

Internally Deposited Radioisotope (III)
Total Dose: Dose Commitment

Generally, if there is more than one compartment, the body burden at any time t after deposition of q_0 units of a radionuclide is given by

$$q(t) = f_1 q_0 e^{-\lambda_1 t} + f_2 q_0 e^{-\lambda_2 t} + \cdots + f_n q_0 e^{-\lambda_n t}, \qquad \textbf{(6.60)}$$

where $f_1, f_2, \ldots, f_n$ = fraction of the total activity deposited in compartments 1, 2, . . . , n, and $\lambda_1, \lambda_2, \ldots, \lambda_n$ = effective clearance rates for compartments 1, 2, . . . , n.

Since the activity in each compartment contributes to the dose to that organ or tissue, Eq. (6.57) becomes, for the multicompartment case,

$$D = \frac{\dot{D}_{10}}{\lambda_{1E}}\left(1 - e^{-\lambda_{1E} t}\right) + \frac{\dot{D}_{20}}{\lambda_{2E}}\left(1 - e^{-\lambda_{2E} t}\right) + \cdots + \frac{\dot{D}_{n0}}{\lambda_{nE}}\left(1 - e^{-\lambda_{nE} t}\right), \qquad \textbf{(6.61)}$$

and when the radionuclide has completely been eliminated, Eq. (6.61) reduces to

$$D(t) = \frac{\dot{D}_{10}}{\lambda_{1E}} + \frac{\dot{D}_{20}}{\lambda_{2E}} + \cdots + \frac{\dot{D}_{n0}}{\lambda_{nE}}. \qquad \textbf{(6.62)}$$

Overall dose commitment

6.18 A child drinks 1 liter of milk per day containing ^{131}I at a mean concentration of 33.3 Bq (900 pCi) per liter over a period of 30 days. Assuming that the child has no other intake of ^{131}I, calculate the dose to the thyroid at the end of the 30 days ingestion period, and the dose commitment.

MIRD Method – Another Example, Cember, 6.18

Step 1: Derive the effective half-live of I-131

First calculate the effective half life of the I–131 in the body, use equation 6.54:

$T_R = 8.05$ d
$T_B = 138$ d (ICRP 28)

$$T_E = \frac{T_R \times T_B}{T_R + T_B} = \frac{8.05\text{ d} \times 138\text{ d}}{8.05\text{ d} + 138\text{ d}} = 7.6\text{ d effective half life of } ^{131}\text{I in body.}$$

Converting to effective elimination constant, using equation 6.52:

$$\lambda = \frac{0.693}{T} = \frac{0.693}{7.6\text{ d}} = 0.091\text{ d}^{-1}$$

Note that we will use λ to symbolize the effective decay constant in the following derivations.

Step 2: Derive the absorbed dose to the thyroid per I-131 decay

The average energy of each ^{131}I beta particle is found (Figure 6.11), and the yield from each decay is also tabulated:

Energy, MeV/t	Yield, f
0.0701	0.016
0.0955	0.069
0.1428	0.005
0.1917	0.904
0.2856	0.006

The mean β energy/transformation is:

$$\bar{E}_e(\beta) = \Sigma \bar{E} \times f_{\beta i} = (0.0701 \times 0.016) + (0.0955 \times 0.069) + (0.1428 \times 0.005) + (0.1917 \times 0.904) + (0.2856 \times 0.006)$$

$$\bar{E}_e(\beta) = 0.184 \text{ MeV/t}$$

Beta energy absorbed in the thyroid per I-131 decay

Absorption fraction

MeV	*f*	Spec. Abs	MeV/t
0.723	0.016	0.00166	1.92E-05
0.637	0.069	0.00166	7.3E-05
0.503	0.003	0.00166	2.5E-06
0.326	0.002	0.00155	1.01E-06
0.177	0.002	0.00155	5.49E-07
0.365	0.853	0.00155	0.000483
0.284	0.051	0.00155	2.25E-05
0.08	0.051	0.0429	0.000175
0.164	0.006	0.00155	1.53E-06
		Sum	0.000778

Gamma ray energy absorbed in the thyroid per I-131 decay

alculating the contribution due to the γ, the specific absorbed fraction is found in ppendix 4 for an adult. Assume all the ^{131}I is deposited in the thyroid. So for this ase, the contribution from the γ is not significant and can be ignored, especially nce the child's thyroid is small (~2-5g, ICRP 53).

Step 3: derive the I-131 activity in the thyroid as a function of t

The intake is $q = 33.3$ Bq/day, however, only one–third of the iodine is directly deposited in the thyroid (ICRP 30).

$$K = 33.3\frac{\text{Bq}}{\text{d}} \times \frac{1 \text{ deposited}}{3 \text{ ingested}} = 11.1\frac{\text{Bq}}{\text{d}}$$ ← rate of intake

Some of the iodine is eliminated daily, so find the concentration in the thyroid at any time:

$$\frac{dq}{dt} = \text{deposition} - \text{disappearance}$$

$$\frac{dq}{dt} = K - \lambda \quad q$$

Separating the variables, we have

$$\int_0^q \frac{dq}{(K - \lambda \quad q)} = \int_0^t dt$$

Note that

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln(ax+b) + C,$$

and consider the following conditions:

$$\begin{cases} a = -\lambda_{eff} \\ b = K \\ q(t=0) = 0 \end{cases}$$

After integration, and solving for q as a function of t;

$$q(t) = \frac{K}{\lambda}\left(1 - e^{-\lambda t}\right) \quad \Rightarrow$$

If the uptake of I-131 continues, the dose rate as a function of time is

$$\dot{D}(t) = \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t}\right)$$
$$= \dot{D}_\infty \cdot \left(1 - \mathrm{e}^{-\lambda t}\right)$$

As $t \to \infty$, q approaches

$$q_\infty = \frac{K}{\lambda} = \frac{11.1 \frac{\text{Bq}}{\text{day}}}{0.091 \text{ day}} = 122 \text{ Bq} \quad \Rightarrow$$

If the uptake of I-131 continues, the saturation dose rate is given by

$$\dot{D}_\infty = \frac{\bar{E}}{m} \cdot \frac{K}{\lambda}$$

$m = 20$ g (for adult, from appendix C), for a child, assume 10% of adult mass (10 CFR 20), 2 g.

Step 4: Derive the accumulated dose received within the first 30 days.

If the uptake continues, the accumulated dose received by a given time t is

$$D = \int_0^t \dot{D}(t') \cdot dt'$$

$$\int_0^t (1 - e^{-\lambda t}) dt = \left[t - \left(-\frac{1}{\lambda}\right) e^{-\lambda t}\right]\Big|_0^t = \left[t + \frac{1}{\lambda} e^{-\lambda t}\right] - \left[0 + \frac{1}{\lambda}\right] = \left[t + \frac{1}{\lambda}(e^{-\lambda t} - 1)\right]$$

where

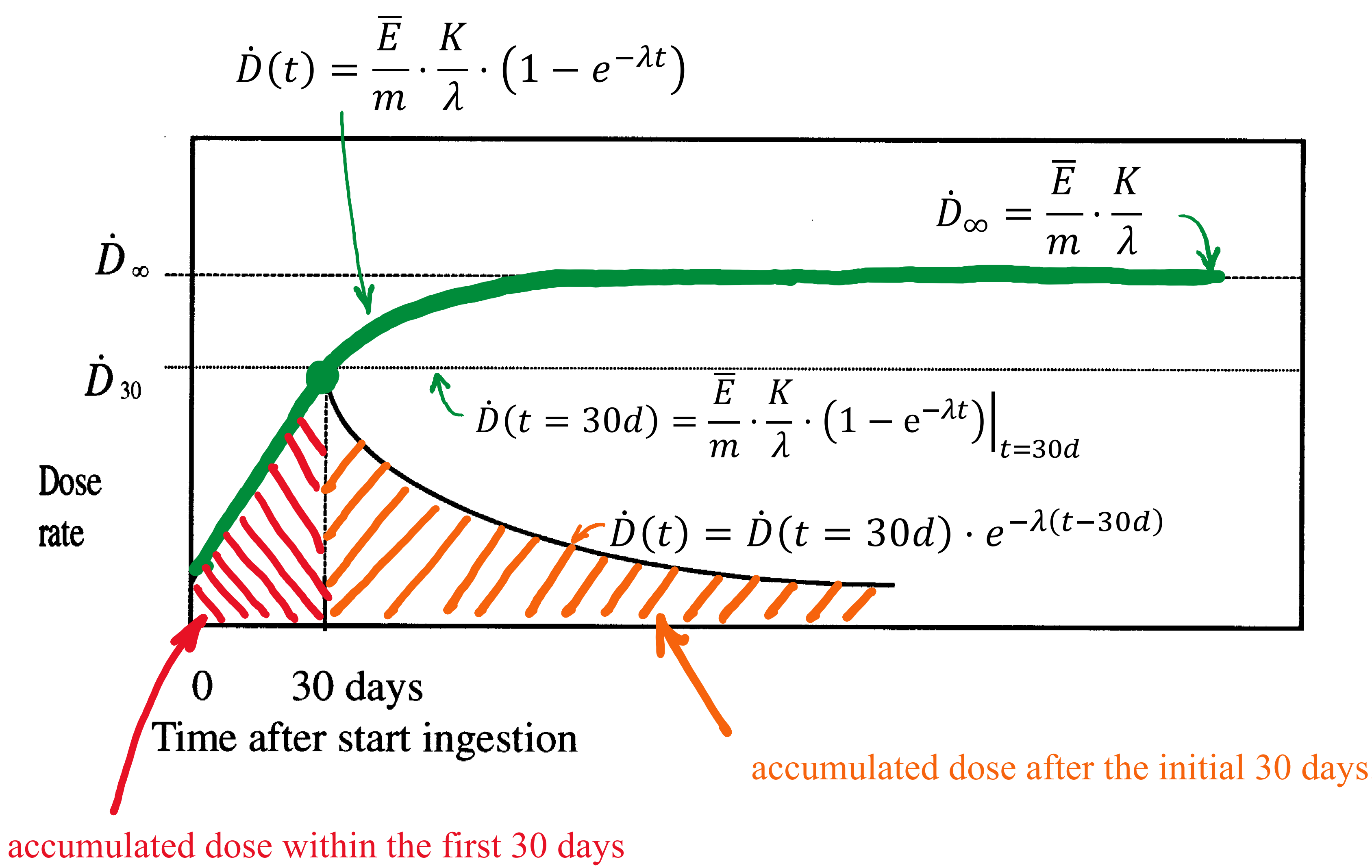
$$\dot{D}(t) = \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t}\right),$$

and $\bar{E}$ is the mean absorbed energy in the organ per decay of I-131 in the thyroid.

So the accumulated dose is given by

$$D = \int_0^t \dot{D}(t') \cdot dt' = \int_0^t \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t'}\right) \cdot dt'$$

$$= \frac{\bar{E}}{m} \cdot \frac{K}{\lambda} \cdot \left[t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right)\right]$$



$\dot{D}(t) = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda} \cdot (1 - e^{-\lambda t})$
$\dot{D}_\infty = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda}$
$\dot{D}_\infty$
$\dot{D}_{30}$
$\dot{D}(t = 30d) = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda} \cdot (1 - e^{-\lambda t})\Big|_{t=30d}$
Dose rate
$\dot{D}(t) = \dot{D}(t = 30d) \cdot e^{-\lambda(t-30d)}$
0
30 days
Time after start ingestion
accumulated dose after the initial 30 days.
accumulated dose within the first 30 days

Step 4: Derive the accumulated dose received within the first 30 days (continued)

Using the following equation for accumulated dose till time t,

$$D = \dot{D}_{\infty}\left[t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right)\right] \quad and \quad \dot{D}_{\infty} = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda},$$

the accumulated dose at 30 days is:

$$\dot{D}_{\infty} = 0.155 \text{ mGy/day}$$
$$\lambda = 0.091 \text{ d}^{-1}$$
$$t = 30 \text{ d}$$

$$D = 0.155\frac{\text{mGy}}{\text{d}} \times \left[30 \text{ d} + \frac{1}{0.091\text{d}^{-1}} \times \left(e^{-0.091\times(30)} - 1\right)\right] = 3 \text{ mGy}$$

3 mGy is the accumulated dose at the end of the 30 day period.

Internally Deposited Radioisotope: Total Dose or Dose Commitment (Revisited)

☞ The total dose received during a time interval t after the deposition of the isotope is

$$D = \dot{D}_0 \int_0^t e^{-\lambda\, t} dt = \frac{\dot{D}_0}{\lambda_E}(1 - e^{-\lambda\, t})$$

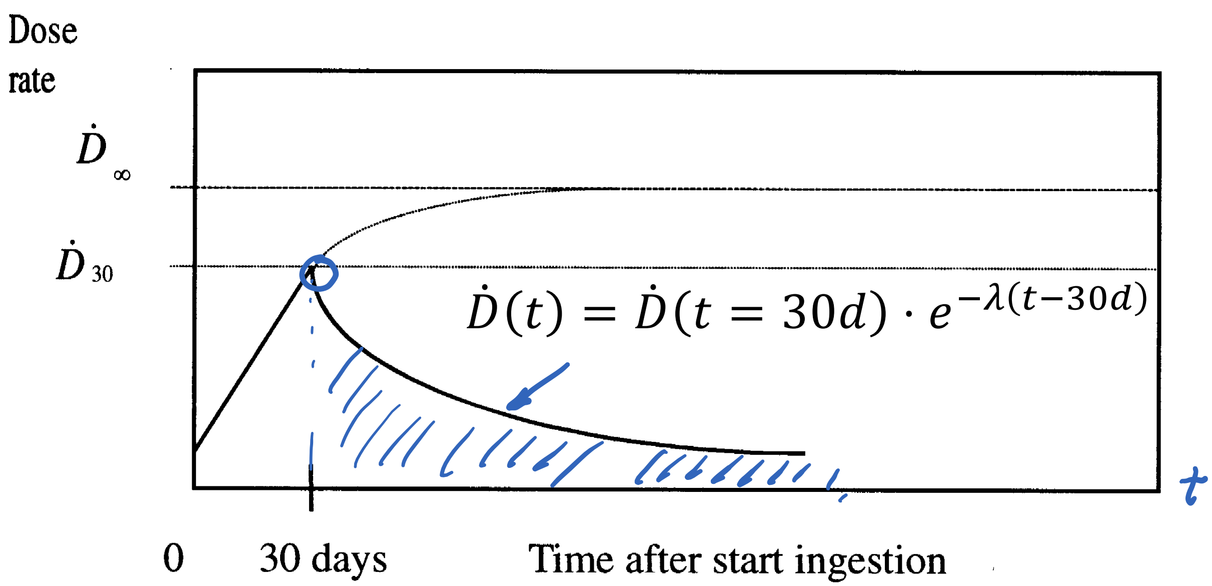
For an infinitely long time—that is, when the isotope is completely gone-

$$D = \frac{\dot{D}_0}{\lambda}$$

☞ For practical purpose, an infinitely long time corresponding to about 6 half-lives. The total dose received from complete decay is called the dose commitment.

Step 5: Derive the total dose (dose commitment) received after t=30d.

The dose commitment is the sum of the dose accumulated during intake, and then during elimination (washout).

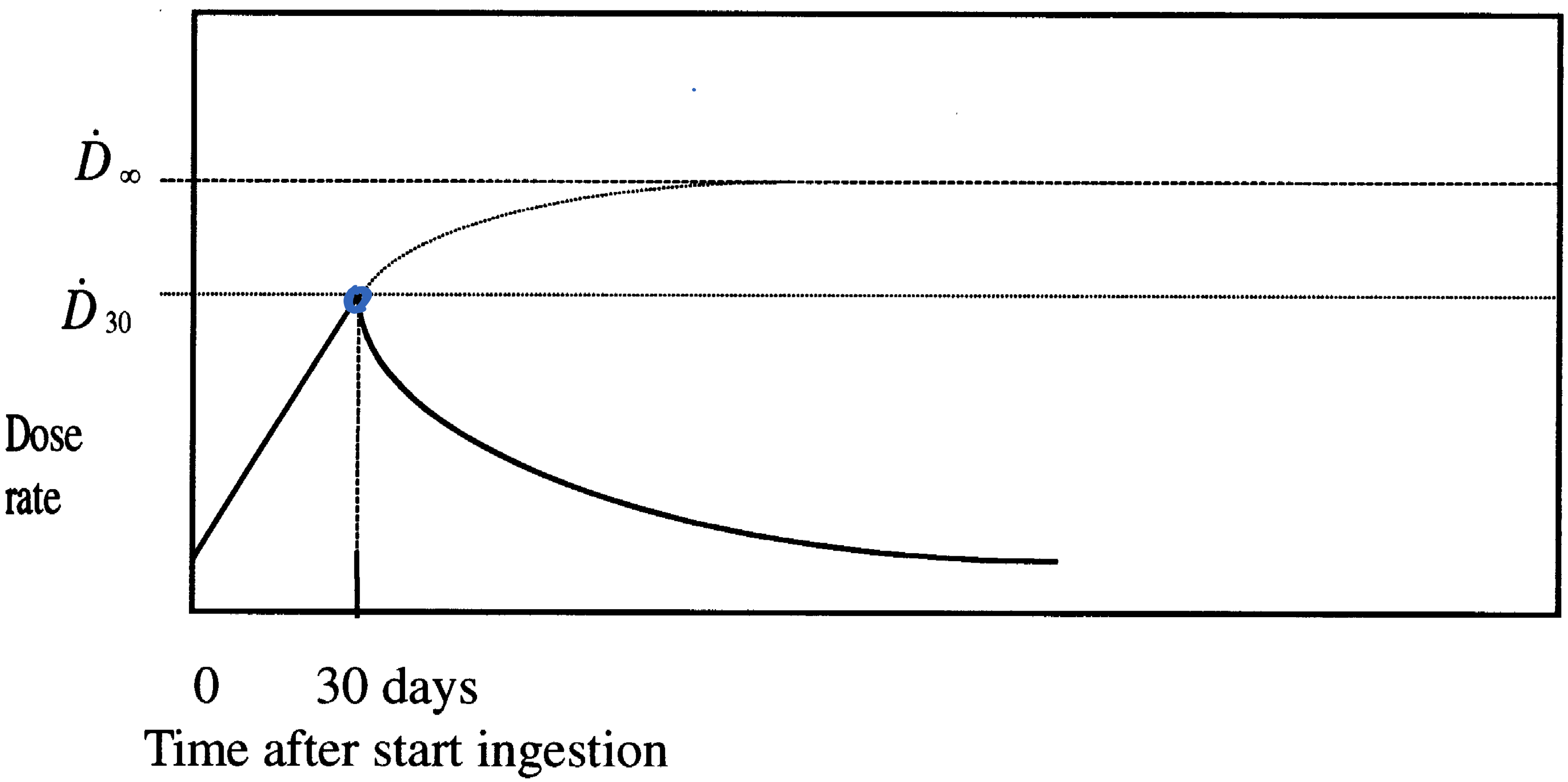


Find the dose rate at $t = 30$ days

$$\dot{D}_{30} = \dot{D}_{\infty}(1 - e^{-\lambda t}) = 0.155\frac{\text{mGy}}{\text{d}} \times \left(1 - e^{-0.091\times(30)}\right) = 0.145\ \frac{\text{mGy}}{\text{day}}$$

$$D = \frac{\dot{D}_{30}}{\lambda} = \frac{0.145\frac{\text{mGy}}{\text{d}}}{0.091\frac{1}{\text{d}}} = 1.59\ \text{mGy}$$ is the dose after ingestion stops.

(Final) Step 6: Derive the total dose (dose commitment) from the initial intake of I-131



The total dose, from the time intake started to the end of the first 30 days, plus the dose after the intake stopped will be;

3 mGy + 1.6 mGy = 4.6 mGy total dose to the childs thyroid.