Limits of Detectability

For a counting system, it is useful to set a detection limit. That is, the amount of activity can be detected reliably.

The basic procedure could be

(1) Setting a certain confidence level – the probability that a decision (on whether or not a source is present) is correct.

(2) Define a quantity based on which the decision can be made. In the source counting case, it is the net count per unit time

\[ n_S = n_g - n_b \]

where

\[ n_S : \text{net counts} \]

\[ n_g : \text{gross counts} \]

\[ n_b : \text{background counts} \]

(3) Finding a critical level, \( L_c \). If \( n_s \) exceeds \( L_c \), we assume source activity is present, otherwise we assume that the source does not contain activity.
False Positive and False Negative Errors

Due to the statistical fluctuation on the counts measured within a given time $t$, there will be

(1) many instances in which a positive $n_S$ is above the critical level even for samples with no activity, which leads to the false positive.

(2) and similarly, measured counts is lower than the critical level even when the source contains non-zero activity, which leads to the false negative.

Figure 3.14 The distributions expected for the net counts $N_S$ for the cases of (a) no activity present, and (b) a real activity present. $L_C$ represents the critical level or “trigger point” of the counting system.
**False Positive and False Negative Errors**

<table>
<thead>
<tr>
<th>Condition (as determined by &quot;Gold standard&quot;)</th>
<th>Test outcome positive</th>
<th>Test outcome negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condition positive</td>
<td>True positive</td>
<td>False positive</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Type I error)</td>
</tr>
<tr>
<td>False negative (Type II error)</td>
<td>False negative</td>
<td>True negative</td>
</tr>
</tbody>
</table>

- **Sensitivity** (also called the true positive rate, or the recall rate in some fields) measures the proportion of actual positives which are correctly identified as such (e.g., the percentage of sick people who are correctly identified as having the condition).
- **Specificity** (sometimes called the true negative rate) measures the proportion of negatives that are correctly identified as such.
Limits of Detectability

In statistics, a receiver operating characteristic (ROC), or ROC curve, is a graphical plot that illustrates the performance of a binary classifier system as its discrimination threshold is varied. The curve is created by plotting the true positive rate against the false positive rate at various threshold settings.

Typical ROC curves

<table>
<thead>
<tr>
<th>Test outcome</th>
<th>Condition (as determined by &quot;Gold standard&quot;)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>True positive, False positive (Type I error)</td>
</tr>
<tr>
<td>Negative</td>
<td>False negative (Type II error), True negative</td>
</tr>
</tbody>
</table>

\[
\text{Sensitivity} = \frac{\Sigma \text{True positive}}{\Sigma \text{Condition positive}} \\
\text{Specificity} = \frac{\Sigma \text{True negative}}{\Sigma \text{Condition negative}} \\
\text{Accuracy} = \frac{\Sigma \text{True positive} + \Sigma \text{True negative}}{\Sigma \text{Condition positive} + \Sigma \text{Condition negative}} \\
\text{Precision} = \frac{\Sigma \text{True positive}}{\Sigma \text{Test outcome positive}} \\
\text{Negative predictive value} = \frac{\Sigma \text{True negative}}{\Sigma \text{Test outcome negative}}
\]
False Positive and False Negative Errors

Type I error (false positive) and Type II error (false negative) are two types of errors that carry different implications.

False positive $\rightarrow$ minimum significant measured activity
False negative $\rightarrow$ minimum detectable true activity
False Positive and False Negative Errors

Type I error (false positive) and Type II error (false negative) are two types of errors that carry different implications.

False positive $\leftrightarrow$ minimum significant measured activity
False negative $\leftrightarrow$ minimum detectable true activity
False Positive Rate and Minimum Significant Net Count Rate – An Example

Example
A sample, counted for 10 min, registers 530 gross counts. A 30-min background reading gives 1500 counts. (a) Does the sample have activity? (b) Without changing the counting times, what minimum number of gross counts can be used as a decision level such that the risk of making a type-I error is no greater than 0.050?

Turner, pp. 315-316
Solution

(a) The numbers of gross and background counts are \( n_g = 530 \) and \( n_b = 1500 \); the respective counting times are \( t_g = 10 \) min and \( t_b = 30 \) min. The gross and background count rates are \( r_g = n_g / t_g = 53 \) cpm and \( r_b = n_b / t_b = 50 \) cpm, giving a net count rate \( r_n = r_g - r_b = 3 \) cpm. The question of whether activity is present cannot be answered in an absolute sense from these measurements. The observed net rate could occur randomly with or without activity in the sample. We can, however, compute the probability that the result would occur randomly when we assume that the sample has no activity. To do this, we compare the net count rate with its estimated standard deviation \( \sigma_{nr} \), given by Eq. (11.51):

\[
\sigma_{nr} = \sqrt{\frac{r_g}{t_g} + \frac{r_b}{t_b}} = \sqrt{\frac{53}{10} + \frac{50}{30}} = 2.64 \text{ cpm.} \tag{11.64}
\]

The observed net rate differs from 0 by \( 3/2.64 = 1.14 \) standard deviations. As found in Table 11.1, the area under the standard normal curve to the right of this value is 0.127. Assuming that the activity \( A \) is zero, as shown in Fig. 11.4, we conclude that an observation giving a net count rate greater than the observed \( r_n = 1.14\sigma_{nr} = 3 \) cpm would occur randomly with a probability of 0.127. This single set of measurements, gross and background, is thus consistent with the conclusion that the sample likely contains little or no activity. However, one does not know where the bell-shaped curve in Fig. 11.4 should be centered. Based on this single measurement, the most likely place is \( r_n = 3 \) cpm, with the sample activity corresponding to that value of the net count rate.
FIGURE 11.4. Probability density $P_n(r_n)$ for measurement of net count rate $r_n$ when no activity is present. See example in text. (Courtesy James S. Bogard.)
Two Potential Decisions from the Previous Measurements

Possible decision #1: If there is no activity in the source, there will be an 87% of chance of observing less than or equal to 3 cpm, the fact that we measured 3 cpm seems consistent with the assumption that there is no activity – So we can conclude that there is NO ACTIVITY in the source.

Possible decision #2: We don’t know where this bell-shaped distribution is. Based on the single measurement and the fact that we see 3 cpm, we may conclude that the source HAS ACTIVITY ...
False Positive Rate and Minimum Significant Net Count Rate – An Example

Example
A sample, counted for 10 min, registers 530 gross counts. A 30-min background reading gives 1500 counts. (a) Does the sample have activity? (b) Without changing the counting times, what minimum number of gross counts can be used as a decision level such that the risk of making a type-I error is no greater than 0.050?

\[
\sigma_{r_n} = \sqrt{\frac{r_g}{t_g} + \frac{r_b}{t_b}}
\]

\( r_n \): measured net count rate.
False Positive Rate and Minimum Significant Net Count Rate – An Example

\[
r_1 = 1.65 \sqrt{\frac{r_g}{t_g} + \frac{r_b}{t_b}} = 1.65 \sqrt{\frac{r_1 + 50}{10} + \frac{50}{30}},
\]

where the substitution \( r_g = r_1 + r_b \) has been made. This equation is quadratic in \( r_1 \). After some manipulation, one finds that

\[
r_1^2 - 0.272r_1 - 18.2 = 0.
\]

The solution is \( r_1 = 4.40 \) cpm. The corresponding gross count rate is \( r_g = r_1 + r_b = 4.40 + 50 = 54.4 \) cpm, and so the critical number of gross counts is \( n_g = r_g t_g = (54.4 \text{ min}^{-1}) \times (10 \text{ min}) = 544 \). Thus, a sample giving \( n_g > 544 \) (i.e., a minimum of 545 gross counts) can be reported as having significant activity, with a probability no greater than 0.05 of making a type-I error.
Minimum Significant Net Count Rate

a) 
P(N_s) 

\[ \sigma_{N_s} \]

\[ L_C \]

NO REAL ACTIVITY
Want to set \( L_C \) high enough to minimize false positives

b) 
P(N_s) 

\[ N_D \]

\[ \sigma_{N_D} \]

ACTIVITY PRESENT
Want to set \( N_D \) high enough to minimize false negatives
Minimum Significant Net Count Rate

Minimum significant measured net count rate \((r_1)\) – the minimum measured net count rate that enables one to confirm the presence of activity and with a probability of false positive of less than a given threshold \(\alpha\).

To derive the minimum significant measured net count rate \((r_1)\), we write

\[
    r_1 = k_\alpha \sqrt{\sigma_{gr}^2 + \sigma_{br}^2} = k_\alpha \sqrt{\frac{r_1 + r_b}{t_g} + \frac{r_b}{t_b}}.
\]

- \(\alpha\): maximum probability for false positive error
- \(k_\alpha\): number of standard deviations of the net count rate that gives a one-tail area (under a Gaussian distribution) equal to \(\alpha\)
- \(r_1\): the minimum significant measured net count rate
Minimum Significant Net Count Rate

To derive the minimum significant measured net count rate \( (r_1) \), we write

\[
 r_1 = k_\alpha \sqrt{\sigma_{gr}^2 + \sigma_{br}^2} = k_\alpha \sqrt{\frac{r_1 + r_b}{t_g}} + \frac{r_b}{t_b}.
\]

Solving for \( r_1 \), we get the minimum significant measured net count rate \( (r_1) \) as

\[
 r_1 = \frac{k_\alpha^2}{2t_g} + \frac{k_\alpha}{2} \sqrt{\frac{k_\alpha^2}{t_g}} + 4r_b \left( \frac{t_g + t_b}{t_g t_b} \right).
\]
Minimum Significant Count Difference

When the gross and background only counting times are equal (t), we can derive the minimum significant count difference, \( \Delta_1 \) as

The minimum difference in the counts measured in both measurements (gross and background) that ensures the probability of having Type I error to be smaller than the threshold \( \alpha \).

\[
\Delta_1 = r_1 t = \frac{1}{2} k_\alpha^2 + \frac{1}{2} k_\alpha \sqrt{k_\alpha^2 + 8n_b},
\]

\[
= k_\alpha \sqrt{2n_b} \left( \frac{k_\alpha}{\sqrt{8n_b}} + \sqrt{1 + \frac{k_\alpha^2}{8n_b}} \right).
\]

In many instances, we have \( k_\alpha / \sqrt{n_b} \ll 1 \).

Then

\[
\Delta_1 \approx k_\alpha \sqrt{2n_b},
\]
False Positive Rate and Minimum Significant Measured Count Difference

Often, the background can be measured accurately. The expected number of background counts $B$ in time $t$ is known.

In such case, if there is no source activity, the standard deviation of the net count is equal to $\sqrt{B}$. It follows that the minimum significant net count difference is

$$
\Delta_1 = k_\alpha \sqrt{B} \quad \text{(Background accurately known)}.
$$

$$
\Delta_1 \equiv k_\alpha \sqrt{2n_b}, \quad k_\alpha / \sqrt{n_b} << 1.
$$

The minimum significant net count difference is lowered by a factor of 1.414 when the background is well known.
Minimum Significant Measured Activity

Consider that the measurements were done with a detector of efficiency $\varepsilon$, then the **minimum significant measured activity** is

$$A_I = \frac{\Delta_1}{\varepsilon t}.$$ 

If the measured net activity $A > A_I$, we state that the source contains activity, with the probability of false positive being $< \alpha$.

$$\Delta_1 = r_1 t = \frac{1}{2} k_\alpha^2 + \frac{1}{2} k_\alpha \sqrt{k_\alpha^2 + 8n_b},$$

$$= k_\alpha \sqrt{2n_b} \left( \frac{k_\alpha}{\sqrt{8n_b}} + \sqrt{1 + \frac{k_\alpha^2}{8n_b}} \right).$$
Minimum Significant Measured Activity

Example
A 10-min background measurement with a certain counter yields 410 counts. A sample is to be measured for activity by taking a gross count for 10 min. The maximum acceptable risk for making a type-I error is 0.05. The counter efficiency is such that 3.5 disintegrations in a sample result, on average, in one net count.

(a) Calculate the minimum significant net count difference and the minimum significant measured activity in Bq.

(b) How much error is made in (a) by using the approximate formula (11.72) in place of (11.69)?

(c) What is the decision level for type-I errors in terms of the number of gross counts in 10 min?
Minimum Significant Measured Activity

Example

A 10-min background measurement with a certain counter yields 410 counts. A sample is to be measured for activity by taking a gross count for 10 min. The maximum acceptable risk for making a type-I error is 0.05. The counter efficiency is such that 3.5 disintegrations in a sample result, on average, in one net count.

(a) Calculate the minimum significant net count difference and the minimum significant measured activity in Bq.

\[ \Delta_1 = r_1 t = \frac{1}{2} k_\alpha^2 + \frac{1}{2} k_\alpha \sqrt{k_\alpha^2 + 8n_b}, \]

Solution

(a) With equal counting times, \( t_g = t_b = t = 10 \text{ min} \), one can use Eq. (11.69) in place of the general expression (11.68). For \( \alpha = 0.05 \), \( k_\alpha = 1.65 \). With \( n_b = 410 \), we obtain

\[ \Delta_1 = \frac{1}{2} (1.65)^2 + \frac{1}{2} (1.65) \sqrt{(1.65)^2 + 8(410)} = 48.6 = 49 \]

(11.74)

for the minimum significant count difference in 10 min (rounded upward to the nearest integer). The counter efficiency is \( \epsilon = 1/3.5 = 0.286 \text{ dpm/cpm} \). It follows from Eq. (11.71) that the minimum significant measured activity is \( A_1 = 48.6/(0.286 \times 10 \text{ min}) = 17.0 \text{ dpm} = 0.283 \text{ Bq} \).

\[
A_1 = \frac{\Delta_1}{\epsilon t}.
\]
Minimum Significant Measured Activity

(b) How much error is made in (a) by using the approximate formula (11.72) in place of (11.69)?

(c) What is the decision level for type-I errors in terms of the number of gross counts in 10 min?

Solution

(c) The decision level for gross counts in 10 min is \( n_1 = n_b + \Delta_1 = 459 \).

\[
\Delta_1 = r_1 t = \frac{1}{2} k_\alpha^2 + \frac{1}{2} k_\alpha \sqrt{k_\alpha^2} + 8n_b,
\]
Minimum Significant Measured Activity

The value $n_1 = 459$ in the last example can serve as a decision level for screening samples for the presence of activity by gross counting for 10 min. A sample showing $n_g < 459$ counts can be reported as having less than the “minimum significant measured activity,” $A_1 = 0.283$ Bq. A sample showing $n_g \geq 459$ counts can be reported as having an activity $(n_g - n_b)/\epsilon t = (n_g - 410)/2.86$ dpm.

Or "having no reportable activity."
Minimum Significant Measured Activity

For samples having zero activity, the probability of making a type-I error is just equal to the value chosen for $\alpha$. For samples having activity, a type-I error cannot occur, by definition. Therefore, when one screens a large collection of samples, some with $A = 0$ and some with $A > 0$, the probability of making a type-I error with any given sample never exceeds $\alpha$. 

NO REAL ACTIVITY

Want to set $L_C$ high enough to minimize false positives
False Negative and Minimum True Activity

Type-II Errors (False Negative) – wrongly conclude that no activity is present when there is actually activity in the source.

ACTIVITY PRESENT
Want to set $N_D$ high enough to minimize false negatives

$P(N_S)$

$N_D$

$\sigma_{N_D}$

$N_S$
False Negative and Minimum True Activity

Type-II Errors (False Negative) – wrongly conclude that “no active source is present” when there is an active source.

If we set a threshold level, \( r_1 \), what would be the minimum true source activity (\( A_{II} \)), so that the decision rule based on the threshold value \( r_1 \) can correctly detect the presence of the source with a probability of \( \geq (1 - \beta) \) \( \) or equivalently with the probability of making Type II error being \( \leq \beta \)?

\[ A > A_{II} : \text{probability of a false negative (type-II error) is less than } \beta. \]

\( A_{II} \) is called the minimum detectable true activity.
False Negative and Minimum True Count Rate, $r_2$

If we use $r_1$ as the critical decision threshold on measured count-rate to ensure that the probability of type-I error (false positive) is less than $\alpha$, what is the **minimum true activity** of the source ($A_{II}$) that would ensure the probability of type-II error (false negative) is less than $\beta$?

Assuming $r_2 \equiv A_{II} \cdot \varepsilon$, where $\varepsilon$ is the detection efficiency of the counting detector.

$r_n$: measured net count rate
False Negative and Minimum True Count Rate, $r_2$

To determine $r_2$, we would write

$$r_1 - r_2 = -k\beta \frac{r_g}{t_g} + \frac{r_b}{t_b}. \quad (1)$$

We would further assume that $r_g = r_1 + r_b$ and substitute into the above equation

$$r_1 - r_2 = -k\beta \frac{r_1 + r_b}{t_g} + \frac{r_b}{t_b}, \quad (2)$$

or

$$r_2 = r_1 + k\beta \sqrt{\frac{r_1}{t_g}} + r_b \left(\frac{t_g}{t_g t_b} + \frac{t_b}{t_g} \right).$$

Substituting for $r_1$ from Eq. (11.68),

$$r_1 = \frac{k^2}{2t_g} + \frac{k}{2} \sqrt{\frac{k^2}{t_g^2} + 4r_b \left(\frac{t_g + t_b}{t_g t_b}\right)}.$$
False Negative and Minimum True Count Rate, $r_2$

we obtain

$$r_2 = k_\alpha \left[ \frac{k_\alpha}{2t_g} + \frac{1}{2} \sqrt{\frac{k_\alpha^2}{t_g^2}} + 4r_b \left( \frac{t_g + t_b}{t_g t_b} \right) \right]$$

$$+ k_\beta \left[ \frac{k_\alpha}{t_g} \left( \frac{k_\alpha}{2t_g} + \frac{1}{2} \sqrt{\frac{k_\alpha^2}{t_g^2}} + 4r_b \left( \frac{t_g + t_b}{t_g t_b} \right) \right) + r_b \left( \frac{t_g + t_b}{t_g t_b} \right) \right]^{1/2}.$$ 

This general result gives the net rate that corresponds to the minimum detectable true activity for a given background rate $r_b$ and arbitrary choices of $\alpha$, $\beta$, and the counting times.
False Negative and Minimum True Activity, $A_{II}$

From $r_2$, we can derive the minimum true activity, $A_{II} = \frac{r_2}{\varepsilon}$.

Decision threshold $r_n$: measured net count rate

Minimum mean net count rate, with given $\alpha, \beta$. 
False Negative and Minimum True Activity

Special case 1:

When the latter are equal \((t_g = t_b = t)\), Eq. (11.78) gives for the number of net counts with the minimum detectable true activity

\[
\Delta_2 = r_2 t = \sqrt{2n_b} \left\{ k_\alpha \left[ \frac{k_\alpha}{\sqrt{8n_b}} + \sqrt{1 + \frac{k_\alpha^2}{8n_b}} \right] + k_\beta \left[ 1 + \frac{k_\alpha^2}{4n_b} + \frac{k_\alpha}{\sqrt{2n_b}} \sqrt{1 + \frac{k_\alpha^2}{8n_b}} \right]^{1/2} \right\}.
\]

\(\Delta_2\): Minimum detectable true count difference

With the help of Eq. (11.70), we can also write

\[
\Delta_2 = \Delta_1 + k_\beta \sqrt{2n_b} \left[ 1 + \frac{k_\alpha^2}{4n_b} + \frac{k_\alpha}{\sqrt{2n_b}} \sqrt{1 + \frac{k_\alpha^2}{8n_b}} \right]^{1/2}.
\]

The minimum detectable true activity is given by

\[
A_{II} = \frac{\Delta_2}{\epsilon t},
\]

(11.81)
False Negative and Minimum Detectable True Count Difference

Special case 2: When $k_\alpha / \sqrt{n_b} \ll 1$, the number of net counts that is corresponding to the minimum detectable true activity is given as

$$\Delta_2 \approx (k_\alpha + k_\beta) \sqrt{2n_b}.$$

$$\Delta_2 = r_2 t = \sqrt{2n_b} \left\{ k_\alpha \left[ \frac{k_\alpha}{\sqrt{8n_b}} + \sqrt{1 + \frac{k_\alpha^2}{8n_b}} \right] + k_\beta \left[ 1 + \frac{k_\alpha^2}{4n_b} + \frac{k_\alpha}{\sqrt{2n_b}} \sqrt{1 + \frac{k_\alpha^2}{8n_b}} \right]^{1/2} \right\}. \quad (11.79)$$

$\Delta_2$: Minimum detectable true count difference
False Negative and Minimum Detectable True Count Difference

Special case 3:

When the **background count** $B$ is accurately known, we have seen by Eq. (11.73) that the minimum significant count difference is $\Delta_1 = k_\alpha \sqrt{B}$. If a sample has exactly the minimum detectable true activity, then the expected number of net counts $\Delta_2$ is just $k_\beta$ standard deviations greater than $\Delta_1$. The standard deviation of the net count rate is $\sqrt{(B + \Delta_2)}$. Thus,

$$\Delta_2 = k_\alpha \sqrt{B} + k_\beta \sqrt{B + \Delta_2}.$$  \hspace{1cm} (11.83)

Solving for $\Delta_2$, we find

$$\Delta_2 = \sqrt{B} \left( k_\alpha + \frac{k_\beta^2}{2 \sqrt{B}} + k_\beta \sqrt{1 + \frac{k_\alpha}{\sqrt{B}} + \frac{k_\beta^2}{4B}} \right)$$ \hspace{1cm} (11.84)

(Background accurately known).
False Negative and Minimum Detectable True Count Rate Difference

Special case 4: When

\[ \frac{k_\alpha}{\sqrt{B}} \leq 1 \quad \text{and} \quad \frac{k_\beta}{\sqrt{B}} \leq 1 \]

the number of net counts that is corresponding to the minimum detectable true activity is given as

\[ \Delta_2 \equiv (k_\alpha + k_\beta) \sqrt{B} \quad \text{(Background accurately known).} \]

\[ \Delta_2 \equiv (k_\alpha + k_\beta) \sqrt{2n_b}. \quad \text{Background not known a priori} \]

As with \( \Delta_1 \) and \( A_I \), accurate knowledge of the background lowers \( \Delta_2 \) and \( A_{II} \) by about a factor of \( \sqrt{2} \).
False Negative and Minimum True Count Rate

Example

The counting arrangement ($\alpha = 0.05$, $\epsilon = 0.286$, $t_g = t_b = 10\ \text{min}$, and $n_b = 410$) and critical gross count number $n_1 = 459$ from the last example are to be used to screen samples for activity. The maximum acceptable probability for making a type-II error is $\beta = 0.025$. (a) Calculate the minimum detectable true activity in Bq.
Limits of Detectability – An Example

(a) Calculate the minimum detectable true activity in Bq. (b) How much error is made by using the approximate formula (11.82) in place of the exact (11.79) or (11).

\[ \Delta_2 = \Delta_1 + k_\beta \sqrt{2n_b} \left[ 1 + \frac{k_\alpha^2}{4n_b} + \frac{k_\alpha}{\sqrt{2n_b}} \sqrt{1 + \frac{k_\alpha^2}{8n_b}} \right]^{1/2}. \]  

\( A_{II} = \frac{\Delta_2}{\epsilon t}, \)  

\( \Delta_2 : \) Minimum detectable true count difference \hspace{1cm} (11.81)

Solution

(a) With equal gross and background counting times, we can use Eqs. (11.80) and (11.81) to find \( A_{II} \). For \( \beta = 0.025, k_\beta = 1.96 \) (Table 11.2). With \( \Delta_1 = 48.6 \) counts from the last example [Eq. (11.74)], Eq. (11.80) gives

\[ \Delta_2 = 48.6 + 1.96 \sqrt{2(410)} \left[ 1 + \frac{(1.65)^2}{4(410)} + \frac{1.65}{\sqrt{2(410)}} \sqrt{1 + \frac{(1.65)^2}{8(410)}} \right]^{1/2} = 106 \]

net counts. The minimum true detectable activity is, from Eq. (11.81),

\[ A_{II} = \frac{106}{0.286 \times 10 \text{ min}} = 37.1 \text{ dpm} = 0.618 \text{ Bq}. \]  

(11.87)
Limits of Detectability – An Example

This example illustrates how a protocol can be set up for reporting activity in a series of samples that are otherwise identical. As shown in Fig. 11.6, the decision level for a 10-min gross count is \( n_1 = 459 \), corresponding to the minimum significant count difference \( \Delta_1 = 49 \) and the minimum significant measured activity, \( A_1 = 0.283 \text{ Bq} \). A sample for which \( n_g < 459 \) is considered as having no reportable activity. When \( n_g \geq 459 \), a sample is reported as having an activity

\[
A = \frac{n_g - n_b}{\epsilon t} = \frac{n_g - 410}{(0.286)(600 \text{ s})} = \frac{n_g - 410}{172} \text{ Bq.} \quad (11.88)
\]

\( \alpha, \quad n, \quad \Delta, \quad r, \quad A_I \)

\( \beta, \quad n_1, \quad \Delta_2, \quad r_2, \quad A_II \)
Limits of Detectability – An Example

and the chance of making type-I error is less than $\alpha$.

\[ n_2 = 516, \text{ minimum true count from the source.} \]

\[ \Delta_1 = 49 \quad \left( A_1 = 0.283 \text{ Bq} \right) \]

\[ \Delta_2 = 106 \quad \left( A_2 = 0.618 \text{ Bq} \right) \]
Limits of Detectability – An Example

Note that $A$ will be greater than the minimum significant measured activity, $A_1 = 0.283$ Bq. From part (a) in the last example, when $n_g = \Delta_2 + 410 = 516$, the reported value of the activity will be $A_{II} = 0.618$ Bq, the minimum detectable true activity. For a sample of unknown activity, the probability of making a type-I error does not exceed $\alpha = 0.05$. (If $A = 0$, the probability equals $\alpha$.) The probability of making a type-II error with a given sample does not exceed $\beta = 0.025$, as long as the activity is greater than $A_{II} = 0.618$ Bq. (If $A = A_{II}$, the probability equals $\beta$.) When $0 < A < A_{II}$, the probability for a type-II error is greater than $\beta$. 
Chapter 3: Counting Statistics

False Negative and Minimum True Activity

From $r_2$, we can derive the minimum true activity, $A_{II} = \frac{r_2}{\varepsilon}$.

Decision threshold

Minimum mean net count rate, with given $\alpha, \beta$. 

$P_n(r_n)$

$A = 0$

$A_{II} > 0$

From $r_2$, we can derive the minimum true activity, $A_{II} = \frac{r_2}{\varepsilon}$. 

what does this width depend on? 

$r_n$: measured net count rate