1. 6.8

(a) From the figure, estimate for a 100-eV electron the probability that a given energy-loss event will result in excitation, rather than ionization, in water.

\[ \mu_{\text{ION}}(100 \, \text{eV}) \simeq 800 \, \frac{1}{\mu\text{m}} \]
\[ \mu_{\text{EXC}}(100 \, \text{eV}) \simeq 70 \, \frac{1}{\mu\text{m}} \]
\[ P(\text{EXC}) = \frac{\mu_{\text{EXC}}}{\mu_{\text{EXC}} + \mu_{\text{ION}}} \]
\[ \simeq 8.05\% \]

(b) What fraction of the collisions at 100 eV are due to elastic scattering?

\[ \mu_{\text{ELA}}(100 \, \text{eV}) \simeq 1000 \, \frac{1}{\mu\text{m}} \]
\[ \mu_{\text{TOTAL}}(100 \, \text{eV}) \simeq 2000 \, \frac{1}{\mu\text{m}} \]
\[ P(\text{ELA}) = \frac{\mu_{\text{ELA}}}{\mu_{\text{TOTAL}}} \]
\[ \simeq 50\% \]
2. 6.11
What is the ratio of the collisional and radiative stopping powers of Al for electrons of the given energies?

For $E$ in MeV:

$$
\frac{-(dE/dx)_{\text{rad}}}{-(dE/dx)_{\text{col}}} \sim \frac{ZE}{800}
$$

$Z_{Al} = 13$

(a) 10 keV

$$E = .01$$

$$
\frac{-(dE/dx)_{\text{rad}}}{-(dE/dx)_{\text{col}}} \approx \frac{27}{800} E
$$

$$= 1.625^{-4}
$$

(b) 1 MeV

$$E = 1$$

$$
\frac{-(dE/dx)_{\text{rad}}}{-(dE/dx)_{\text{col}}} \approx \frac{27}{800} E
$$

$$= 1.625^{-2}
$$

(c) 100 MeV

$$E = 100$$

$$
\frac{-(dE/dx)_{\text{rad}}}{-(dE/dx)_{\text{col}}} \approx \frac{27}{800} E
$$

$$= 1.625$$
To protect the cell culture from 10-MeV beta rays, what advantage would be gained if the positions of the Lucite and the lead were swapped, so that the Lucite was on top?

It would greatly improve the shielding efficiency. With this configuration, the fast electrons will be primarily absorbed in Lucite. Compared to lead, the lower effective Z of Lucite would greatly reduce the fraction of electron energy being converted into bremsstrahlung radiation. Then the addition of the lead would help to stop the bremsstrahlung X-rays.
(a) Use the figure to estimate the probability that a normally incident, 740-keV electron will penetrate a water phantom to a maximum depth between 1,500µm and 2,000µm.

The "Maximum Depth" \( f \) function is normalized, so \( \int_{-\infty}^{\infty} f \, dx = 1 \). Assume that the \( f \) is approximately linear for distances between 1500 and 2000 µm.

\[
f(1500) \approx 0.00055 \\
f(2000) \approx 0.000275
\]

\[
P(1500 \leq x \leq 2000) \approx 500 \cdot 0.000275 + \frac{500 \cdot (0.00055 - 0.000275)}{2} \\
\approx 0.20625
\]

(b) What is the probability that the pathlength will be between these two distances? Make similar assumptions on "Pathlength" \( g \) as were made on \( f \) in part 4a.

\[
f(1500) \approx 0.00005 \\
f(2000) \approx 0.000125
\]

\[
P(1500 \leq x \leq 2000) \approx 500 \cdot 0.00005 + \frac{500 \cdot (0.000125 - 0.00005)}{2} \\
\approx 0.04375
\]
(a) For 0.05-MeV protons in water, what is the smallest value of $\Delta$ for which
\[(\frac{-dE}{dx})_\Delta = (\frac{-dE}{dx})_\infty?\]
$\Delta$ is minimal at $Q_{\text{max}}$. The maximum energy will be transferred when the photon collides with an electron.

\[
Q_{\text{max}} = \frac{4mME}{(m+M)^2}
\]
\[m = 9.1093837 \times 10^{-31} \text{ kg}\]
\[M = 1.67262192 \times 10^{-27} \text{ kg}\]
\[Q_{\text{max}} = .0001088 \text{ MeV}\]
\[Q_{\text{max}} = 108.8 \text{ eV}\]

(b) Repeat for 0.10-MeV protons.
Use the same assumptions and equations as in part 5a.

\[
Q_{\text{max}} = \frac{4mME}{(m+M)^2}
\]
\[Q_{\text{max}} = .0002176 \text{ MeV}\]
\[Q_{\text{max}} = 217.6 \text{ eV}\]

(c) Are your answers consistent with the textbook’s Table 7.1 on page 161?
Yes, the answers are consistent. For a .05 MeV photon, all values of mass stopping power listed in Table 7.1 are equal to that of $\Delta = \infty$, but the $\Delta$ values are all greater than the answer determined for part 5a. Similarly, for the .1 MeV proton, for all entries with $\Delta > Q_{\text{max}}$, they are equal to the entry for $\Delta = \infty$. 
6. 7.11

What is the specific ionization of a 12-MeV proton in tissue if an average of 22 eV is needed to produce an ion pair? Tissue will be approximated as water for the purposes of this exercise. Denote specific ionization with $\sigma$.

$$\sigma = \frac{-(dE/dx)}{22 \text{ eV}}$$

$$-(dE/dx) = -(dE/\rho dx) \cdot \rho$$

$$\rho = .997$$

From textbook Eq. 5.3 (on page 127), for a 12 MeV proton;

$$-(dE/\rho dx) = 39.5 \text{ MeV} \cdot \text{cm}^2 / \text{g}$$

$$-(dE/dx) = 39.3815$$

$$\sigma = 1.79 \times 10^6 \text{ cm}^{-1}$$
7. 8.12
Derive equation 1 from equations 2 - 4.

\[ hv' = \frac{hv}{1 + (\frac{hv}{mc^2})(1 - \cos(\theta))} \]  \hspace{1cm} (1)

\[ hv + mc^2 = hv' + E' \]  \hspace{1cm} (2)

\[ \frac{hv}{c} = \frac{hv'}{c} \cos(\theta) + P' \cos(\phi) \]  \hspace{1cm} (3)

\[ \frac{hv'}{c} \sin(\theta) = P' \sin(\phi) \]  \hspace{1cm} (4)

\[ \frac{hv}{c} = \frac{hv'}{c} \cos(\theta) + P' \cos(\phi) \]

\[ P' \cos(\phi) = \frac{hv}{c} - \frac{hv'}{c} \cos(\theta) \]

\[ P'^2 \cos^2(\phi) = \left( \frac{hv}{c} - \frac{hv'}{c} \cos(\theta) \right)^2 \]
\[ = \left( \frac{hv}{c} \right)^2 + \left( \frac{hv'}{c} \right)^2 \cos^2(\theta) - 2 \frac{hv}{c} \frac{hv'}{c} \cos(\theta) \]

\[ \frac{h v'}{c} \sin(\theta) = P' \sin(\phi) \]

\[ P' \sin(\phi) = \frac{h v'}{c} \sin(\theta) \]

\[ P'^2 \sin^2(\phi) = \left( \frac{h v'}{c} \sin(\theta) \right)^2 \]
\[ = \left( \frac{h v'}{c} \right)^2 \sin^2(\theta) \]

\[ P'^2 \left( \cos^2(\phi) + \sin^2(\phi) \right) = \left( \frac{h v}{c} \right)^2 + \left( \frac{h v'}{c} \right)^2 \left( \cos^2(\theta) + \sin^2(\theta) \right) - 2 \frac{h v}{c} \frac{h v'}{c} \cos(\theta) \]
\[ P'^2 = \left( \frac{h v}{c} \right)^2 + \left( \frac{h v'}{c} \right)^2 - 2 \frac{h v}{c} \frac{h v'}{c} \cos(\theta) \]

\[ E'^2 = (mc^2)^2 + (P' c)^2 \]

\[ hv + mc^2 = h v' + E' \]

\[ E' = hv + mc^2 - h v' \]

\[ E'^2 = (hv + mc^2 - h v')^2 \]
\[ = (h v)^2 + (mc^2)^2 - (h v')^2 + 2 h v m c^2 - 2 h v' h v - 2 h v' m c^2 \]
\[ P'^2 c^2 = (hv)^2 + (hv')^2 + 2hvmc^2 - 2hv'hv - 2hv'mc^2 \]

\[ P'^2 = \left( \frac{hv}{c} \right)^2 + \left( \frac{hv'}{c} \right)^2 - 2 \frac{hv hv'}{c} \cos(\theta) \]

\[ P'^2 c^2 = (hv)^2 + (hv')^2 - 2hvhv' \cos(\theta) \]

\[ (hv)^2 + (hv')^2 - 2hvhv' \cos(\theta) = (hv)^2 + (hv')^2 + 2hvmc^2 - 2hv'hv - 2hv'mc^2 \]

\[ -2hv hv' \cos(\theta) = 2hvmc^2 - 2hv'hv - 2hv'mc^2 \]

\[ 0 = hvmc^2 - hv'hv - hv'mc^2 + hvhv' \cos(\theta) \]

\[ 0 = hv - \frac{hv hv'}{mc^2} - hv' + \frac{hv hv'}{mc^2} \cos(\theta) \]

\[ hv' + \frac{hv hv'}{mc^2} - \frac{hv hv'}{mc^2} \cos(\theta) = hv \]

\[ \left( 1 + \frac{hv}{mc^2} - \frac{hv}{mc^2} \cos(\theta) \right) hv' = hv \]

\[ hv' = \frac{hv}{1 + \frac{hv}{mc^2} - \frac{hv}{mc^2} \cos(\theta)} \]

\[ hv' = \frac{hv}{1 + \left( \frac{hv}{mc^2} \right) (1 - \cos(\theta))} \]
The Klein–Nishina cross section for the collision of a 1-MeV photon with an electron is $2.11 \times 10^{-25}$ cm$^2$. Calculate, for Compton scattering on aluminum,

(a) the energy-transfer cross section (per electron cm$^{-2}$)

\[
\sigma_{tr} = \sigma \frac{T_{avg}}{hv}
\]

From Table 8.1 (Textbook page 184):

\[
\frac{T_{avg}}{hv} = 0.440 \\
\sigma_{tr} = 9.284 \times 10^{-26}
\]

(b) the energy-scattering cross section (per electron cm$^{-2}$)

\[
\sigma = \sigma_s + \sigma_{tr} \\
\sigma_s = \sigma - \sigma_{tr} = 1.182 \times 10^{-25}
\]

(c) the atomic cross section

\[
\sigma_a = \left(\frac{\mu}{\rho}\right)\left(\frac{A}{N_0}\right)
\]

From Figure 8.8 (Textbook page 189):

\[
\frac{\mu}{\rho} \approx 0.06 \\
A = 26.98153 \\
N_0 = 6.02 \times 10^{23} \\
\sigma_a \approx 2.689 \times 10^{-24}
\]

(d) the linear attenuation coefficient.

\[
\mu = \sigma_a \rho \left(\frac{N_0}{A}\right) \\
\rho \approx 2.7 \\
\mu = 0.1619
\]