Q1. Bernoulli Process and Poisson Process
Consider the following counting measurement.

The detector detected \( k \) counts during the measurement

Detector:
Covering 1% of solid angle,
Detection efficiency: \( \lambda = 55\% \)

**mean(\( N \)) = \( m \)**

1) Assuming the average (or mean) number of photons reaching the detector during the measurement is \( m \), should the number of photons reaching the detector, \( N \), follow Poisson distribution or binomial distribution? and please explain why.

2) If there is \( N \) photons reaching the detector, should the number of photons detected by the detector, \( k \), follow Poisson or Binomial distribution? and please explain why.

3) Please write down the probability of detecting \( k \) counts during the measurement.

Note:
Poisson distribution with mean of \( m \) is characterized by the following probability distribution function,

\[
P(N|m) = \frac{m^N}{N!} e^{-m}.
\]

Both the mean and variance (square of standard deviation) of the distribution is \( m \).

The binomial distribution is characterized by

\[
P_k = \binom{N}{k} p^k q^{N-k}.
\]
Q2: Error Propagation

In a typical counting measurement, the net count rate from a source is obtained by subtracting the background count rate $r_b = \frac{n_b}{t_b}$ from gross count rate $r_g = \frac{n_g}{t_g}$,

$$r_n = r_g - r_b = \frac{n_g}{t_g} - \frac{n_b}{t_b},$$

where $n_b$ and $n_g$ are measured background and gross counts, $t_b$ and $t_g$ are the time used in the gross and background counting measurements.

Please show that the standard deviation of the net count rate can be approximately given by

$$\sigma_{r_n} = \sqrt{\frac{n_g^2}{t_g^2} + \frac{n_b^2}{t_b^2}} = \sqrt{\frac{r_g^2}{t_g^2} + \frac{r_b^2}{t_b^2}}.$$
Q3: Detection Limits

(a) What is false positive (or type-I error), and what is false-negative (type-II error)?

(b) A sample, counted for 5 mins, registers 270 gross counts. A 20-min background reading gives 840 counts. What minimum number of gross counts can be used as a decision level such that the risk of making a type-I error is no greater than 0.1?

Hint:
1. The detected number of counts with a given time interval follows Gaussian distribution

\[ p(n|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left[ -\frac{(n - \mu)^2}{2\sigma^2} \right] \]

where \( \mu \) is the mean value of the distribution, and \( \sigma^2 = \mu \) is the standard deviation.

2. One-tail areas \( \alpha \) under the standard Gaussian distribution from \( z = k_\alpha \) to \( \infty \) are given in the table below.

<table>
<thead>
<tr>
<th>Area, ( \alpha )</th>
<th>( k_\alpha )</th>
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