Part 1
1. Ch. 3 Q. 27

A parent nuclide decays by beta-particle emission into a stable daughter. The major radiations, energies (MeV), and frequencies are, in the notation of Appendix D:
\[ \beta^- : 3.92 \text{ max (7%), } 3.10 \text{ max (5%), } 1.60 \text{ max (88%)} \]
\[ \gamma : 2.32 \text{ (34%), } 1.50 \text{ (54%), } 0.820 \text{ (88%)} \]
\[ e^- : 0.818 \text{, } 0.805 \]

(a) Draw the decay scheme

Solution:

(b) What is the maximum energy that the antineutrino can receive in this decay?

Solution:

\[ E_{\text{max}, \bar{\nu}} = E_{\text{max}, \beta^-} = 3.92 \text{ MeV} \]

(c) What is the value of the internal-conversion coefficient?

Solution:

\[ \alpha = \frac{N_e}{N_\gamma} = \frac{10\%}{49\%} = 0.204 \]

The reciprocal of the solution is also accepted.

(d) Estimate the L-shell electron binding energy of the daughter nuclide.
Solution:

\[ E_e^- = E^* - E_B \]
\[ E^* = 0.82 \text{ MeV} \]
\[ E_e^- = 0.818, 0.805 \text{ MeV} \]
\[ E_B = 0.82 - 0.818 \text{ MeV} = 0.002 \text{ MeV} \]
\[ E_B = 0.82 - 0.805 \text{ MeV} = 0.015 \text{ MeV} \]

The larger \( E_B \) is for K-shell, so the binding energy for L-shell electron is 0.002 MeV

(e) Would daughter X rays be expected also? Why or why not?

Solution:

Yes. The daughter atom would be missing a K-shell or L-shell electron after internal conversion. When an outer shell electron moves in to fill this hole, the energy difference could be released as X rays (or Auger electrons).

2. Ch.3 Q. 33

The nuclide \(^{65}_{30}\text{Zn}\) decays by electron capture (98.5%) and by positron emission (1.5%).

(a) Calculate the Q value for both modes of decay

Solution

Electron capture:

\[^{65}_{30}\text{Zn} + {}_1^0\text{e}^- \rightarrow ^{65}_{29}\text{Cu} + \gamma\]

\[ Q = \Delta P - \Delta D - E_B \]
\[ = (-65.911) - (-67.273) - 0.009 \text{ MeV} \]
\[ = 1.343 \text{ MeV} \]

Positron emission:

\[^{65}_{30}\text{Zn} \rightarrow ^{65}_{29}\text{Cu} + {}_1^0\text{B}^+ + \gamma\]

\[ Q = \Delta P - \Delta D - 2mc^2 \]
\[ = (-65.911) - (-67.273) - 1.022 \text{ MeV} \]
\[ = 0.33 \text{ MeV} \]

(b) Draw the decay scheme for \(^{65}_{30}\text{Zn}\)

Solution
(c) What are the physical processes responsible for each of the major radiations listed in Appendix D?

**Solution:**
Positrons are produced by the positron decay of the parent atoms. When these positrons annihilate with electrons in the surrounding material, two annihilation photons of ~511 keV are emitted per annihilation event.
At least 51% of the decay is via Electron Capture that populates the 1.116 MeV energy level. From this energy level, the excited daughter atoms either decay to the ground state via the 1.116 MeV gamma emissions (51%) or via Internal Conversion that ejects (most probably) the K-shell electron with an energy of 1.107 MeV.
The remaining Electron Capture events towards the nuclear ground-state of the daughter atoms as well as the above Internal Conversion events contribute to the population of excited daughter atoms (missing K-shell or L-shell electrons). These excited atoms de-excite when the outer shell electrons move in to fill the hole, producing daughter X-rays, or when the atoms eject auger electrons.

(d) Estimate the binding energy of a K-shell electron in copper.

**Solution:**

\[ E_B = E^* - E_e \]
\[ = 1.116 - 1.107 \text{ MeV} \]
\[ = 0.009 \text{ MeV} \]

3. Ch.3 Q.35

Show that \(^{55}\text{Fe}\), which decays by electron capture, cannot decay by positron decay.

**Solution:**

Q value of decay of \(^{55}\text{Fe}\): \(^{55}\text{Fe} + 0^- e^- \rightarrow ^{55m}_{26}\text{Mn} + 0^0\gamma\)
\[ Q = \Delta P - \Delta D - E_B \]
\[ = (-57.474) - (-57.705) - E_B \text{ MeV} \]
\[ = 0.231 \text{ MeV} - E_B < 2 \times m_e c^2 \]

Since the Q value of the decay of $^{55}_{26}Fe$ is less than two times of the mass of electrons required for positron decay, the decay cannot proceed via positron decay.

4. Ch.3 Q.36

The isotope $^{126}_{53}I$ can decay by EC, $\beta^-$, $\beta^+$ transitions.

(a) Calculate the Q values for the three modes of decay to the ground states of the daughter nuclei,

**Solution:**

EC: $^{126}_{53}I + ^0_1 e^- \rightarrow ^{126}_{52}Te + ^0_0 \gamma$

\[ Q_{EC} = \Delta P + \Delta D - E_B \]
\[ = -87.9 - (-90.05) - 0.033 \text{ MeV} \]
\[ = 2.12 \text{ MeV} \]

$\beta^+$: $^{126}_{53}I \rightarrow ^{126}_{52}Te + ^0_1 \beta^+ + ^0_0 \nu$

\[ Q_{\beta^+} = \Delta P + \Delta D - 2m_e c^2 \]
\[ = -87.9 - (-90.05) - 0.033 \text{ MeV} \]
\[ = 1.13 \text{ MeV} \]

$\beta^-$: $^{126}_{53}I \rightarrow ^{126}_{54}Xe + ^0_1 \beta^- + ^0_0 \nu$

\[ Q_{\beta^-} = \Delta P + \Delta D \]
\[ = -87.9 - (-89.15) \text{ MeV} \]
\[ = 1.25 \text{ MeV} \]

(b) Draw the decay scheme

**Solution:**
(c) What kinds of radiation can one expect from a $^{126}_{53}I$ source?

**Solution:** X-ray, Gamma-rays, $\beta^-$, $\beta^+$, as well as 511 keV annihilation photons can be expected.
Part 2
1. Ch.4 Q. 17
Consider the following $\beta^-$ nuclide decay chain with the half-lives indicated:

\[
{^{210}\text{Pb}} \xrightarrow{\beta^-} {^{210}\text{Bi}} \xrightarrow{\beta^-} {^{210}\text{Po}}
\]

A sample contains 30 MBq of $^{210}\text{Pb}$ and 15 MBq of $^{210}\text{Bi}$ at time $t = 0$.
(a) Calculate the activity of $^{210}\text{Bi}$ at time $t = 0$

Secular Equilibrium ($T_1 >> T_2$)
Eq. (4.37)
\[
A_2 = A_1 (1 - e^{-\lambda_2 t}) + A_20 e^{-\lambda_2 t},
\]
Where $\lambda_2 = \frac{\ln(2)}{T_2} = 0.1386 \text{ d}^{-1}$

Solution:
\[
A_2 = (30 \text{ MBq}) (1 - e^{-(0.1386 \text{ d}^{-1})(10 \text{ d})}) + (15 \text{ MBq}) e^{-(0.1386 \text{ d}^{-1})(10 \text{ d})}
\]
\[
A_2 = 26.25 \text{ MBq}
\]
(b) If the sample was originally pure $^{210}\text{Pb}$, then how old is it at time $t = 0$?
For initial pure sample, $A_{20} = 0$. Using Eq. (4.37)

Solution:
\[
15 \text{ MBq} = (30 \text{ MBq}) (1 - e^{-(0.1386 \text{ d}^{-1})\Delta t})
\]
\[
\Delta t = 5 \text{ d}
\]
2. Ch.4 Q.24
A 40-mg sample of pure $^{226}\text{Ra}$ is encapsulated
(a) How long will it take for the activity of $^{222}\text{Rn}$ to build up to 10 mCi?
Specific Activity of Radium-226
Eq. (4.24):
\[ SA = \frac{6.02 \times 10^{23} \lambda}{M} = \frac{4.17 \times 10^{23}}{MT} \]

From Appendix D, \( T = 1600 \) y and \( M = A = 226 \). Converting \( T \) to seconds, we have Eq. (4.25):

\[ SA = \frac{4.17 \times 10^{23}}{226 \times 1600 \times 365 \times 24 \times 3600} \]

Eq. (4.26):

\[ = 3.66 \times 10^{10} \text{s}^{-1} g^{-1} = 3.7 \times 10^{10} Bq \cdot g^{-1} \]

This, by definition, is an activity of 1 Ci. For an arbitrary nuclide of half-life \( T \) in years:

Eq. (4.27):

\[ SA = \frac{1600}{T} \times \frac{226}{A} \text{Ci} \cdot g^{-1} \]

Secular Equilibrium (\( T_1 > > T_2 \))

For initial pure sample, \( A_2 = 0 \). Using Eq. (4.37):

\[ A_2 = A_1 (1 - e^{-\lambda_2 t}) \]
\[ e^{\frac{t}{T_2} \ln \left( \frac{1}{2} \right)} = 1 - \frac{A_2}{A_1} \]
\[ t = T_2 \frac{\ln \left( \frac{1 - \frac{A_2}{A_1}}{\frac{1}{2}} \right)}{\ln \left( \frac{1}{2} \right)} \]

Solution:

\[ A_1 = 40 \text{ mg} \times 1 \text{ Ci} \cdot g^{-1} = 40 \text{ mCi} \]
\[ t = \frac{(3.82 \text{ d}) \ln \left( 1 - \frac{10}{40} \right)}{\ln \left( \frac{1}{2} \right)} \]
\[ t = 1.6 \text{ d} \]

(b) What will be the activity of \(^{222}\text{Rn}\) after 2 years?

After about seven daughter half-lives (\( t \ll 7 T_2 \)), \( e^{-\lambda_2 t} \ll 1 \), secular equilibrium is met. Since \( 7 \times 3.82 \text{ d} = 26.74 \text{ d}, t=2 \text{ y} >> 26.74 \text{ d} \). So \( A_2 = A_1 \). However, \( t=2 \text{ y} << T_1 = 1600 \text{ y} \). Therefore,

Solution:

\[ A_2 = A_1 = A_{10} \]
\[ A_2 = 40 \text{ mCi} \]
(c) What will be the activity of $^{222}\text{Rn}$ after 1000 years?

Now $t=1000\ y \lesssim T_1 = 1600\ y$. Following Eq. (4.13):

Eq. (4.13):

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^{t/T}$$

**Solution:**

$$A_2 = A_1 = (40\ mCi)(0.5)^{\frac{1000\ y}{1600\ y}}$$

$$A_2 = 26\ mCi$$

(d) What is the ratio of the specific activity of $^{222}\text{Rn}$ to that of $^{226}\text{Ra}$?

The ratio is the specific activity of $^{222}\text{Rn}$, since the unit of curie represents the activity of 1-gram of $^{226}\text{Ra}$. Using Eq. (4.27):

**Solution:**

$$SA_2 = \frac{1600}{3.82\ d} \times \frac{226}{222} \frac{\text{Ci}}{\text{g}^{-1}}$$

$$\frac{SA_2}{365\ d} = \frac{SA_1}{156,000}$$

3. Ch.4 Q.26

(a) Verify Eq. (4.44)

**Solution:**

Total Activity:

$$A_1 + A_2 = \lambda_1 N_1 + \lambda_2 N_2$$

Maximum total activity:

$$\frac{d(A_1 + A_2)}{dt} = \lambda_1 \frac{dN_1}{dt} + \lambda_2 \frac{dN_2}{dt} = 0$$

$$\frac{dN_1}{dt} = -\lambda_1 N_1, \quad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$$

$$(\lambda_1 \lambda_2 - \lambda_1^2) N_1 = \lambda_2^2 N_2$$

Eq. (4.40):
\[ N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \]

Substitution:

\[ \lambda_1 (\lambda_2 - \lambda_1) e^{-\lambda_1 t} = \frac{\lambda_1 \lambda_2^2}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t}) \]

\[ \frac{(\lambda_2 - \lambda_1)^2}{\lambda_2^2} = 1 - e^{-(\lambda_2 - \lambda_1) t} \]

\[ -(\lambda_2 - \lambda_1) t = \ln \left( 1 - \frac{(\lambda_2 - \lambda_1)^2}{\lambda_2^2} \right) = \ln \left( \frac{2\lambda_1 \lambda_2 - \lambda_1^2}{\lambda_2^2} \right) \]

Eq. (4.44)

\[ t = \frac{1}{\lambda_2 - \lambda_1} \ln \left( \frac{\lambda_2^2}{2\lambda_1 \lambda_2 - \lambda_1^2} \right) \]

(b) Show that the time of maximum total activity occurs earlier than the time of maximum daughter activity in Fig. 4.5.

The inequality of maximum \( A_2 \) (Eq. 4.43) > maximum \( A_1 + A_2 \) (Eq. 4.44) must satisfy the condition that \( \lambda_1 < \lambda_2 \) for Transient Equilibrium \( (T_1 \gtrsim T_2) \).

Solution:

\[ \frac{1}{\lambda_2 - \lambda_1} \ln \left( \frac{\lambda_2}{\lambda_1} \right) > \frac{1}{\lambda_2 - \lambda_1} \ln \left( \frac{\lambda_1^2}{2\lambda_1 \lambda_2 - \lambda_1^2} \right) \]

\[ \frac{\lambda_2}{\lambda_1} > \frac{\lambda_1^2}{2\lambda_1 \lambda_2 - \lambda_1^2} \]

\[ 2\lambda_1 \lambda_2^2 - \lambda_1^2 \lambda_2 > \lambda_1 \lambda_2^2 \]

\[ 2\lambda_2 - \lambda_1 > \lambda_2 \]

\[ \lambda_2 > \lambda_1 \]

(c) Does Eq. 4.43 apply to \( A_2 \) when there is no equilibrium (Fig. 4.6)?

Solutions:

When there is no equilibrium \( (T_1 < T_2) \), then \( \lambda_1 > \lambda_2 \). Since Eq. (4.43) was derived for the general case, then it should always apply to \( A_2 \) at the point where \( A_1 = A_2 \), so long that you can show that \( t > 0 \) and is real. A more proper form for the “No Equilibrium” case is:

\[ \frac{1}{\lambda_1 - \lambda_2} \ln \left( \frac{\lambda_1}{\lambda_2} \right) \]

which is always positive and real. This is not the case for Eq. (4.44), since the quantity inside the natural logarithm is negative for \( \lambda_1 > \lambda_2 \).
4. Ch.4 Q.28
(a) The average mass of potassium in the human body is about 140 g. From the abundance and half-life given in Appendix D, estimate the average activity of $^{40}K$ in the body.

The natural abundance of $^{40}K$ is 0.0118% and the half-life is $1.28 \times 10^9$ years. The specific activity is needed, given by Eq. (4.27):

**Solution:**

$$SA = \frac{1600}{1.28 \times 10^9} \times \frac{226}{40} = 7.06 \, \muCi \, g^{-1}$$

$$A = 0.000118 \times 140 \, g \times 7.06 \, \muCi \, g^{-1}$$

$$A = 0.117 \, \muCi$$