Chapter 1.2: Transformation Kinetics

Serial Transformation

In many situations, the parent nuclides produce one or more radioactive offsprings in a chain. In such cases, it is important to consider the radioactivity from both the parent and the daughter nuclides as a function of time.

\[ {^{90}}Kr \rightarrow {^{90}}Rb \rightarrow {^{90}}Sr \rightarrow {^{90}}Y \rightarrow {^{90}}Zr. \]

- Due to their short half lives, \(^{90}Kr\) and \(^{90}Rb\) will be completely transformed, resulting in a rapid build up of \(^{90}Sr\).
- \(^{90}Y\) has a much shorter half-life compared to \(^{90}Sr\). After a certain period of time, the instantaneous amount of \(^{90}Sr\) transformed per unit time will be equal to that of \(^{90}Y\).
- In this case, \(^{90}Y\) is said to be in a secular equilibrium.

• Different isotopes are characterized by their different rate of transformation (decay).
• The activity of a pure radionuclide decreases exponentially with time. For a given sample, the number of decays within a unit time window around a given time \(t\) is a Poisson random variable, whose expectation is given by

\[ Q = Q_0 e^{-\lambda t} \]

- The decay constant \(\lambda\) is the probability of a nucleus of the isotope undergoing a decay within a unit period of time.
Why Exponential Decay?

The activity of a pure radionuclide decreases exponentially with time, as we now show. If \( N \) represents the number of atoms of a radionuclide in a sample at any given time, then the change \( dN \) in the number during a short time \( dt \) is proportional to \( N \) and to \( dt \). Letting \( \lambda \) be the constant of proportionality, we write

\[
\frac{dN}{dt} = -\lambda N dt.
\]

The decay rate, \( A \), is given by

\[
A = -\frac{dN}{dt} = \lambda N.
\]

Separate variables in above equation, we have

\[
\frac{dN}{N} = -\lambda dt.
\]

Integration of both sides gives

\[
\ln N = -\lambda t + c,
\]

The decay constant \( \lambda \) is the probability of a nucleus of the isotope undergoing a decay within a unit period of time.

Characteristics of Exponential Decay – Half-life

Half-life

The time required for any given radioisotope to decrease to one-half of its original quantity is defined as the half-life, \( T \).

\[
\lambda = \frac{0.693}{T}
\]

Characteristics of Exponential Decay – half-life

The relationship between half-life \( T \) and decay constant \( \lambda \) can be derived by writing

\[
\frac{1}{2} = e^{-\lambda T}.
\]

Taking the natural logarithm of both sides gives

\[
-\lambda T = \ln\left(\frac{1}{2}\right) = -\ln 2,
\]

and therefore

\[
T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}.
\]
Characteristics of Exponential Decay – Average or Mean Life

It is sometimes useful to characterize a radioactive source in terms of the average or mean life of the given isotope, \( \tau \). It can be understood as the sum of the lifetimes of the individual atoms divided by the total number of atoms originally present.

\[
\tau = \frac{1}{\lambda}
\]

(Continued)

Characteristics of Exponential Decay – Average or Mean Life (Continued)

Since

\[
\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)
\]

we have

\[
\tau = \frac{1}{N_0} \int_0^\infty \lambda N_0 e^{-\lambda t} dt.
\]

Remember that

\[
\int xe^{ax} dx = \frac{e^{ax}}{a^2} (ax - 1)
\]

Then

\[
\tau = \frac{\lambda e^{-\lambda t}}{\lambda^2} (-\lambda t - 1) \bigg|_0^\infty = \frac{1}{\lambda}
\]

Units for Radioactivity

The Becquerel (Bq) – SI standard unit for radioactivity

The Becquerel is the quantity of radioactive material in which one atom is transform per second.

\[
1 \text{Bq} = 1 \text{pps}
\]

\[
1 \text{Curie (Ci)} = 3.7 \times 10^{10} \text{Bq}
\]

Note that a Becquerel is not the number of particles emitted by the radioactive isotope in 1 s.
Specific Activity (SA)

Specific activity of a sample is defined as its activity per unit mass, given in units of Bq/g or Ci/g.

Specific activity for pure radioisotopes is defined as the number of Becquerels per unit mass.

\[
\text{Specific Activity} = \frac{6.03 \times 10^{23} \text{(atoms/mole)}}{A \text{(g/mole)}} \times \lambda \quad \text{Bq/g}
\]

SA can be related to the half-life (T) of the radionuclide by

\[
SA = \frac{4.18 \times 10^{13}}{A \cdot T} \quad \text{Bq/g}
\]

Specific Activity (Continued)

(a) what is the specific activity of the mercury?

Solution:

\[
\text{SA(Hg)} = \frac{\text{activity from Hg per mL}}{\text{weight of Hg per mL}} = \frac{1.5 \times 10^9 \text{ Bq/mL}}{5 \text{ mg Hg/mL}} = 0.3 \times 10^9 \text{ Bq/mg Hg}
\]

and the specific activity of $^{203}\text{Hg}$ is calculated from

\[
SA = \frac{4.18 \times 10^{13}}{A \cdot T} \quad \text{Bq/g} = \frac{4.18 \times 10^{13}}{203 - 46.5} \cdot \frac{\text{Bq}}{24 \cdot 3600 \text{ s/h}} = 5.2 \times 10^4 \text{ Bq/g}
\]

(b) what fraction of the mercury in the Hg(NO$_3$)$_2$ is $^{203}\text{Hg}$?

Solution:

The weight-fraction of mercury that is tagged is given by

\[
\frac{\text{SA(Hg)}}{\text{SA($^{203}\text{Hg}$)}}
\]

and the specific activity of $^{203}\text{Hg}$ is calculated from

\[
SA = \frac{4.18 \times 10^{13}}{A \cdot T} \quad \text{Bq/g} = \frac{4.18 \times 10^{13}}{203 - 46.5} \cdot \frac{\text{Bq}}{24 \cdot 3600 \text{ s/h}} \cdot \frac{\text{Bq/g}}{5.2 \times 10^4} = 5.2 \times 10^4 \text{ Bq/g}
\]

The weight fraction of $^{203}\text{Hg}$, therefore, is

\[
\frac{\text{SA(Hg)}}{\text{SA($^{203}\text{Hg}$)}} \quad \frac{0.3 \times 10^9 \text{ Bq/mg Hg}}{5.2 \times 10^4 \text{ Bq/g}} \cdot \frac{\text{g}}{\text{g Hg}} = 5.8 \times 10^{-4} \text{ g}^{203}\text{Hg}/\text{g Hg}
\]
Specific Activity (Continued)

(e) what is the specific activity of the Hg(NO$_3$)$_2$?

Solution:

Since an infinitesimal small fraction of the mercury is tagged with $^{208}$Hg, it may be assumed that the formula weight of the tagged Hg (NO$_3$)$_2$ is 324.63 and that the concentration of Hg (NO$_3$)$_2$ is

$$\frac{324.63 \text{ mg Hg (NO$_3$)$_2$}}{200.61 \text{ mg Hg}} \times \frac{5 \text{ mg Hg}}{\text{mL}} = 8.1 \text{ mg Hg (NO$_3$)$_2$/mL.}$$

The specific activity,

$$1.5 \times 10^5 \text{ Bq/mL} \times \frac{8.1 \text{ mg Hg (NO$_3$)$_2$/mL}}{0.5 \text{ Hg (NO$_3$)$_2$/mL}} = 1.9 \times 10^6 \text{ Bq/mg Hg (NO$_3$)$_2$/mL.}$$

Serial Transformation

In many situations, the parent nuclides produce one or more radioactive offsprings in a chain. In such cases, it is important to consider the radioactivity from both the parent and the daughter nuclides as a function of time.

- Due to their short half lives, $^{90}$Kr and $^{90}$Rb will be completely transformed, results in a rapid building up of $^{90}$Sr.
- $^{90}$Y has a much shorter half-life compared to $^{90}$Sr. After a certain period of time, the instantaneous amount of $^{90}$Sr transformed per unit time will be equal to that of $^{90}$Y.
- In this case, $^{90}$Y is said to be in a secular equilibrium.

Indoor Radon

Naturally Occurring Radioactivity – Health Concerns of Radon Gas
Consider a more general case, in which (a) the half-life of the parent can be of any conceivable value and (b) no restrictions are applied on the relative half-lives of both the parent and the daughter.

The number of atoms of the parent A and the daughter B at any given time $t$ are therefore related by

$$N_B = \frac{\lambda_A N_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})$$

**Proof of the Previous Serial Decay Equation**

From Cember, p123-124

of the daughter, it follows that secular equilibrium is a special case of a more general situation in which the half-life of the parent may be of any conceivable magnitude but greater than that of the daughter. For this general case, where the parent activity is not relatively constant,

$$A \to \lambda_A B \to \lambda_B C,$$

the time rate of change of the number of atoms of species $B$ is given by the differential equation

$$\frac{dN_A}{dt} = \lambda_A N_A - \lambda_B N_B.$$  \hspace{1cm} (4.42)

In this equation, $\lambda_A N_A$ is the rate of transformation of species $A$ and is exactly equal to the rate of formation of species $B$, the rate of transformation of isotope $B$ is $\lambda_B N_B$, and the difference between these two rates at any time is the instantaneous rate of growth of species $B$ at that time.

According to Eq. (4.18), the value of $\lambda_A$ in Eq. (4.42) may be written as

$$N_A = N_A e^{-\lambda_A t}.$$  \hspace{1cm} (4.43)

Equation (4.42) may be rewritten, after substituting the expression above for $N_A$ and transposing $\lambda_B N_B$, as

$$\frac{dN_A}{dt} + \lambda_B N_B = \lambda_A N_A e^{-\lambda_A t}.$$  \hspace{1cm} (4.44)

Equation (4.44) is a first-order linear differential equation of the form

$$\frac{dP}{dx} + F(x) g(x) = Q(x),$$

and may be integrable by multiplying both sides of the equation by $e^{\int F dx}$ as

$$e^{\int F dx} (\int e^{\int F dx} Q dx).$$

and the solution to Eq. (4.45) is

$$\int e^{\int F dx} g(x) dt = \int e^{\int F dx} Q dx.$$  \hspace{1cm} (4.46)

Since $N_A$, $\lambda_A$, and $\lambda_A N_A e^{-\lambda_A t}$ from Eq. (4.44) are represented in Eq. (4.46) by $y$, $F$, and $Q$, respectively, the solution of Eq. (4.44) is

$$N_A e^{-\lambda_A t} = \int \lambda_A N_A e^{-\lambda_A t} dt + C.$$  \hspace{1cm} (4.47)

or, if the two exponentials are combined, we have

$$N_A e^{-\lambda_A t} = \int \lambda_A N_A e^{(1-e^{-\lambda_A t})} dt + C.$$  \hspace{1cm} (4.48)
Proof of The Serial Decay Equation (Continued)

\[ N_{A,e}^{-1} \delta t = \int \lambda_{A} N_{A} e^{\lambda_{A} \delta t} dt + C, \quad (4.48) \]

If the integrand in Eq. (4.48) is multiplied by the integrating factor \( \lambda_{B} - \lambda_{A} \), then Eq. (4.48) is in the form

\[ e^{-\lambda_{B} \delta t} = e^C + C \quad (4.49) \]

and the solution is

\[ N_{A,e}^{-1} \delta t = \frac{1}{\lambda_{B} - \lambda_{A}} \lambda_{A} N_{A} e^{\lambda_{A} \delta t} + C. \quad (4.50) \]

The constant \( C \) may be evaluated by applying the boundary conditions

\[ N_{A} = 0 \quad \text{when} \quad t = 0 \]

\[ 0 = \frac{1}{\lambda_{B} - \lambda_{A}} \lambda_{A} N_{A} + C \]

\[ C = -\frac{\lambda_{B}}{\lambda_{B} - \lambda_{A}} N_{A} \quad (4.51) \]

If the value for \( C \), from Eq. (4.51), is substituted into Eq. (4.50), the solution for \( N_{B} \)

\[ N_{B} = \frac{\lambda_{A} N_{A}}{\lambda_{B} - \lambda_{A}} (e^{-\lambda_{A} \delta t} - e^{-\lambda_{B} \delta t}) \]

General Case

Consider a more general case, in which (a) the half-life of the parent can be of any conceivable value and (b) no restrictions are applied on the relative half-lives of both the parent and the daughter.

\[ A \xrightarrow{\lambda_{A}} B \xrightarrow{\lambda_{B}} C, \]

The number of atoms of the parent \( A \) and the daughter \( B \) at any given time \( t \) are therefore related by

\[ N_{B} = \frac{\lambda_{A} N_{A}}{\lambda_{B} - \lambda_{A}} \left( e^{-\lambda_{A} t} - e^{-\lambda_{B} t} \right) \]

Activity Peaking Times Under General Case

\[ Q_{\text{tot}}(t) = Q_{A}(t) + Q_{B}(t) = \lambda_{A} N_{A} e^{-\lambda_{A} t} + \frac{\lambda_{B} \lambda_{A} N_{A}}{\lambda_{B} - \lambda_{A}} (e^{-\lambda_{A} t} - e^{-\lambda_{B} t}). \]

\[ Q_{A}(t) = \lambda_{A} N_{A} = N_{A} e^{-\lambda_{A} t} \]

\[ Q_{B}(t) = \lambda_{B} N_{B} = \frac{\lambda_{B} \lambda_{A} N_{A}}{\lambda_{B} - \lambda_{A}} (e^{-\lambda_{A} t} - e^{-\lambda_{B} t}) \]
Activity Peaking Time Under General Case

The peak-reaching-time for the activity from the daughter can be derived as the following:

Start from the equation for the general case

\[ \lambda_B N_B = \frac{\lambda_B \lambda_A N_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \]

Differentiate respect to \( t \) and set to zero

\[ \frac{d(\lambda_B N_B)}{dt} = \frac{\lambda_B \lambda_A N_A}{\lambda_B - \lambda_A} (-\lambda_A e^{-\lambda_A t} + \lambda_B e^{-\lambda_B t}) = 0, \]

and therefore

\[ \ln \frac{\lambda_B}{\lambda_A} = (\lambda_B - \lambda_A) t \]

\[ t_{\text{end}} = t_{\text{end}} = \ln \left( \frac{\lambda_B}{\lambda_A} \right) = \frac{2.3 \log(\lambda_B/\lambda_A)}{\lambda_B - \lambda_A}. \]

Activity Peaking Times Under General Case

Similarly, the peak reaching times for the total activity is ...

The total activity is

\[ Q(t) = \lambda_B N_B(t) + \lambda_A N_A(t) \]

Since

\[ Q_B(t) = \lambda_B N_B(t) = \frac{\lambda_B \lambda_A N_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \]

and

\[ Q_A(t) = \lambda_A N_A(t) = \frac{\lambda_B \lambda_A N_A}{\lambda_B - \lambda_A} (e^{-\lambda_B t} - e^{-\lambda_A t}) \]

Differentiate respect to \( t \) and set to zero, we have

\[ \frac{dQ_{\text{tot}}}{dt} = -\lambda_A^2 N_A e^{-\lambda_A t} + \frac{\lambda_B \lambda_A N_A}{\lambda_B - \lambda_A} (e^{-\lambda_B t} - e^{-\lambda_A t}) = 0. \]

Solving for \( t \), it can be shown that

\[ t = t_{\text{end}} = \frac{1}{\lambda_B - \lambda_A} \ln \left( \frac{\lambda_B^2}{2 \lambda_A \lambda_B - \lambda_A^2} \right). \]

Further Discussions on Serial Transformations

Now, I know the question burning in your mind is, “What if species \( C \) is also radioactive?” This is certainly possible; in fact some of the most important and interesting problems in health physics involve long chains of products, one decaying to the next until a stable species is reached. So let’s solve for the situation:

\[ A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C \xrightarrow{\lambda_C} D \]
Activity Peaking Times Under General Case

\[ \frac{d N_C}{dt} = \lambda_B N_B - \lambda_C N_C \]

The number of atoms of C can be found by substitution into the above equation. The activity of C at any time is found in

\[ A_C(t) = A_B(t) \left( \frac{\lambda_B - \lambda_C}{\lambda_B - \lambda_A} e^{-\lambda_C t} + \frac{\lambda_C}{\lambda_B - \lambda_C} e^{-\lambda_B t} \right) + \frac{\lambda_B}{\lambda_B - \lambda_C} e^{-\lambda_B t} \]

Chapter 1: Radioactivity

Further Discussions on Serial Transformations

Figure 3.11 The $^{226}$Ra decay series.
Several Special Cases

For the following serial transformation:

\[ A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C, \]

where \( \lambda_A \ll \lambda_B \) and \( T_A \gg T_B \), \( B \) is said to be in secular equilibrium. For example:

\[ ^{90}\text{Kr} \xrightarrow{\frac{1}{33} \text{s}} ^{89}\text{Rb} \xrightarrow{2.74 \text{min}} ^{90}\text{Sr} \xrightarrow{28.8 \text{years}} ^{90}\text{Y} \xrightarrow{64.2 \text{h}} ^{90}\text{Zr}. \]

Activity Peaking Times Under General Case

\[ Q_{\text{tot}}(t) = \lambda_A N_A e^{-\lambda_A t} + \frac{\lambda_B \lambda_A N_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}). \]

\[ Q_A(t) = N_A e^{-\lambda_A t} \]

\[ Q_B(t) = \frac{\lambda_B N_A}{\lambda_B - \lambda_A} \left( e^{-\lambda_A t} - e^{-\lambda_B t} \right). \]

\[ t = t_0 = \frac{1}{\lambda_B - \lambda_A} \ln \left( \frac{\lambda_B}{\lambda_B - \lambda_A} \right). \]

General Case

\[ N_B = \frac{\lambda_A N_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}). \]

Secular Equilibrium

\[ N_B = \frac{\lambda_A N_A}{\lambda_B} (1 - e^{-\lambda_B t}). \]
Secular Equilibrium: $T_A >> T_B (\lambda_A \ll \lambda_B)$

From this relationship,

$$N_B = \frac{\lambda_A}{\lambda_B} N_A (1 - e^{-\lambda_B t})$$

one can see that

1. As the time goes by, $e^{-\lambda_B t}$ decreases and $Q_B$ approaches $Q_A$.

At equilibrium, we have

$$\lambda_A N_A = \lambda_B N_B \quad \text{and} \quad Q_A = Q_B$$

2. Since $A$ has a relatively long half-life, $Q_A$ may be considered as a constant. So the total activity converges to a constant.

No Equilibrium: When $T_A < T_B$ and $\lambda_A > \lambda_B$

- The half-life of the daughter exceeds that of the parent, no equilibrium is possible.
- The number of parent atoms gradually decay to zero.
- The activity of the daughter rises to the maximum and then decays at its own characteristic rate.

<table>
<thead>
<tr>
<th>$^{90}$Kr</th>
<th>$^{89}$Sr</th>
<th>$^{90}$Sr</th>
<th>$^{90}$Y</th>
<th>$^{90}$Zr</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>37</td>
<td>38</td>
<td>39</td>
<td>83</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$\beta$</td>
<td>$\beta$</td>
<td>$\beta$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>2.74 min</td>
<td>28.8 years</td>
<td>64.2 h</td>
<td>39 Zr</td>
<td></td>
</tr>
</tbody>
</table>

Summary of Serial Transformations

$$A \xrightarrow{\lambda_A} B \xrightarrow{\lambda_B} C,$$

<table>
<thead>
<tr>
<th>$T_A &gt; T_B$</th>
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<th>$T_A &gt; T_B$</th>
<th>$T_A &lt; T_B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transient Equilibrium</td>
<td>Secular Equilibrium</td>
<td>No Equilibrium</td>
<td>Transient Equilibrium</td>
</tr>
<tr>
<td>$Q_A = Q_B$</td>
<td>$Q_B = \frac{\lambda_A}{\lambda_B} Q_A$</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>
Secular Equilibrium: $T_A >> T_B$ ($\lambda_A << \lambda_B$)

From "Radiation Protection and Dosimetry", by Michael Stabin.

Figure 3.11 The $^{228}$Ra decay series.

We can continue on with a species D, E, F, and so on, but the relationships among the species obviously become more complicated and are difficult to categorize. If Species A is very long-lived, however, relative to other members of the chain, after a long time (seven to ten half-lives of the longest-lived progeny species), all the members of the chain will be in secular equilibrium and decaying with the half-life of Species A, and all having the same activity as Species A. An important example is the $^{228}$Ra decay series (Figure 3.11).

Secular Equilibrium: $T_A >> T_B$ ($\lambda_A << \lambda_B$)

Now this assumes that all of the species stay in the same place. If we are talking about uranium ores buried deep in the earth and of large size, this may be reasonable. Radon-222 is a noble gas, so it will tend to diffuse through most structures. If a uranium deposit is large, throughout most of the ore, radon diffusion out of a region will be balanced by other radon diffusion in, and all of the species will have about the same activity. If we dig up that ore, however, the digging and excavation process will disturb the radon and it will be dispersed, so the equilibrium will be maintained only from $^{228}$U down to $^{228}$Ra, and we will need another 30 days or so with the ore in an enclosed space before all of the progeny through $^{214}$Po will come back into equilibrium. In phosphate fertilizers from Florida phosphate deposits (which are rich in uranium), chemical separation processes cause the uranium and similar products to be separated from the radium species, so the $^{228}$Ra becomes the controlling parent. The uranium portions go with the fertilizer (these are not a significant radiation hazard, but they are present), and the radium remains behind in the leftovers ("tails"). If this material is left on reclaimed lands, and people build houses on it, seepage of $^{228}$Ra into the homes can result in significant radiation exposure, from a cancer risk perspective.

From "Radiation Protection and Dosimetry", by Michael Stabin.

An Example

A sample contains 1 mCi of $^{191}$Os at time $t = 0$. The isotope decays by $\beta^-$ emission into metastable $^{191m}$Ir, which then decays by $\gamma$ emission into $^{191}$Ir. The decay and half-lives can be represented by writing:

\[
^{191}\text{Os} \rightarrow \beta^- \rightarrow ^{191m}\text{Ir} \rightarrow ^{191}\text{Ir}.
\]

(c) How many atoms of $^{191}$Ir decay between $t = 100$ s and $t = 102$ s?
(d) How many atoms of $^{191}$Ir decay between $t = 30$ d and $t = 40$ d?
Secular Equilibrium: $T_A >> T_B$

(c) How many atoms of $^{191m}$Ir decay between $t = 100$ s and $t = 102$ s?

\[ \begin{align*}
^{191}_{76} \text{Os} & \rightarrow^{191m}_{77} \text{Ir} \quad \beta^- \quad \lambda_A = 15.4 \text{ d} \quad \lambda_B = 4.94 \text{ s} \\
\end{align*} \]

Since $T_A >> T_B$ and $t = 100-102$ s is longer than 7 times $T_B$, we are looking at a secular equilibrium ...

Therefore, the activity from $^{191m}$Ir is roughly equal to the activity from a constant number of $^{191}$Os.

\[ \lambda_A N_A = \lambda_B N_B \quad \text{and} \quad Q_A = Q_B \]

From this relationship, one can see that $Q_A$ decreases and $Q_B$ approaches $Q_A$.

At equilibrium, we have

2. Since $A$ has a relatively long half life, $Q_A$ may be considered as a constant. So the total activity converges to a constant.

Secular Equilibrium: $T_A >> T_B (\lambda_A << \lambda_B)$

A sample contains 1 mCi of $^{191}$Os at time $t = 0$. The isotope decays by $\beta^-$ emission into metastable $^{191m}$Ir, which then decays by $\gamma$ emission into $^{191}$Ir. The decay and half-lives can be represented by writing

\[ \begin{align*}
^{191}_{76} \text{Os} & \rightarrow^{191m}_{77} \text{Ir} \quad \beta^- \quad \lambda_A = 15.4 \text{ d} \\
^{191m}_{77} \text{Ir} & \rightarrow^{191}_{77} \text{Ir} \quad \gamma \quad \lambda_B = 4.94 \text{ s} \\
\end{align*} \]

(d) How many atoms of $^{191m}$Ir decay between $t = 30$ d and $t = 40$ d?

Solution:

This part is like (c), except that the activities $A_1$ and $A_2$ do not stay constant during the time between 30 and 40 d. Since transient equilibrium exists, the numbers of atoms of $^{191m}$Ir and $^{191}$Os that decay are equal. The number of $^{191m}$Ir atoms that decay, therefore, is equal to the integral of the $^{191}$Os activity during the specified time ($t$ in days):

\[ \int_0^{40} 3.7 \times 10^7 e^{-0.0450t} \, dt = 3.7 \times 10^7 \left[ e^{-0.0450t} \right]_0^{40} = -8.22 \times 10^6 \left( 0.165 - 0.259 \right) = 7.73 \times 10^5. \]
### Summary of Serial Transformations

\[
A \xrightarrow{j_A} B \xrightarrow{j_B} C,
\]

<table>
<thead>
<tr>
<th>General case</th>
<th>Secular Equilibrium</th>
<th>Transient Equilibrium</th>
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<tr>
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<td>(T_A &lt; T_B)</td>
</tr>
<tr>
<td>(t &gt; T_B)</td>
<td>(t &gt; T_B)</td>
<td>(t &gt; T_{md})</td>
<td>---</td>
</tr>
</tbody>
</table>

\[
\frac{dN_A}{dt} = j_A N_A e^{-j_B t} - j_B N_B e^{-j_A t}
\]

\[
Q_a = Q_B
\]

\[
Q_1 = \frac{j_A}{j_B - j_A} Q_B
\]