Neutron Shielding Design
Considerations for Neutron Shielding Design

- Radiological assessment of radiation hazards associated with neutron sources is normally a complex task.
  - Involves both neutron radiation and secondary gamma-ray radiation.
  - Few problems can be solved with elementary techniques.

- General treatment involves considerations on the dose delivered by the elastic scattering of fast neutrons and the thermal neutron capture gamma-ray radiations.
- Neutron removal cross-section. Under special circumstances, the fast neutron dose after the shielding can be derived with an exponential attenuation factor.
Radiation Dose from Fast Neutrons

- Neutron dose is deposited through scattering and neutron induced nuclear reactions.
- In cases of elastic scattering, the scattered nuclei dissipate their energy in the immediate vicinity of the primary neutron interaction. The radiation dose absorbed locally in this way is called the first collision dose. The scattered neutron is not considered after this primary interaction.
- For fast neutrons, the first collision dose rate is given by

\[
\dot{D}_n(E) = \frac{\phi(E)E}{1 \text{ J/kg} \cdot \text{Gy}} \sum_i N_i \sigma_i f,
\]

where
- \( \phi(E) \) = flux of neutrons whose energy is \( E \), in neutrons/cm\(^2\) s,
- \( E \) = neutron energy, in joules,
- \( N_i \) = atoms per kilogram of the \( i \)th element,
- \( \sigma_i \) = scattering across section of the \( i \)th element for neutrons of energy \( E \), in barns \( \times 10^{-24} \) cm\(^2\),
- \( f \) = mean fractional energy transferred from neutron to scattered atom during collision with neutron.
Energy Distributions of Scattered Neutrons

The fraction of energy carried by the scattered neutron is

\[ \alpha = \frac{(M-m)^2}{(M+m)^2} \]

\[ \frac{E'}{E_0} = \frac{M^2 + m^2 + 2Mm \cos \theta}{(M+m)^2} \]

The distribution of the energy of the scattered neutrons is given by

\[ p(E) = \frac{1}{1-\alpha} \frac{1}{E_0}, \ E \in [\alpha E_0, E_0]. \]
Radiation Dose from Thermal Neutrons

Two reactions are normally considered, namely $^{14}\text{N}(n,p)^{14}\text{C}$ and $^{1}\text{H}(n,r)^{2}\text{H}$ reactions.

For the $^{14}\text{N}(n,p)^{14}\text{C}$ reaction, the dose is given by

$$D_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}}{1 \text{ J/kg} \cdot \text{Gy}},$$

where

- $\phi$ = thermal flux, neutrons per cm$^2$ per second,
- $N_N$ = number of nitrogen atoms per kg tissue, $1.49 \times 10^{24}$,
- $\sigma_N$ = absorption cross section for nitrogen, $1.75 \times 10^{-24}$ cm$^2$,
- $Q$ = energy released by the reaction = 0.63 MeV.
Radiation Dose from Thermal Neutrons

For the $^1\text{H}(n, \gamma)^2\text{H}$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

$$A = \phi N_H \sigma_H \text{“Bq”/kg,}$$

where $\phi$ = thermal flux, neutrons per cm$^2$ per second,
$N_H$ = number of hydrogen atoms per kg tissue = $5.98 \times 10^{25}$,
$\sigma_H$ = absorption cross section for hydrogen = $0.33 \times 10^{-24}$ cm$^2$.

The resulting gamma ray dose is illustrated with the following example.
Neutron Shielding – An Example

Example 10.11

Design a shield for an $18.5 \times 10^4$ MBq (5 Ci) Pu-Be neutron source that emits $5 \times 10^6$ neutrons per second, such that the dose rate at the outside surface of the shield will not exceed $15 \, \mu$Sv/h (1.5 mrems/h). The mean energy of the neutrons produced in this source is 4 MeV.

Æ Cember, P452.
Step 1: Shielding for fast neutrons (continued)

Let’s arbitrarily allow for a maximum dose from fast neutrons leaking from the shielding to be 10 μSv/h. We could use either the Table 9.5 (given on the next page) or the equation for fast neutron dose to derive the fast neutron flux that leads to this dose rate would be 3.7 n/s·cm².

Then what is the thickness of a shielding needed to bring the fast neutron flux to this level?

Consider a point source of neutrons, the fast neutron flux after passing through a thickness of nT (cm) could be calculate approximately as

\[
\dot{\phi} = \frac{B \cdot S}{4\pi(nT)^2} \frac{1}{2^n} \left(\frac{\text{neutrons}}{cm^2 \cdot s}\right).
\]

B: the build-up factor, (5 for this case).
T: Half-Valued-Layer (HVL), 3.71 cm.
S: Source strength in neutrons/s, (5×10⁻⁶ neutrons/s).
Radiation Dose from Fast Neutrons

- Neutron dose is deposited through scattering and neutron induced nuclear reactions.
- In cases of elastic scattering, the scattered nuclei dissipate their energy in the immediate vicinity of the primary neutron interaction. The radiation dose absorbed locally in this way is called the first collision dose. The scattered neutron is not considered after this primary interaction.
- For fast neutrons, the first collision dose rate is given by

$$\dot{D}_n(E) = \frac{\phi(E)E \sum_{i} N_i \sigma_i f}{1 \text{ J/kg} \cdot \text{Gy}},$$

where
- $\phi(E) = \text{flux of neutrons whose energy is } E, \text{ in neutrons/cm}^2 \cdot \text{s}$,
- $E = \text{neutron energy, in joules}$,
- $N_i = \text{atoms per kilogram of the } i\text{th element}$,
- $\sigma_i = \text{scattering cross section of the } i\text{th element for neutrons of energy } E, \text{ in barns } \times 10^{-24} \text{ cm}^2$,
- $f = \text{mean fractional energy transferred from neutron to scattered atom during collision with neutron}$. 
The 4MeV fast neutron flux that could introduce a dose equivalent rate of 1mSv/40h (25 μSv/h).

We would need ~ 3.7n/s·cm² to deliver 10 μSv/h or 1 mRem/h dose from fast neutrons.
**Step 1: Shielding for fast neutrons**

Assuming that the shielding is made of water.

For the 4 MeV fast neutrons produced by the Pu-Be source, the cross sections of H and O atoms for elastic scattering are 1.9 barns and 1.7 barns, respectively.

So the linear scattering coefficient of water is given by

\[
\Sigma = 1.9 \times 10^{-24} \text{ cm}^2 \text{ atom}^{-1} \times 6.7 \times 10^{22} \text{ atoms cm}^{-3} + 1.7 \times 10^{-24} \text{ cm}^2 \text{ atom}^{-1} \times 3.35 \times 10^{22} \text{ atoms cm}^{-3}
\]

\[
= 0.186 \text{ cm}^{-1}
\]

which is corresponding to a Half-Valued Layer (HVL) of T=3.71 cm.
Step 1: Shielding for fast neutrons (continued)

Consider a point source of neutrons, the fast neutron flux after passing through a thickness of $nT$ (cm) could be calculated approximately as

$$\phi = \frac{B \cdot S}{4\pi (nT)^2} \frac{5 \times 10^6 \text{ n/s}}{2^n} \left(\frac{\text{ neutrons}}{\text{ cm}^2 \cdot \text{s}}\right).$$

B: the build-up factor, (5 for this case).
T: HVL (3.71 cm).
S: Source strength in neutrons/s, (5 × 10^{-6} neutrons/s).

Solving the above equation for $n$, it gives $n=9$. We would need $9 \times 3.71 = 33$ cm of water to achieve the neutron flux of 3.7 n/s/cm², which ensures the fast neutron dose to stay below 10 $\mu$Sv/h at the surface of the shielding.
Step 2: Shielding for thermal neutron radiation

Considering that the radius of the water filled spherical shielding is 34 cm, which is corresponding to 9 times of the HVL. We could assume that most of the fast neutrons will be attenuated and thermalized at close to the center. So we could **assume the source-shielding volume as a point-source of thermal neutrons placed at the center of a spherical shielding volume with approximately 34 cm radius.**

The thermal neutrons escaping from the surface of the spherical shielding volume could be calculated as

\[
\phi = \frac{S}{4\pi RD} e^{-R/L} \left( \frac{\text{neutrons}}{cm^2 \cdot s} \right) = 0.55 \frac{n}{s \cdot cm^2} \Rightarrow \text{Dose negligible}
\]

- **S**: Strength of the thermal neutron source.
- **R**: Radius of the spherical shielding volume.
- **L**: Thermal diffusion length.
- **D**: Diffusion coefficient.
Fast- and Thermal-Diffusion Lengths

The **fast-diffusion length**: the average straight-line distance covered by fast neutrons traveling in a given medium.

The **thermal-diffusion length**: the average distance covered by thermalized neutrons before it is absorbed. It is measured by the thickness of a slowing down medium that attenuates the beam of thermal neutrons by a factor of $e$. Thus the attenuation of a beam of thermal neutrons by a substance of thickness $t$ (cm), whose thermal diffusion length is $L$ (cm) is given by

$$n = n_0 e^{-t/L}$$

<table>
<thead>
<tr>
<th>Substance</th>
<th>Fast Diffusion Length, cm</th>
<th>Thermal Diffusion Length, cm</th>
<th>Thermal Diffusion Coefficient, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$O</td>
<td>5.75</td>
<td>2.88</td>
<td>0.16</td>
</tr>
<tr>
<td>D$_2$O</td>
<td>11</td>
<td>171</td>
<td>0.87</td>
</tr>
<tr>
<td>Be</td>
<td>9.9</td>
<td>24</td>
<td>0.50</td>
</tr>
<tr>
<td>C (graphite)</td>
<td>17.3</td>
<td>50</td>
<td>0.84</td>
</tr>
</tbody>
</table>
Neutron Induced Reactions

\[ _0^1n + ^{14}_7N \rightarrow ^{14}_6C + ^1_1p \]

- Cross section for thermal neutron is 1.70 barns.
- Q=0.626MeV.
- Since the range of the proton and the $^{14}$C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant dose to soft tissue.
Radiation Dose from Thermal Neutrons

Two reactions are normally considered, namely $^{14}\text{N}(n,p)^{14}\text{C}$ and $^{1}\text{H}(n,r)^{2}\text{H}$ reactions.

For the $^{14}\text{N}(n,p)^{14}\text{C}$ reaction, the dose is given by

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}}{1 \text{ J/kg} \cdot \text{Gy}},$$

where $\phi = \text{thermal flux, neutrons per cm}^2 \text{ per second}$,
$N_N = \text{number of nitrogen atoms per kg tissue, } 1.49 \times 10^{24}$,
$\sigma_N = \text{absorption cross section for nitrogen, } 1.75 \times 10^{-24} \text{ cm}^2$,
$Q = \text{energy released by the reaction } = 0.63 \text{ MeV}.$
Neutron Induced Reactions

\[ ^{1}_{0}n + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{0}γ \]

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy \( Q=2.22\text{MeV} \) released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.
Radiation Dose from Thermal Neutrons

For the $^1\text{H}(n, \gamma)^2\text{H}$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

$$A = \phi N_H \sigma_H \text{“Bq”/kg},$$

where $\phi$ = thermal flux, neutrons per cm$^2$ per second,
$N_H$ = number of hydrogen atoms per kg tissue = $5.98 \times 10^{25}$,
$\sigma_H$ = absorption cross section for hydrogen = $0.33 \times 10^{-24}$ cm$^2$.

The resulting gamma ray dose is illustrated with the following example.
Step 2: Shielding for thermal neutron radiation (continued)

Considering that the radius of the water filled spherical shielding is 34 cm, which is corresponding to 9 times of the HVL. We could assume that most of the fast neutrons will be attenuated and thermalized at close to the center. So we could assume the source-shielding volume as a point-source of thermal neutrons placed at the center of a spherical shielding volume with approximately 34 cm radius.

The thermal neutrons escaping from the surface of the spherical shielding volume could be calculated as

\[ \dot{\phi} = \frac{S}{4\pi RD} e^{-\frac{R}{L}} \left( \frac{\text{neutrons}}{cm^2 \cdot s} \right) = 0.55 \frac{n}{s \cdot cm^2} \]

\( \Rightarrow \) Dose negligible

S: Strength of the thermal neutron source.
R: Radius of the spherical shielding volume.
L: Thermal diffusion length.
D: Diffusion coefficient.
Step 3: Shielding for gamma rays from the water shielding volume

- The energy of each gamma-ray is 2.26 MeV.
- With the spherical water volume of 34 cm radius, there will be 3.7 n/s/cm², or $5 \times 10^4 \text{n/s}$ escaping from the surface.
- We assume that the remaining neutrons will eventually be absorbed by hydrogen atoms, giving rise to the 2.26 MeV photons. Considering the apparent gamma-ray activity is uniformly distributed across the spherical volume, then the gamma-ray activity is

$$A = \frac{5 \times 10^6 (\text{n/s}) - 5.4 \times 10^4 (\text{n/s})}{\frac{4}{3} \pi \cdot (34 \text{cm})^3} = 30 (\text{Bq/cm}^3)$$
Step 3: Shielding for gamma rays (continued)

Now consider a spherical volume filled with uniform radioactivity of $A$ (Bq/cm³), the dose rate at the surface of the sphere is given by

$$D = \frac{1}{2} \cdot C \cdot \Gamma \cdot \frac{4\pi}{\mu} \cdot (1 - e^{-\mu r}) = 9 \times 10^{-3} (\text{mGy/h})$$

A spherical shielding of 34 cm radius filled with water would lead to a fast neutron dose of $10 \ \mu\text{Gy/h}$, a negligible thermal neutron dose, and gamma-ray dose of $9 \ \mu\text{Gy/h}$.

The total is $19 \ \mu\text{Gy/h}$, which is still greater than the $15 \ \mu\text{Gy/h}$ target. What should we do now?
Thermal Neutron Capture by Boron

\[ ^{10}\text{B} + ^{1}\text{n} \rightarrow ^{7}\text{Li} + ^{4}\text{He} + \gamma (0.48 \text{ MeV}) \]

- Capture cross section: 755 barn.
- The 0.48 MeV gamma ray is emitted in 93% of the capture.
Step 4: Improve the shielding efficiency by adding boric acid \((H_3BO_3)\) in water

Consider that we can add boric acid in water, whose formula weight is 61.84 and solubility is 63 g/L, and the thermal absorption coefficient is 775 Barns.

If we add the maximum soluble concentration of boric acid in water, then the concentration of boron atoms is

\[
\frac{63.2 (g/L) \times 10^{-3} (L/mL) \cdot 6.02 \times 10^{23} (molecules/mole)}{61.84 (g/mole)} = 6.17 \times 10^{20} \text{ (atoms/mL)}
\]

Compute the ratio of linear thermal absorption coefficients due to boron and hydrogen atoms,

\[
\frac{\Sigma_H}{\Sigma_B} = \frac{1.9 \times 10^{-24} (cm^2/atom) \cdot 6.7 \times 10^{22} (atoms/cm^3)}{775 \times 10^{-24} (cm^3/atom) \cdot 6.17 \times 10^{20} (atoms/cm^3)} = 0.31
\]

Therefore, for every 1 thermal neutron captured by a hydrogen atom, there will be about 3.23 thermal neutrons each captured by a boron atom.
Step 4: Improve the shielding efficiency by adding boric acid ($H_3BO_3$) in water

Gamma-ray dose due to the 2.2 MeV gamma-rays from **thermal neutron capture by hydrogen atoms** in the water tank can be derived as the following:

* For every 1 thermal neutron captured by a hydrogen atom, there will be about 3.23 thermal neutrons each captured by a boron atom.

* If we assume that all thermal neutrons are captured by either hydrogen or boron atoms, then there will be \( [5 \times 10^6 (n/s) - 5.4 \times 10^4 (n/s)] \times \frac{1}{1+3.23} \) thermal neutrons being **captured by hydrogen atoms** per second.

* The gamma ray dose due to thermal neutrons captured by hydrogen will be reduced by a factor of \( \frac{1}{1+3.23} \) to 2.1 $\mu$Gy/h. (from 9 $\mu$ Gy/h previously with pure water).
Step 4: Improve the shielding efficiency by adding boric acid ($H_3BO_3$) in water

Gamma-ray dose due to gamma rays from **thermal neutron capture by boron atoms** in the water tank can be derived as the following:

Therefore, for every 1 thermal neutron captured by a hydrogen atom, there will be about 3.23 thermal neutrons each captured by a boron atom.

If we assume that all thermal neutrons are captured by either hydrogen or boron atoms, then there will be $[5 \times 10^6 (n/s) - 5.4 \times 10^4 (n/s)] \times \frac{3.23}{1+3.23}$ thermal neutrons being captured by boron atoms per second, leading to an apparent gamma-ray activity of

$$\frac{[5 \times 10^6 (n/s) - 5.4 \times 10^4 (n/s)] \times \frac{3.23}{1+3.23}}{\frac{4}{3} \pi \cdot (34 \text{cm})^3} = 22 \left( \frac{Bq}{cm^3} \right)$$

Therefore the dose rate at the surface of the spherical volume due to thermal neutron capture by boron is

$$\dot{D} = \frac{1}{2} \cdot C \cdot \Gamma \cdot \frac{4\pi}{\mu} \cdot (1 - e^{-\mu r}) = 0.8 \times 10^{-3} \left( m \text{ Gy/h} \right)$$
Step 4: Improve the shielding efficiency by adding boric acid ($H_3BO_3$) in water

- A spherical shielding of 34 cm radius filled with water would lead to a fast neutron dose of 10 μGy/h, a negligible thermal neutron dose, a gamma ray dose of 2.1 μGy/h from thermal neutron capture by hydrogen, and a gamma ray dose of 0.8 μGy/h from thermal neutron capture by boron.
- The total is 12.9 μGy, which is smaller than the 15 μGy/h target.