1. (10 points) (a) From Fig. 6.2, estimate for a 100-eV electron the probability that a given energy-loss event will result in excitation, rather than ionization, in water.

![Image: Fig. 6.2 Attenuation coefficients for excitation, ionization, elastic scattering, and total interaction for electrons in liquid water as functions of energy.]

**Solution:**

\[
\mu_{\text{exc}} \approx 70 \mu m^{-1} \\
\mu_{\text{ion}} \approx 700 \mu m^{-1} \\
P(\text{exc}) = \frac{\mu_{\text{exc}}}{\mu_{\text{exc}} + \mu_{\text{ion}}} \\
= \frac{70}{70 + 700} \\
= 9.09\%
\]

(b) What fraction of the collision at 100 eV are due to elastic scattering?

**Solution:**

\[
R(\text{ela}) = \frac{\mu_{\text{ela}}}{\mu_{\text{total}}} \\
= \frac{1000}{2000} \\
= 50\%
\]

2. (10 points) What is the ratio of the collisional and radiative stopping powers of Al for electrons of energy (a) 10 keV?

(a) 10 keV

**Solution:**

\[
\frac{(-dE/dx)_{\text{rad}}}{(-dE/dx)_{\text{col}}} \approx \frac{ZE}{800} \\
= \frac{13 \times 10^{-2}}{800} \\
= 1.625 \times 10^{-4}
\]
(b) 1 MeV

**Solution:**
\[
\frac{(-dE/dx)_{\text{rad}}}{(-dE/dx)_{\text{col}}} \approx \frac{ZE}{800} = \frac{13 \times 1}{800} = 0.01625
\]

(c) 100 MeV

**Solution:**
\[
\frac{(-dE/dx)_{\text{rad}}}{(-dE/dx)_{\text{col}}} \approx \frac{ZE}{800} = \frac{13 \times 100}{800} = 1.625
\]

3. (10 points) Estimate the range in cm of the maximum-energy beta ray \((2.28 \text{ MeV})\) from \(^{90}\text{Y}\) in bone of density \(1.9 \text{ g/cm}^3\)

**Solution:** Bone can be considered as low-Z materials, so:
\[
R = 0.412T^{1.27 - 0.0954\ln T}
\]
\[= 0.412 \times 2.28^{1.27 - 0.0954\times\ln(2.28)}\]
\[= 1.10 \text{g/cm}^2\]
\[
d = \frac{R}{\rho} = \frac{1.10 \text{g/cm}^2}{1.9 \text{g/cm}^3} = 0.58 \text{cm}
\]

4. (5 points) To protect the cell culture in the last problem from radiation, what advantage would be gained if the positions of the Lucite and the lead were swapped, so that the Lucite was on top?

**Solution:** It would greatly improve the shielding efficiency. With this configuration, the fast electrons will be primarily absorbed in Lucite. Compared to lead, the lower effective Z of Lucite would greatly reduce the fraction of electron energy being converted into bremsstrahlung radiation. Then the addition of the lead would help to stop the bremsstrahlung X-rays.

5. (15 points) (a) Use Fig.6.8 to estimate the probability that a normally incident, 740-keV electron will penetrate a water phantom to a maximum depth between 1,500 \(\mu\text{m}\) and 2,000 \(\mu\text{m}\).
Solution: We can assume the probability density of maximum depth changes linearly from distance 1500 µm to 2000 µm. The corresponding function is given as:

\[ f(x) = 0.00105 - 3.5 \times 10^{-7}x \]

The probability is given as:

\[ P = \int_{1500}^{2000} f(x)dx = \int_{1500}^{2000} (0.00105 - 3.5 \times 10^{-7}x)dx = 0.21875 \]

(b) What is the probability that the pathlength will be between these two distance?

Solution: Similarly, we can assume the probability density of pathlength changes linearly from distance 1500 µm to 2000 µm. The corresponding function is given as:

\[ f(x) = -0.0001 + 1 \times 10^{-7}x \]

The probability is given as:

\[ P = \int_{1500}^{2000} f(x)dx = \int_{1500}^{2000} (-0.0001 + 1 \times 10^{-7}x)dx = 0.0375 \]

6. (20 points) (a) For 0.05-MeV protons in water, what is the smallest value of \( \Delta \) for which \((-dE/dx)_\Delta = (-dE/dx)_\infty \)?

Solution: The minimum value of \( \Delta \) is \( Q_{max} \), which given as:

\[ Q_{max} = \frac{4mME}{(m + M)^2} \]
\[ \Delta_{\text{min}} = Q_{\text{max}} = \frac{4mME}{(m+M)^2} \]
\[ = \frac{4 \times 9.10 \times 10^{-31} \times 1.67 \times 10^{-27} \times 0.05 \text{MeV}}{(9.10 \times 10^{-31} + 1.67 \times 10^{-27})^2} \]
\[ = 109 \text{eV} \]

(b) Repeat for 0.10-MeV protons.

\[ \Delta_{\text{min}} = Q_{\text{max}} = \frac{4mME}{(m+M)^2} \]
\[ = \frac{4 \times 9.10 \times 10^{-31} \times 1.67 \times 10^{-27} \times 0.10 \text{MeV}}{(9.10 \times 10^{-31} + 1.67 \times 10^{-27})^2} \]
\[ = 218 \text{eV} \]

(c) Are your answers consistent with Table 7.1?

\[ \text{Solution}: \text{ In Table 7.1 } Q_{\text{max}} \text{ of 0.05-MeV and 0.10-MeV protons are given as 109 eV and 220 eV respectively. Therefore, it is properly to conclude that the answers we obtained are consistent with Table 7.1} \]

7. (20 points) What is the specific ionization of a 12-MeV proton in tissue if an average of 22 eV is needed to produce an ion pair?

\[ \text{Solution}: \text{ Tissue can be considered as water. } \beta^2 \text{ and } F(\beta) \text{ are:} \]
\[ \beta^2 = 0.02099 \times \frac{12}{10} = 0.025188 \]
\[ F(\beta) = \ln \frac{1.02 \times 10^6 \beta^2}{1 - \beta^2} - \beta^2 \]
\[ = \ln \frac{1.02 \times 10^6 \times 0.025188}{1 - 0.025188} - 0.025188 \]
\[ = 10.15425 \]

The stopping power of tissue for a 12-MeV proton is:
\[ - \frac{dE}{dx} = \frac{0.170}{\beta^2} [F(\beta) - 4.31] \text{MeV/cm} \]
\[ = \frac{0.170}{0.025188} (10.15425 - 4.31) \text{MeV/cm} \]
\[ = 39.44 \text{MeV/cm} \]

The specific ionization is:
\[ \frac{39.44 \text{MeV/cm}}{22 \text{eV}} = 1.8 \times 10^6 \text{cm}^{-1} \]

8. (10 points) (a) From the numerical analysis of Fig. 7.3 given in Section 7.5, how many total ionizations per proton would be expected at a pressure of 0.7 atm?
Solution: According to Page 163 of Turner’s textbook, an energy loss of 34 keV by a proton would produce 1360 ionizations in the gas at 0.2 atm pressure. So a proton of the same energy traveling through the same gas volume but at a higher pressure of 0.7 atm would produce

\[1360 \times 3.5 = 4760\] ionizations

(b) How many of these ionizations would be produced by secondary electrons?

Solution: Collisions produced by proton:

\[570 \times 3.5 = 1995\]

Collisions produced by secondary electrons:

\[4760 - 1995 = 2765\]