Chapter 2:

Introduction to Physical Principles in Radiological Imaging

Reading Material:

Chapter 1, Radiation Detection and Measurements,

Forth Edition, by G. F. Knoll

Beta Decay Sources for Nuclear Imaging

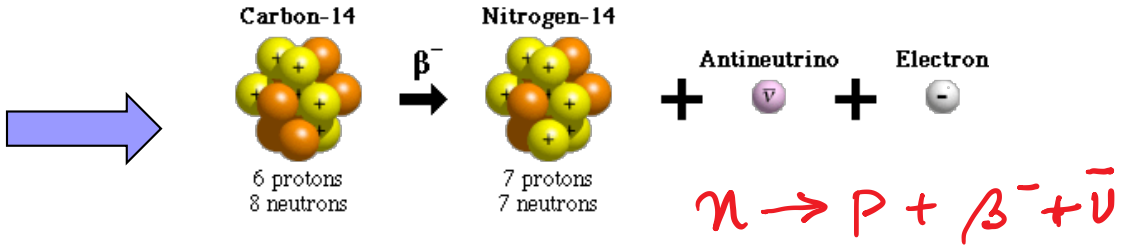
- Beta decay process and subsequent gamma-ray emission
- What are the desired properties of radionuclides (e.g., Tc-99m) used for diagnostic imaging?
- Positron decay
  - What is positron decay?
  - What are the positron range and non-collinearity of annihilation gamma rays?
  - How do they affect the image quality of PET?

Beta Emission

- Beta particle is an ordinary electron. Many atomic and nuclear processes result in the emission of beta particles.
- One of the most common source of beta particles is the beta decay of nuclides, in which

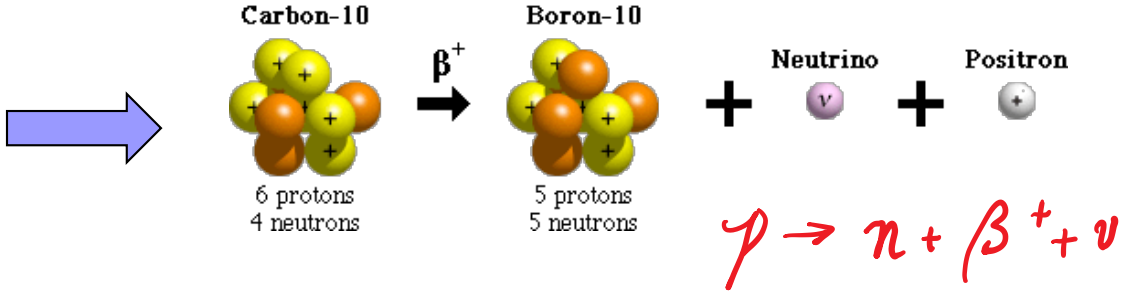
Beta decay

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\upsilon} $$



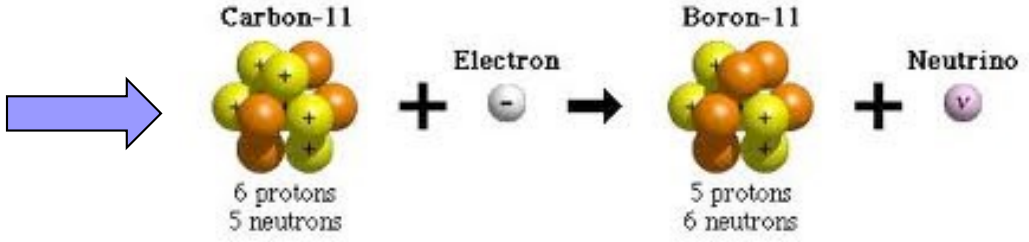
Beta-plus decay

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}\beta + \upsilon $$



Electron capture

$$ {}^{A}_{Z}X + e^{-} \rightarrow {}^{A}_{Z-1}Y + \upsilon $$

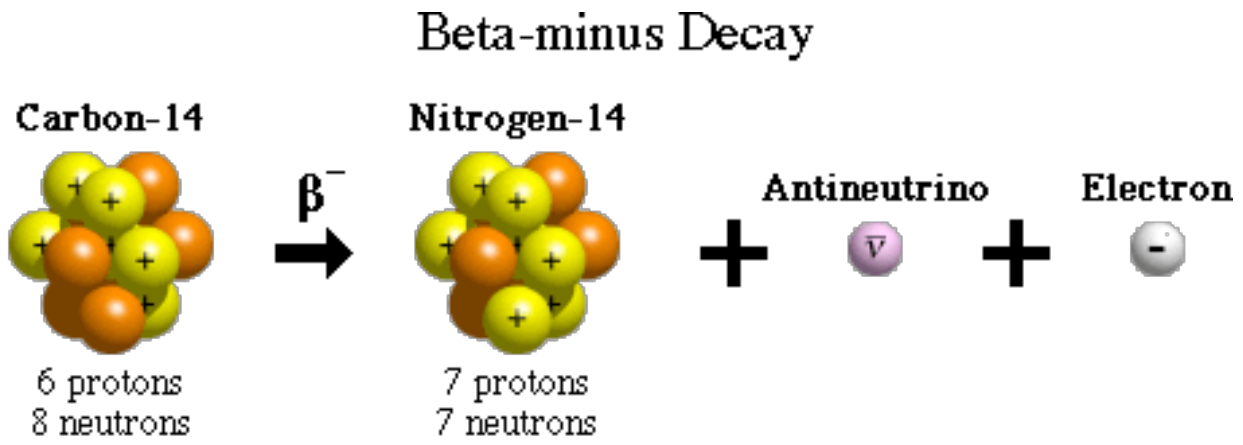


Energy Release of Beta Decay

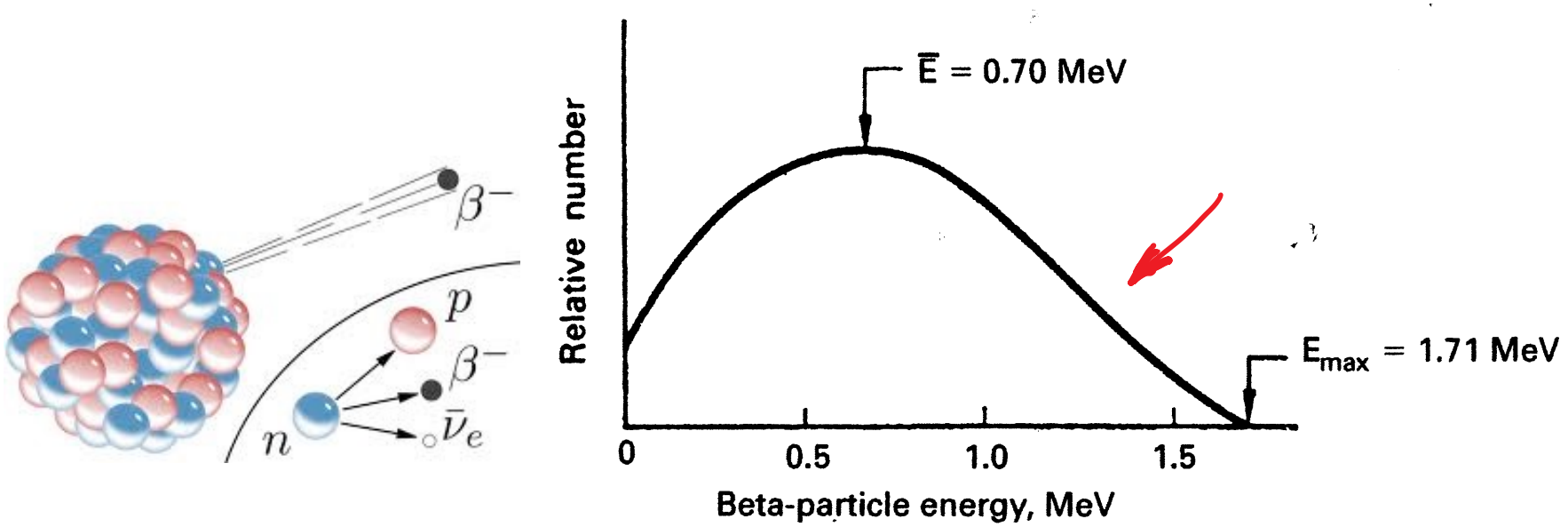
The energy release in a beta decay is given as

$$Q = M_p - (M_d + M_e)$$

- The energy release is once again given by the conversion of a fraction of the mass into energy. Note that atomic electron bonding energy is neglected.
- For a beta decay to be possible, the energy release has to be positive.



Typical Energy Spectrum of Beta Particles

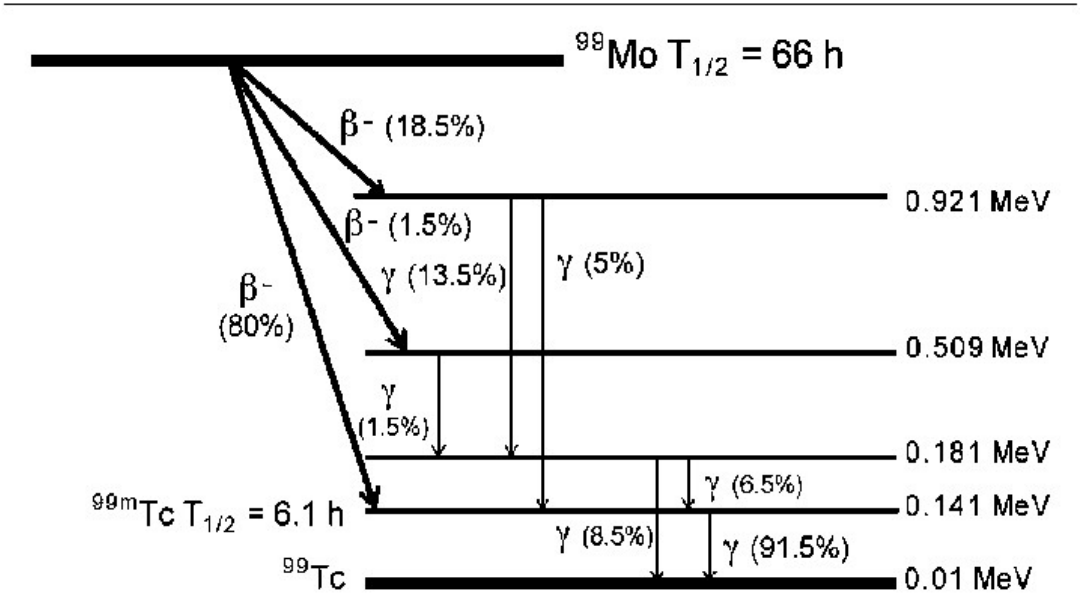


- The energy release is shared by all three daughter products. Due to the relatively large mass of the daughter nucleus, it attains only a small fraction of the energy. Therefore, the kinetic energy of the beta particle is

$$E_{\beta^-} \approx Q - E_{\bar{\nu}}$$

Metastable Nuclear States and Gamma Ray Emission

Another example of a metastable nuclide is $^{99m}_{43}Tc$, which results from the beta decay of the molybdenum isotope $^{99}_{42}Mo$.
an isomeric transition (IT) to the ground



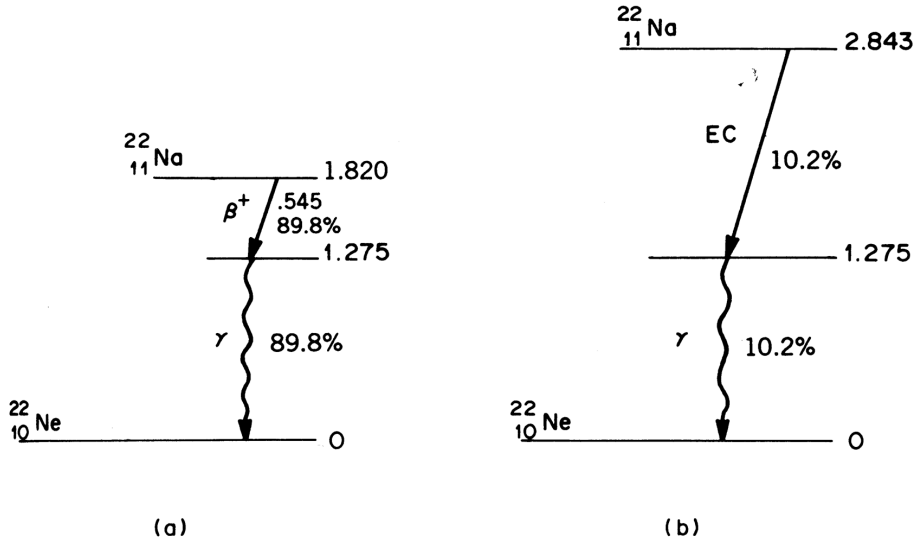
Why is Tc-99m attractive for emission tomography and for clinical studies?

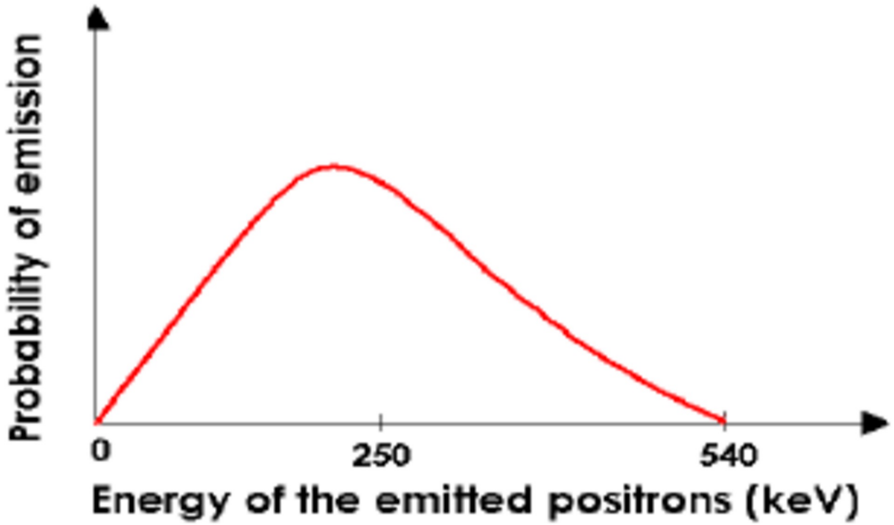
- The 6-hour half-life is appropriate for imaging.
- The energy of the emitted gamma ray (140 keV) is reasonable for detection
- Bio- and chemical-compatibility.
- Easy to produce at a reasonably low cost, typically with a Tc-99m generator.

Positron Decay

Orbital Electron Capture and Positron Decay

- Electron capture and positron decay are normally competing processes through which a neutron deficient nucleus may attain an increased stability.
- Both the emission of a positron and the capture of an electron, a neutrino is always emitted in order to conserve energy.
- In positron decay, the neutrino carries the difference between the energy release and the energy of the resultant positron. In electron capture, however, the neutrino must be mono-energetic.





Annihilation Radiation following Positron Emission

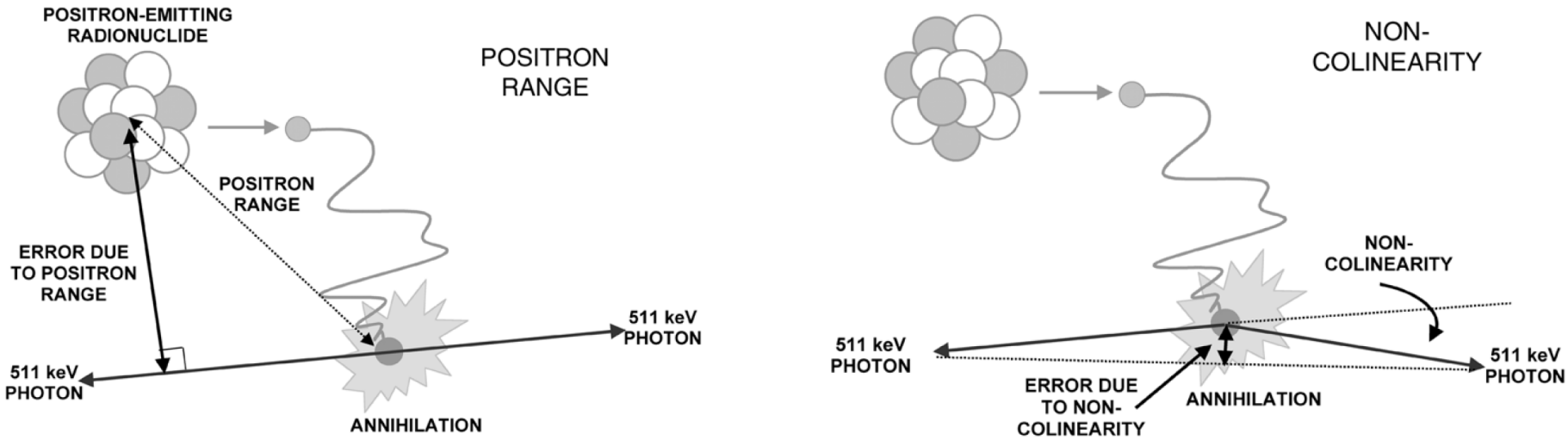


FIGURE 4. Error in determining the location of the emitting nucleus due to positron range (top) and noncolinearity (bottom). The positron range error is dependent on the energy of the emitted positrons. Noncolinearity is independent of radionuclide, and the error is determined by the separation of the detectors. The deviation from noncolinearity is highly exaggerated in the figure; the average angular deviation from 180° is about ± 0.25°. (Reproduced with permission from Cherry SR, Sorenson JA, Phelps ME. *Physics in Nuclear Medicine*, W.B. Saunders, New York, 2003.)

Physics in Nuclear Medicine, 4th Edition by Simon R. Cherry, James A. Sorenson, Michael E. Phelps

Positron Range

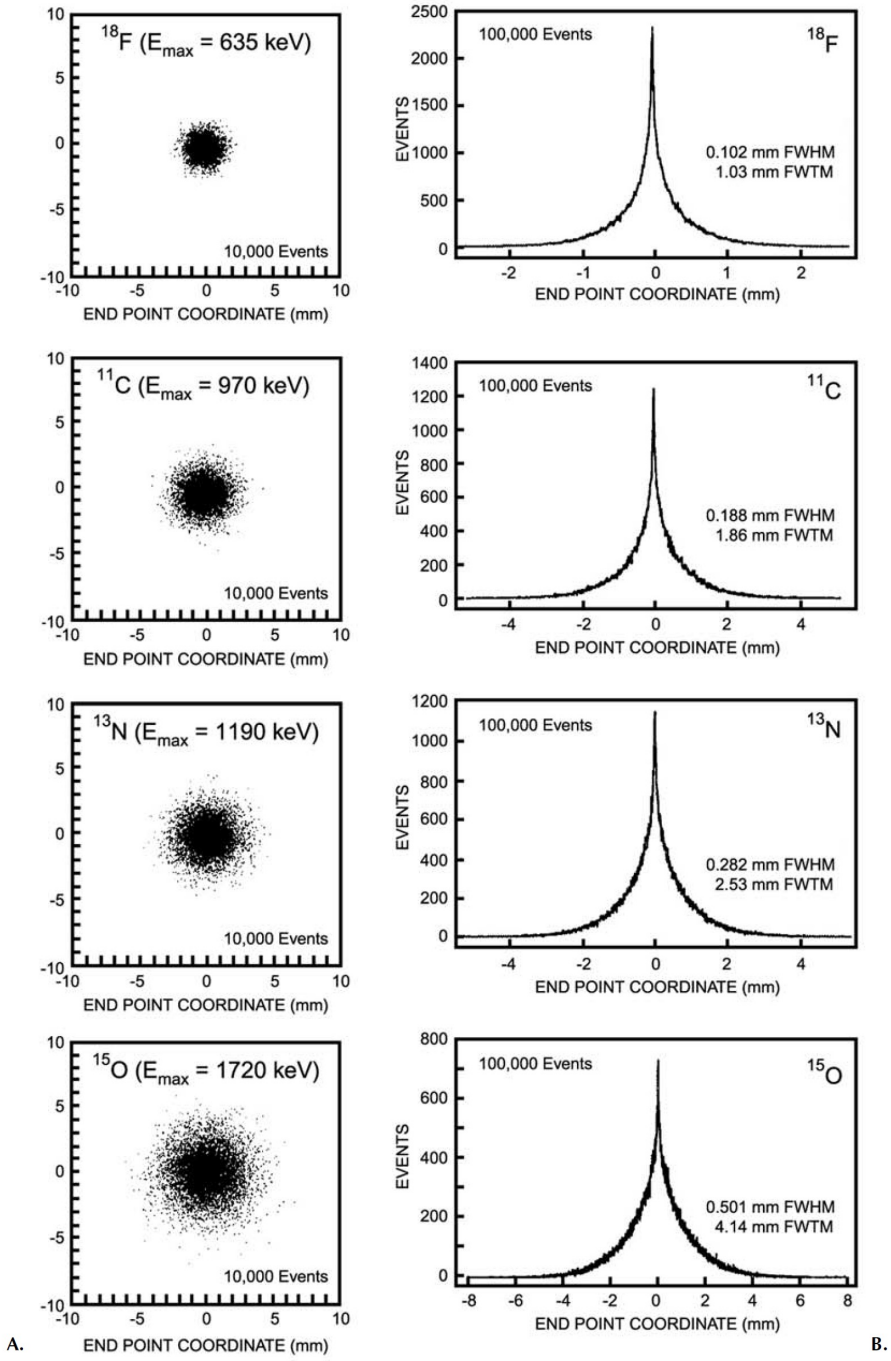


FIGURE 5. A: Simulations for several PET radionuclides showing the distribution of positron annihilation sites in water for positrons emitted at the center of the image (position 0.0 mm). B: Profiles through the simulated distributions showing measured FWHM and FWTM of the distributions. Abbreviations: FWHM, full width at half maximum; FWTM, full width at tenth maximum. (Reproduced with permission from Levin C, Hoffman EJ. *Phys Med Biol* 1999, 44: 781–799.)

Noncollinearity of the Annihilation Gamma Rays

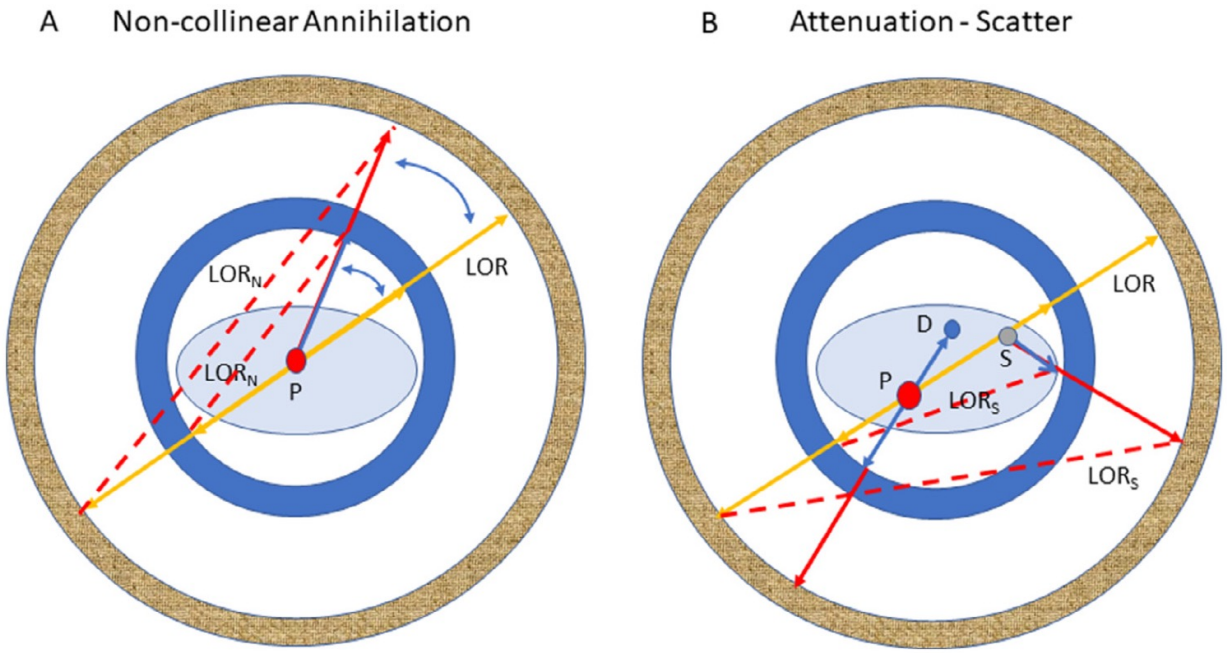


Figure 5 Illustration of the effects of noncolinear annihilation (**A**) and attenuation/scatter (**B**) in two PET scanners with large (brown) and small (blue) ring diameters. P: annihilation site; LOR_N, erroneous line-of-response for a noncolinear annihilation; LOR, correct line-of-response for a colinear annihilation; S, location of a 511 keV scattering event; LOR_S, erroneous line-of-response for a scatter event; D: absorption of either gamma ray will distort the true activity distribution in the reconstructed image of the object. Because of the very large mass difference, scatter and attenuation are much reduced in rodents compared to adult human subjects but are not ignorable. While attenuation for a single mouse plus bed is generally negligible, multiple mice and rats require attenuation correction.

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Seminars in
NUCLEAR
MEDICINE

Advances in Preclinical PET

Stephen S. Adler, PhD,*,‡ Jurgen Seidel, PhD,†,‡ and Peter L. Choyke, MD‡

Noncollinearity

noncolinearity. This effect is independent of radionuclide because the positrons must lose most of their energy before they can annihilate; hence, the initial energy is irrelevant. The distribution of emitted angles is roughly Gaussian in shape, with a full width at half maximum (FWHM)* of ~0.5°. After detecting the annihilation photons, PET assumes that the emission was exactly back to back, resulting in a small error in locating the line of annihilation (Figure 4 bottom). Assuming a Gaussian distribution and using the fact that the angles are small, the blurring effect due to noncolinearity, Δ_{nc}, can be estimated as:

$$\Delta_{nc} = 0.0022 \times D \tag{10}$$

where D is the diameter of the PET scanner. The error increases linearly as the diameter of the PET scanner increases. Once again, the effect is relatively small compared with the detector resolution in most clinical PET scanners. In PET scanners used for animals, D generally is small, and as illustrated in Example 4, noncolinearity is not a major limiting factor at the present time.

EXAMPLE 4

Calculate the blurring due to photon noncolinearity in an 80-cm diameter PET scanner designed for imaging humans and in a 15-cm diameter PET scanner designed for imaging small animals.

ANSWER

From Equation 10, the blurring is calculated as:

80-cm human scanner:

$$\Delta_{nc} = 0.0022 \times D = 0.0022 \times 800 \text{ mm} = 1.76 \text{ mm}$$

15-cm small-animal scanner:

$$\Delta_{nc} = 0.0022 \times D = 0.0022 \times 150 \text{ mm} = 0.33 \text{ mm}$$

A Few Details on Positron Decay and Their Implications to PET

Isotope	half-life (min)	Maximum positron energy (MeV)	Positron range in water (FWHM in mm)	Production method
^{11}C	20.3	0.96	1.1	cyclotron
^{13}N	9.97	1.19	1.4	cyclotron
^{15}O	2.03	1.70	1.5	cyclotron
^{18}F	109.8	0.64	1.0	cyclotron
^{68}Ga	67.8	1.89	1.7	generator
^{82}Rb	1.26	3.15	1.7	generator

Table 2. Properties of commonly used positron emitting radio-isotopes

Element of life.

Beta Emitters for Radiotherapy

A few things to consider:

- Interactions of beta particles in tissue
- What are the ideal characteristics for a beta-emitting radionuclide to be used for therapeutic applications?
- What are the major challenges and limitations associated with the use of beta-emitting radionuclides for therapy?
- How do internally deposited beta particles deliver dose to tissue?
  - What are the types of radiation involved in the dose delivery (e.g., beta particles, internal conversion electrons, auger electrons, X-rays and gamma rays)?, and
  - Where do these particles come from?

Common isotopes used in nuclear medicine [9][10]

isotope	symbol	Z	$T_{1/2}$	decay	gamma (keV)	positron (keV)
Imaging:						
fluorine-18	^{18}F	9	109.77 m	β^+	511 (193%)	249.8 (97%)[11]
gallium-67	^{67}Ga	31	3.26 d	ec	93 (39%), 185 (21%), 300 (17%)	-
krypton-81m	^{81m}Kr	36	13.1 s	IT	190 (68%)	-
rubidium-82	^{82}Rb	37	1.27 m	β^+	511 (191%)	3.379 (95%)
nitrogen-13	^{13}N	7	9.97 m	β^+	511 (200%)	1190 (100%)[12]
technetium-99m	^{99m}Tc	43	6.01 h	IT	140 (89%)	-
indium-111	^{111}In	49	2.80 d	ec	171 (90%), 245 (94%)	-
iodine-123	^{123}I	53	13.3 h	ec	159 (83%)	-
xenon-133	^{133}Xe	54	5.24 d	β^-	81 (31%)	0.364 (99%)
thallium-201	^{201}Tl	81	3.04 d	ec	69–83* (94%), 167 (10%)	-
Therapy:						
yttrium-90	^{90}Y	39	2.67 d	β^-	-	2.280 (100%)
iodine-131	^{131}I	53	8.02 d	β^-	364 (81%)	0.807 (100%)

Z = atomic number, the number of protons; $T_{1/2}$ = half-life; decay = mode of decay

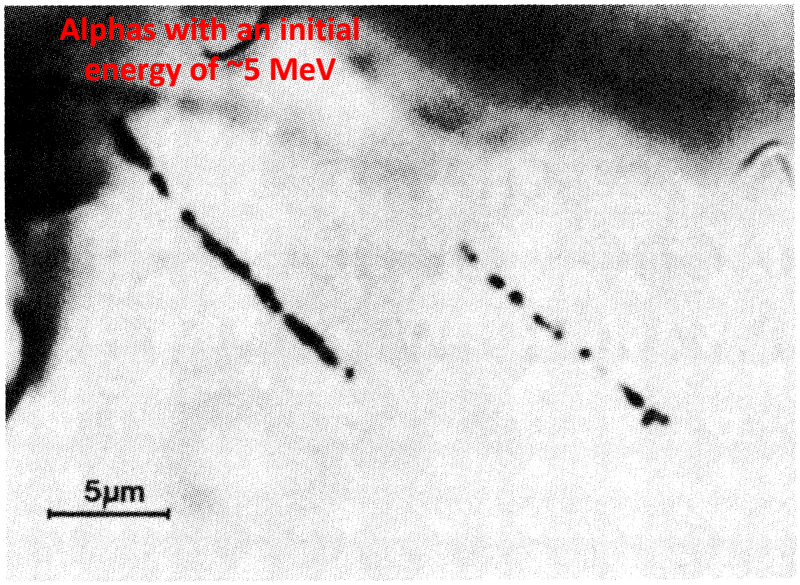
photons = principle photon energies in kilo-electron volts, keV, (abundance/decay)

β = beta maximum energy in mega-electron volts, MeV, (abundance/decay)

β^+ = β^+ decay; β^- = β^- decay; IT = isomeric transition; ec = electron capture

* X-rays from progeny, mercury, Hg

Energy Loss Mechanisms



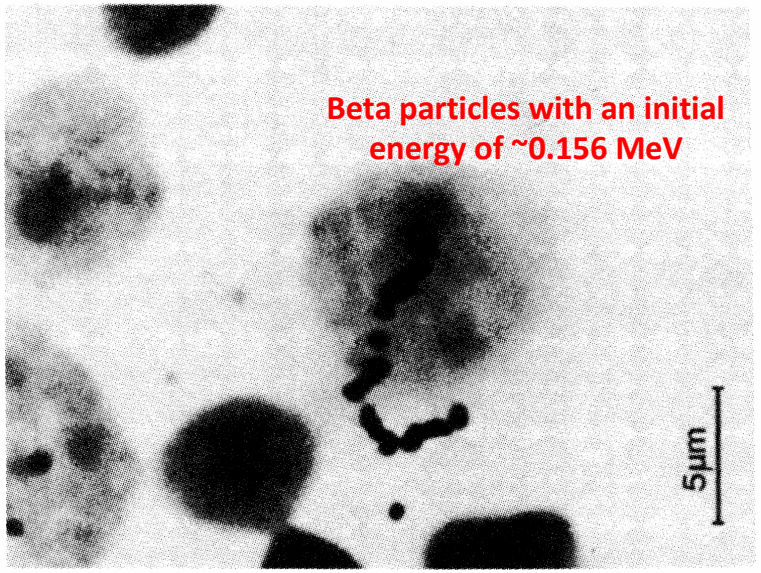


FIGURE 5.1. (Top) Alpha-particle autoradiograph of rat bone after inhalation of ^{241}Am. Biological preparation by R. Masse and N. Parmentier. (Bottom) Beta-particle autoradiograph of isolated rat-brain nucleus. The ^{14}C-thymidine incorporated in the nucleolus is located at the track origin of the electron emitted by the tracer element. Biological preparation by M. Wintzerith and P. Mandel. (Courtesy R. Rechenmann and E. Wittendorp-Rechenmann, Laboratoire de Biophysique des Rayonnements et de Methodologie INSERM U.220, Strasbourg, France.)

Single Collision Energy-Loss Spectrum

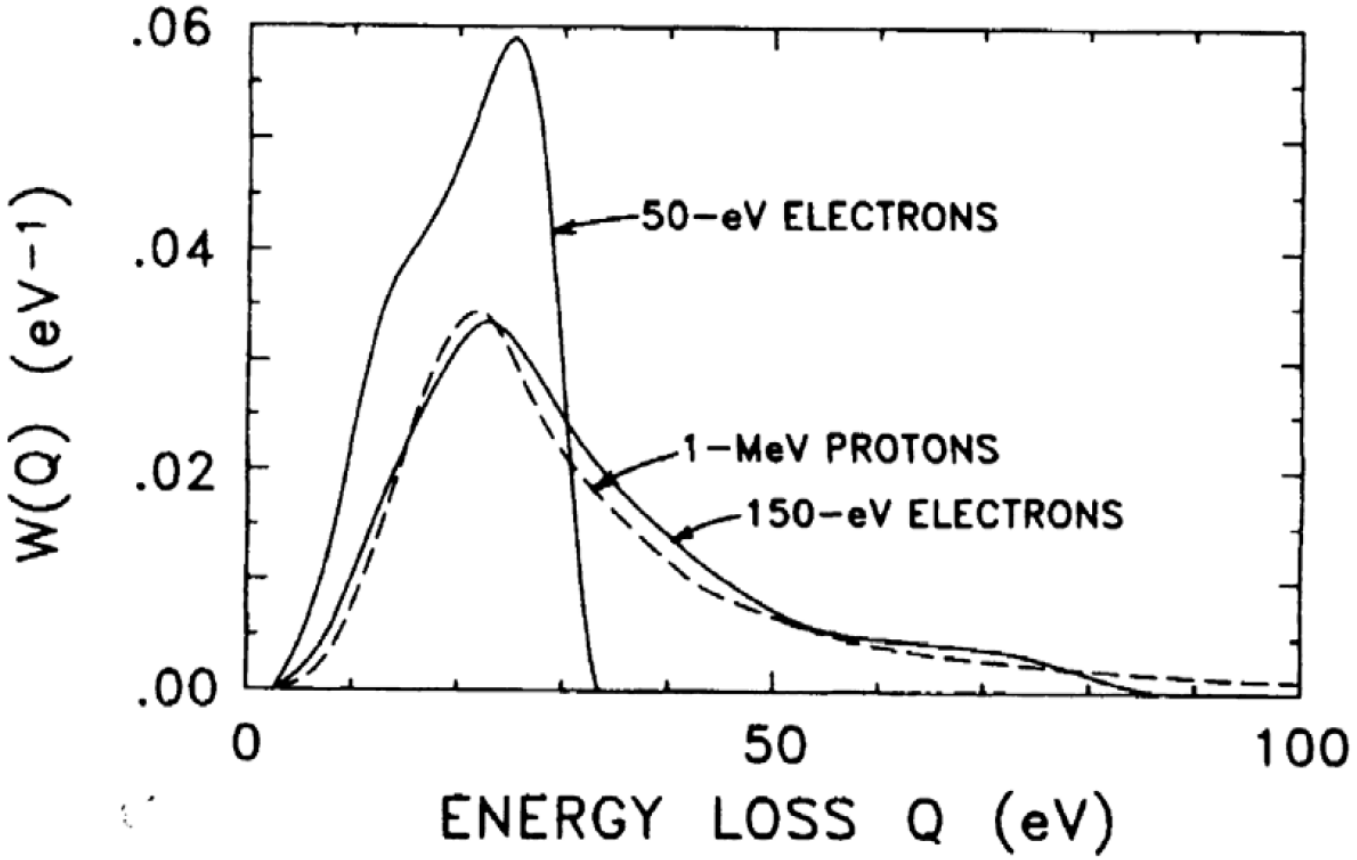
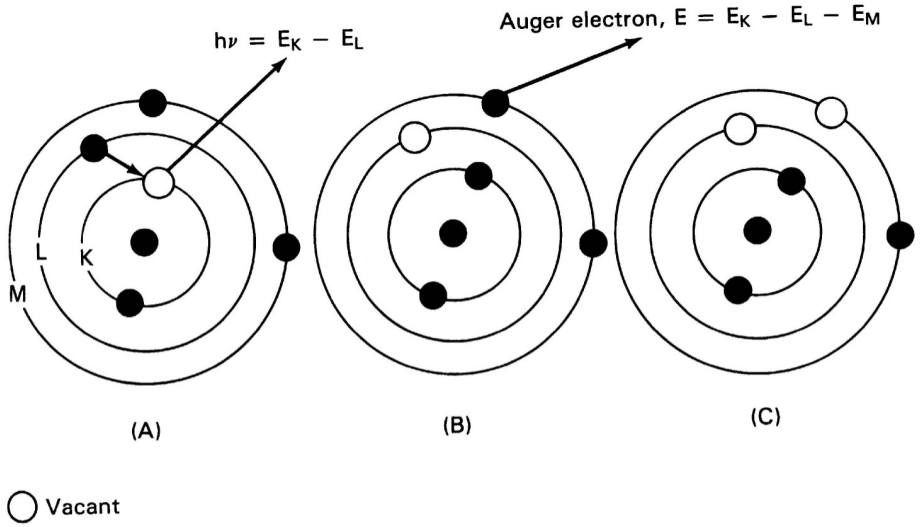


Fig. 5.3 Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)

Auger Electrons

- The excitation energy of the atom may be transferred to one of the outer electrons, causing it to be ejected from the atom.
- Auger electrons are roughly the analogue of internal conversion electrons when the excitation energy originates in the atom rather than in the nucleus.



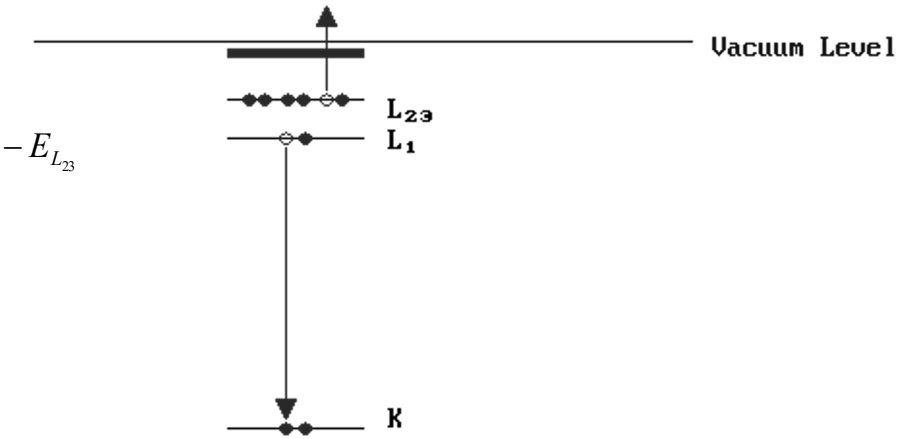


Figure 3.7 (A) The usual emission of a K characteristic X-ray, $h\nu$, energy equal to $E_K - E_L$, the difference in binding energy for the two orbital electrons, K and L. (B) $h\nu$ has been absorbed and a monoenergetic Auger electron is emitted, in the example shown, from the M shell, the energy of which is $E_K - E_L - E_M$. (C) In its final state the atom has vacancies in the L and M orbitals.

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EJNMMI Radiopharmacy and Chemistry

REVIEW Open Access

Auger electrons for cancer therapy – a review

Anthony Ku[1†], Valerie J. Facca[1†], Zhongli Cai[1] and Raymond M. Reilly[1,2,3,4*]

Table 1 Properties of Auger electron-emitting radionuclides [a]

Auger electrons (AEs)					Internal conversion (IC) electrons		
Radionuclide	Half-life	AEs/decay	Average AE energy per decay (keV)	Average energy per AE (keV)	IC electrons/decay	Average IC electron energy released per decay (keV)	Average energy per IC electron (keV)
^{125}I	57 d	23.0	12.0	0.5	0.9	7.3	7.7
^{123}I	13 h	13.7	7.2	0.5	0.2	21.0	222.6
^{67}Ga	78 h	5.0	6.6	1.3	0.3	29.7	14.1
^{99m}Tc	6 h	4.4	0.9	0.2	1.1	15.2	13.8
^{111}In	67 h	7.4	6.9	0.9	0.2	27.9	176.1
^{201}Tl	73 h	20.9	14.8	0.7	0.9	29.9	32.9
^{191}Pt	2.8 d	14	17.8	1.3	304	57.1	0.2
^{193m}Pt	4.3 d	27.4	10.9	0.4	3.0	126.8	42.4
^{195m}Pt	4.0 d	36.6	23.1	0.6	2.8	161.4	58.1
^{197}Hg	64.1 h	23.2	16.1	0.7	0.8	54.1	67.0
^{197m}Hg	23.8 h	19.4	13.5	0.7	1.6	203.5	127.0
^{119}Sb	38.2 h	23.7	8.9	0.4	0.8	17.0	20.2
^{161}Tb [b]	6.9 d	0.9 [c]	5.1 [c]	5.7	1.4	36.7	26.2

[a] The number of AEs and IC electrons were obtained from MIRD Radionuclide and Decay Schemes (Eckerman and Endo 2008)
[b] The number of AEs and IC electrons were obtained from the National Nuclear Data Center for ^{161}Tb (65-Terbium-161 2011)
[c] Calculation based on K and L shell Auger electrons only

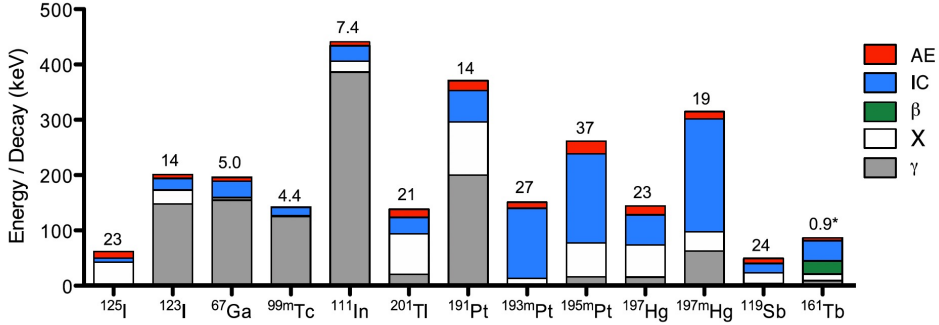


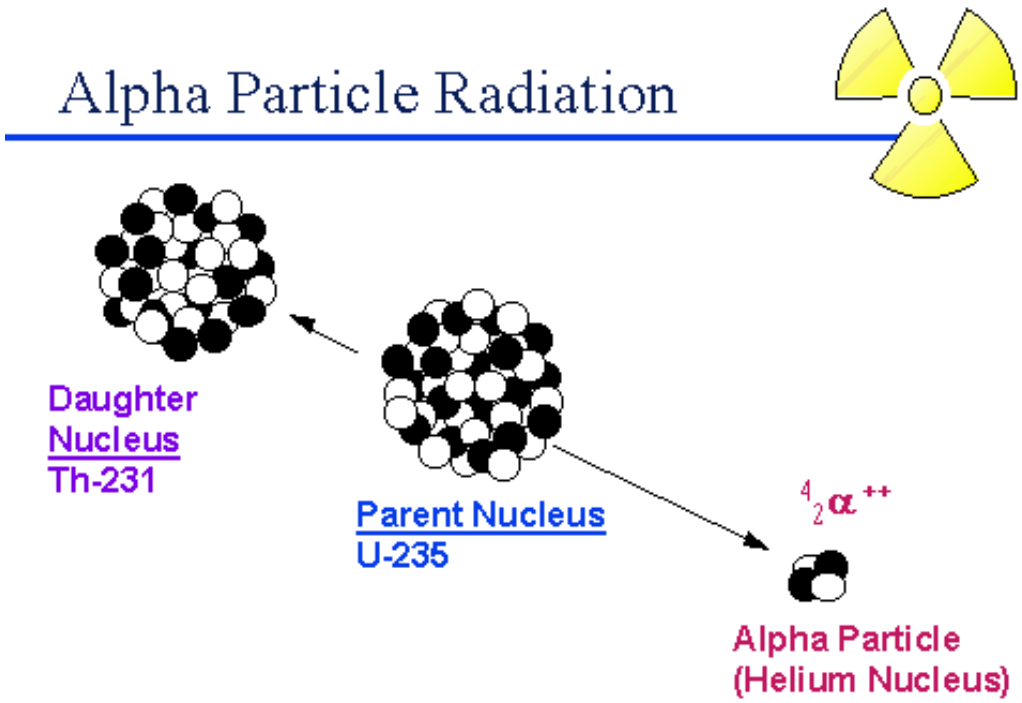
Fig. 1 Energy contribution of γ-photons, X-rays, β-particles, internal conversion (IC) electrons, and Auger electrons (AEs) per decay event for several radionuclides. Energy estimates are based on MIRD Radionuclide Data and Decay Schemes (Eckerman and Endo 2008), and the National Nuclear Data Center for ^{161}Tb (65-Terbium-161 2011). * Number of K- and L- shell Auger electrons only

Alpha Emitters for Radiotherapy

- Alpha-decay process and the interactions of alpha particles in tissue
- Why is alpha therapy attractive?
- What challenges must alpha therapy overcome to be an effective therapeutic modality?

Alpha Emission

- An alpha particle is a highly energetic helium nucleus consisting of two neutrons and 2 protons.
- It is normally emitted from isotopes when the neutron-to-proton ratio is too low – through the so-called alpha decay.
- Atomic number and atomic mass number are conserved in alpha decays



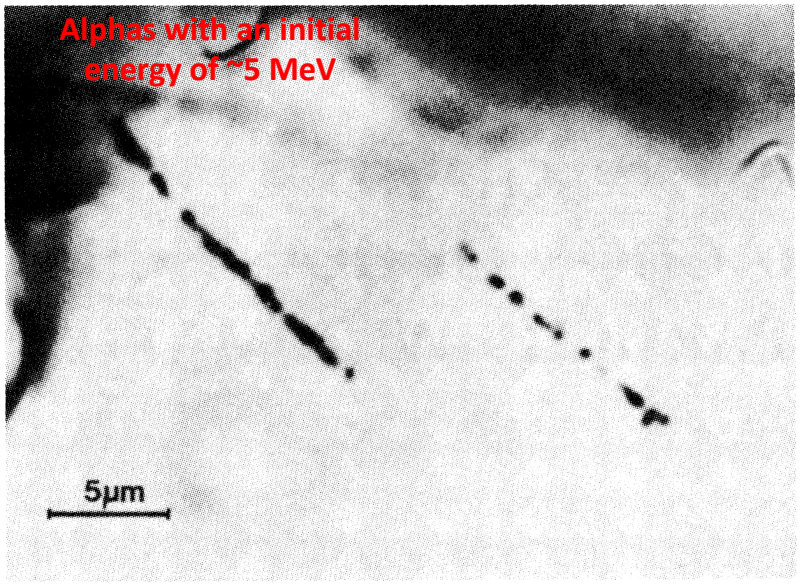
Energy Release in Alpha Emission

- The required kinetic energy has to come from the decrease in mass following the decay process.
- The relationship between mass and energy associated with an alpha emission is given as

$$M_p = M_d + M_\alpha + 2M_e + Q, \tag{4.1}$$

where M_p, M_d, M_α, and M_e are respectively equal to the masses of the parent, the daughter, the emitted alpha particle, and the two orbital electrons that are lost during the transition to the lower atomic numbered daughter, while Q is the total energy release associated with the radioactive transformation.

Energy Loss Mechanisms



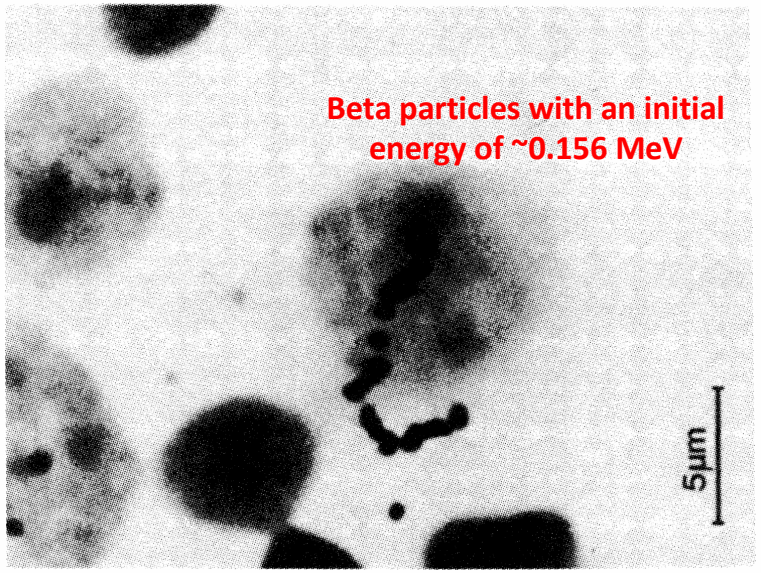
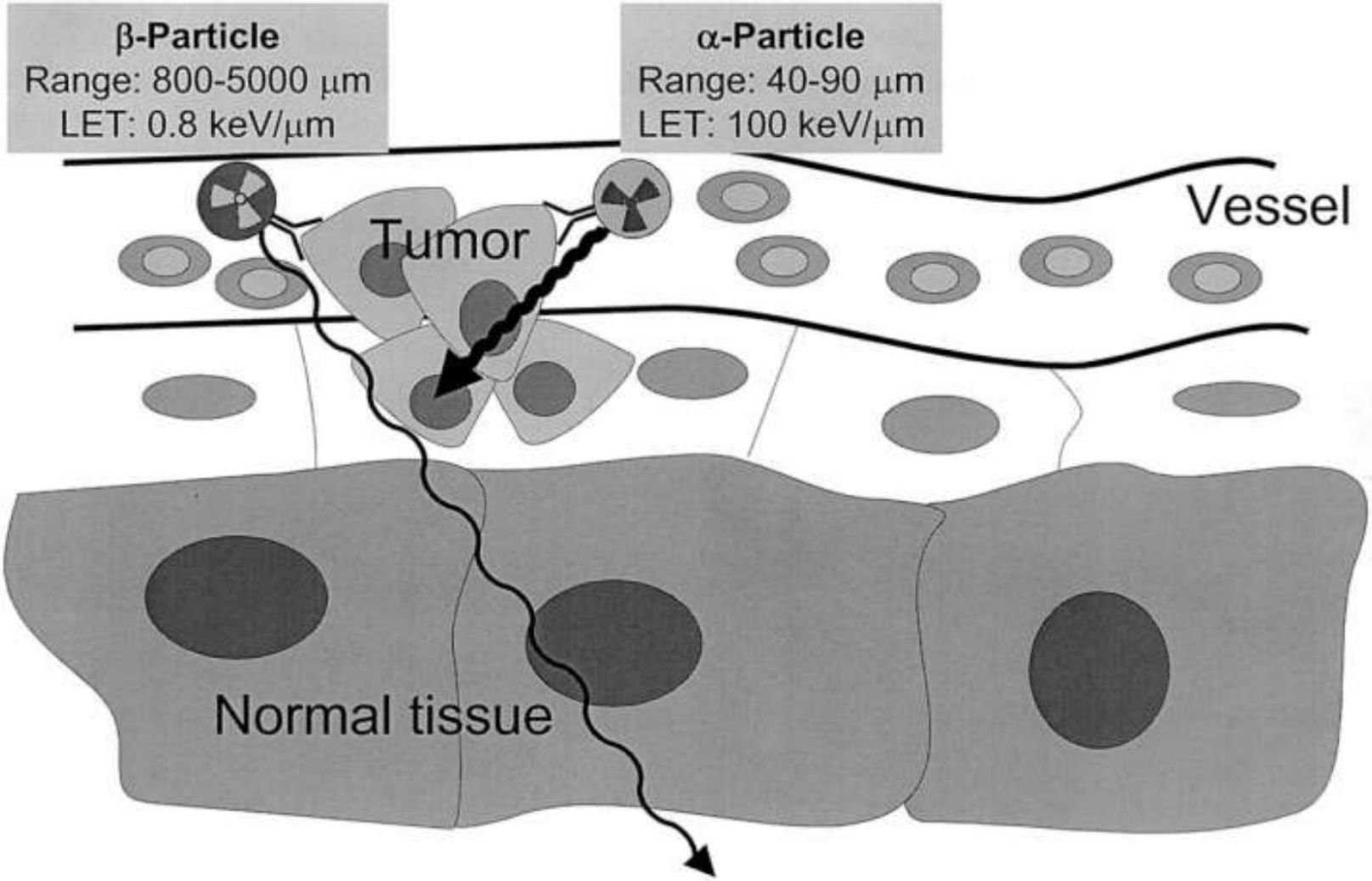


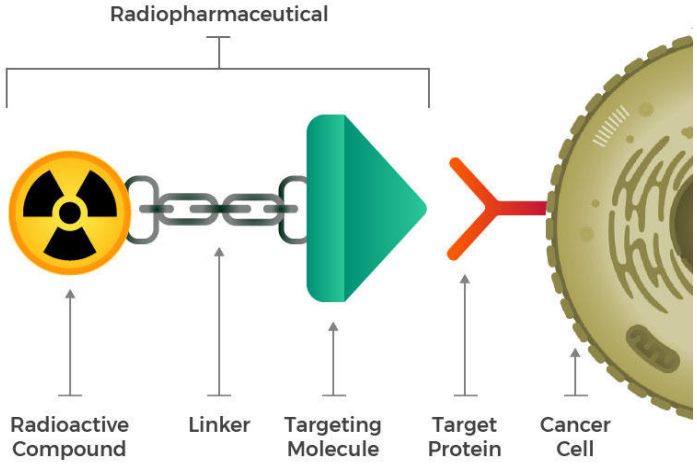
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Targeted Therapy – Basic Principles



http://www.icrp.org/docs/icrp2017/18%20Lassman%20Presentation.pdf

Radiopharmaceuticals:
Radiation Therapy Enters the Molecular Age



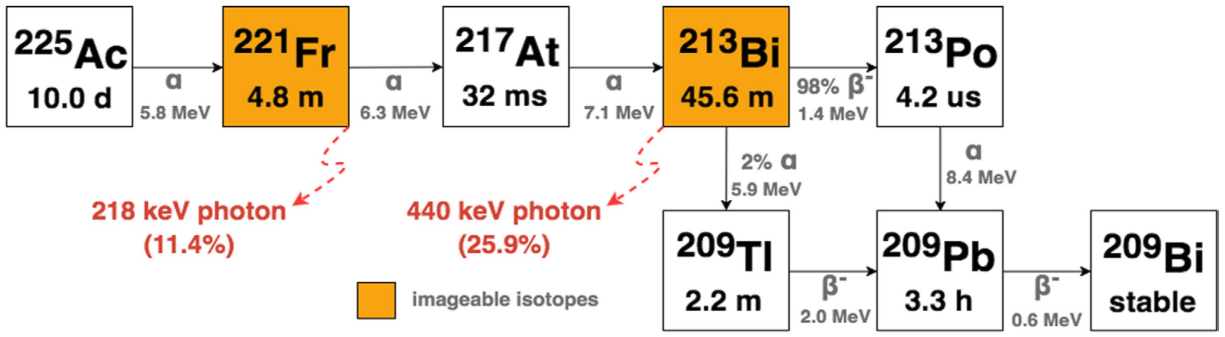
https://www.cancer.gov/news-events/cancer-currents-blog/2020/radiopharmaceuticals-cancer-radiation-therapy

Table 1 | **Radionuclide properties**

Radionuclide	Therapeutic emission	Approximate emission range in tissue (mm)	Radionuclide half-life
Yttrium-90	β^-	5.30	64.1 hours
Iodine-131	β^-	0.8	8.0 days
Samarium-153	β^-	0.4	46.5 hours
Lutetium-177	β^-	0.62	6.6 days
Astatine-211	α	0.05	7.2 hours
Lead-212/bismuth-212	β^-/α	<0.1/0.05	10.6 hours/1.0 hours
Radium-223	α	0.05–0.08	11.4 days
Actinium-225	α	0.05–0.08	10.0 days
Thorium-227	α	0.05–0.08	18.7 days

Half-lives from REF.[297], ranges from REFS[298–300].

"Radiopharmaceutical therapy in cancer: clinical advances and challenges", George Sgouroset al., Nature Reviews Drug Discovery volume 19, pages 589–608 (2020)



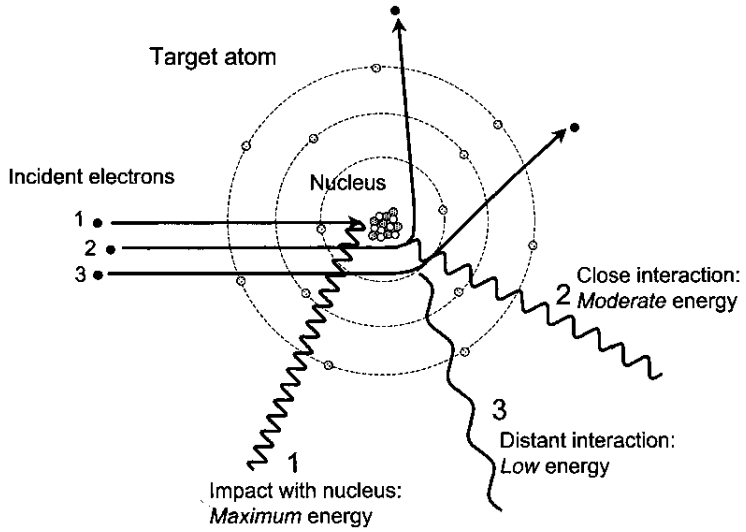
The above figure was extracted from Robertson et al. 2017, PMB

X-Rays Sources

- Could you please explain the operational principles of an X-ray generator?
- Physical properties of the X-rays from an X-ray generator
- What characteristics should an ideal X-ray generator for imaging possess?
  - What are the physical aspects of the regular X-ray generator that prevent us from achieving these ideal properties?
  - How do we improve?

X-ray Generation – Bremmstrahlung

- Target nucleus positive charge ($Z \cdot p^+$) attracts incident e^-
- Deceleration of an incident e^- occurs in the proximity of the target atom nucleus
- E lost by e^- is gained by the EM photon (x-ray) generated
  - The impact parameter distance, the closest approach to the nucleus by the e^- determines the amount of E loss
  - The Coulomb force of attraction varies strongly with distance ($\propto 1/r^2$); ↓ distance → ↑ deceleration and E loss → ↑ photon E
  - Direct impact on the nucleus determines the maximum x-ray E (E_{max})



X-ray Generators



https://en.wikipedia.org/wiki/X-ray_tube

“Regular” X-ray tube

LONGCARE MEDICAL

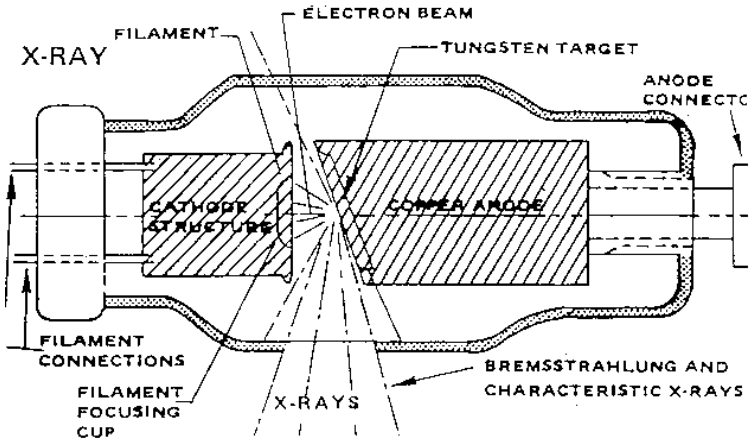
https://longcaremed.en.made-in-china.com/product/EfqpkTiCanhd/China-X-Ray-Tube-Generator-160kv-200-Kv-X-Ray-Scanner-Spare-Part.html



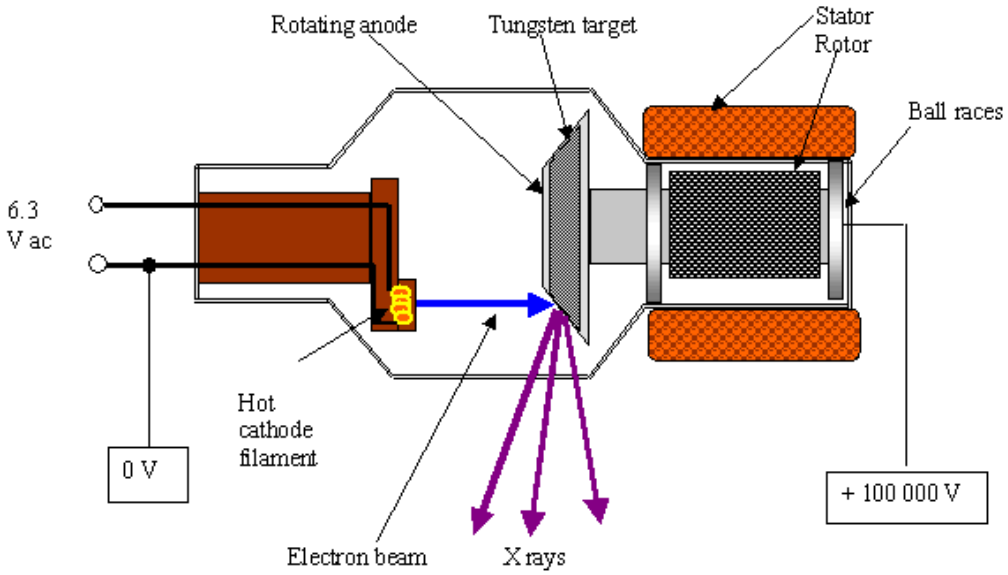
Rotating anode X-ray tube

Physical Designs Considerations

How do we deal with the heat generated from electron bombardment?



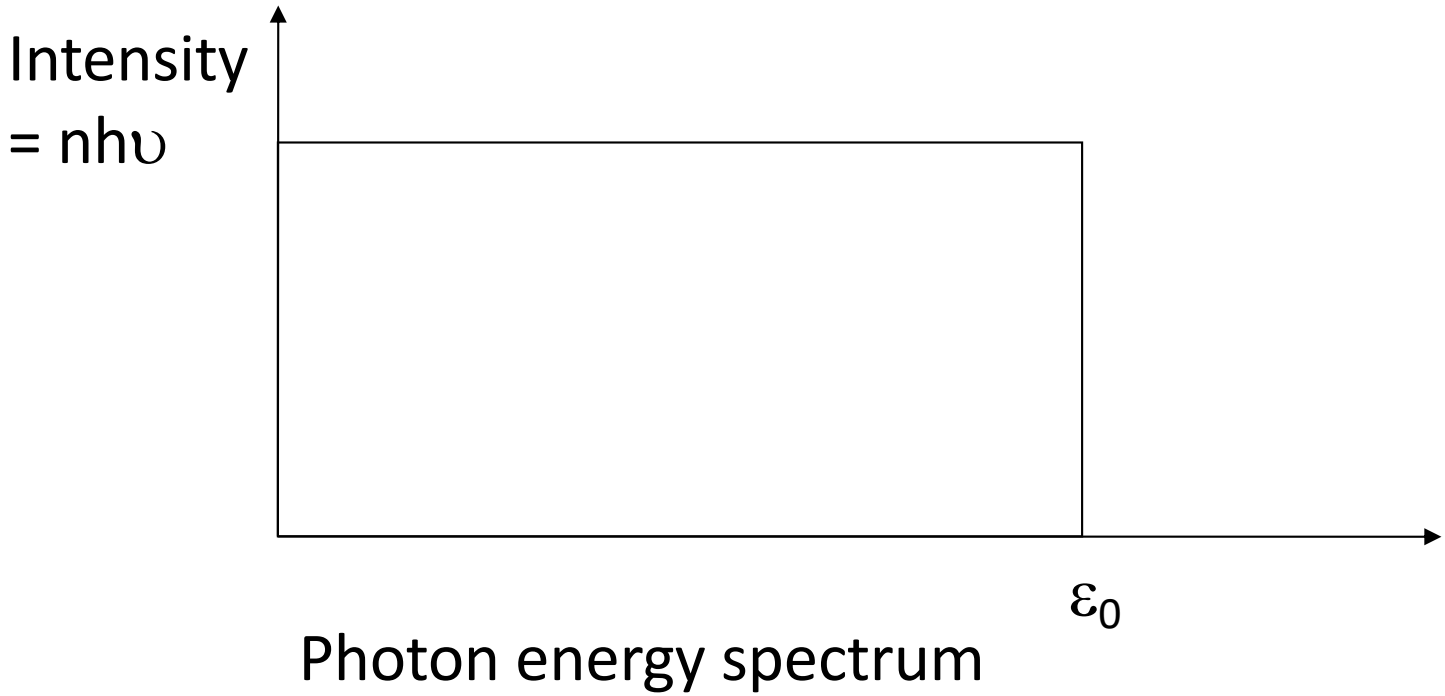
Typical micro-focus X-ray source



Rotating anode X-ray source

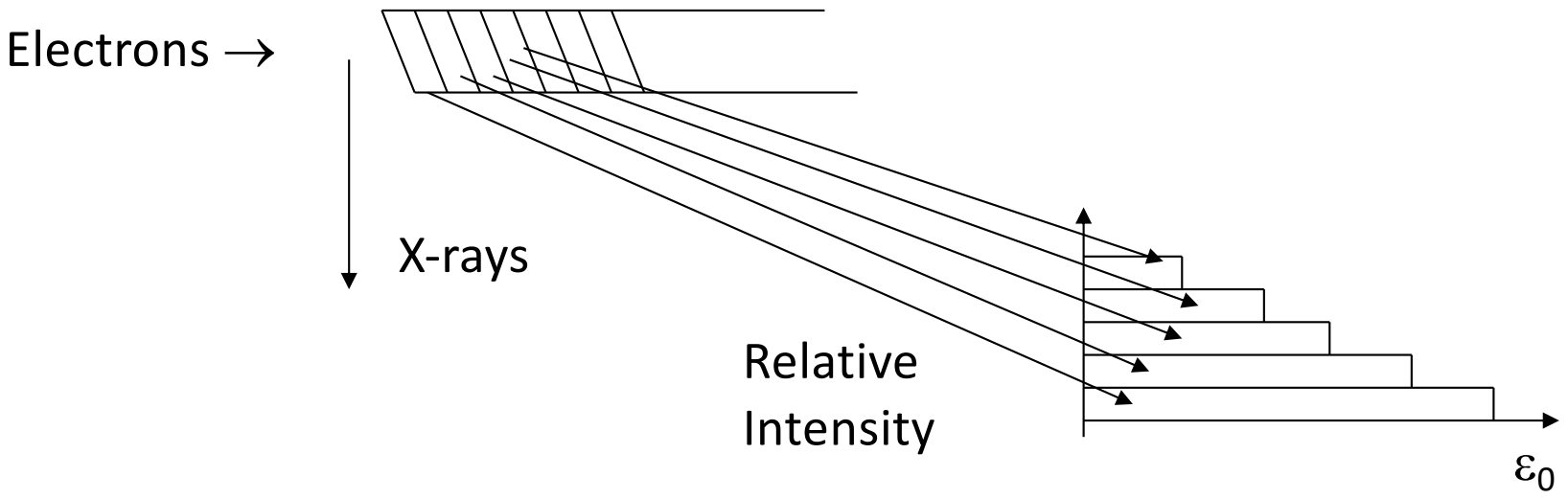
Bremmstrahlung X-ray Energy Spectrum

Interestingly, this process creates a relatively uniform spectrum.



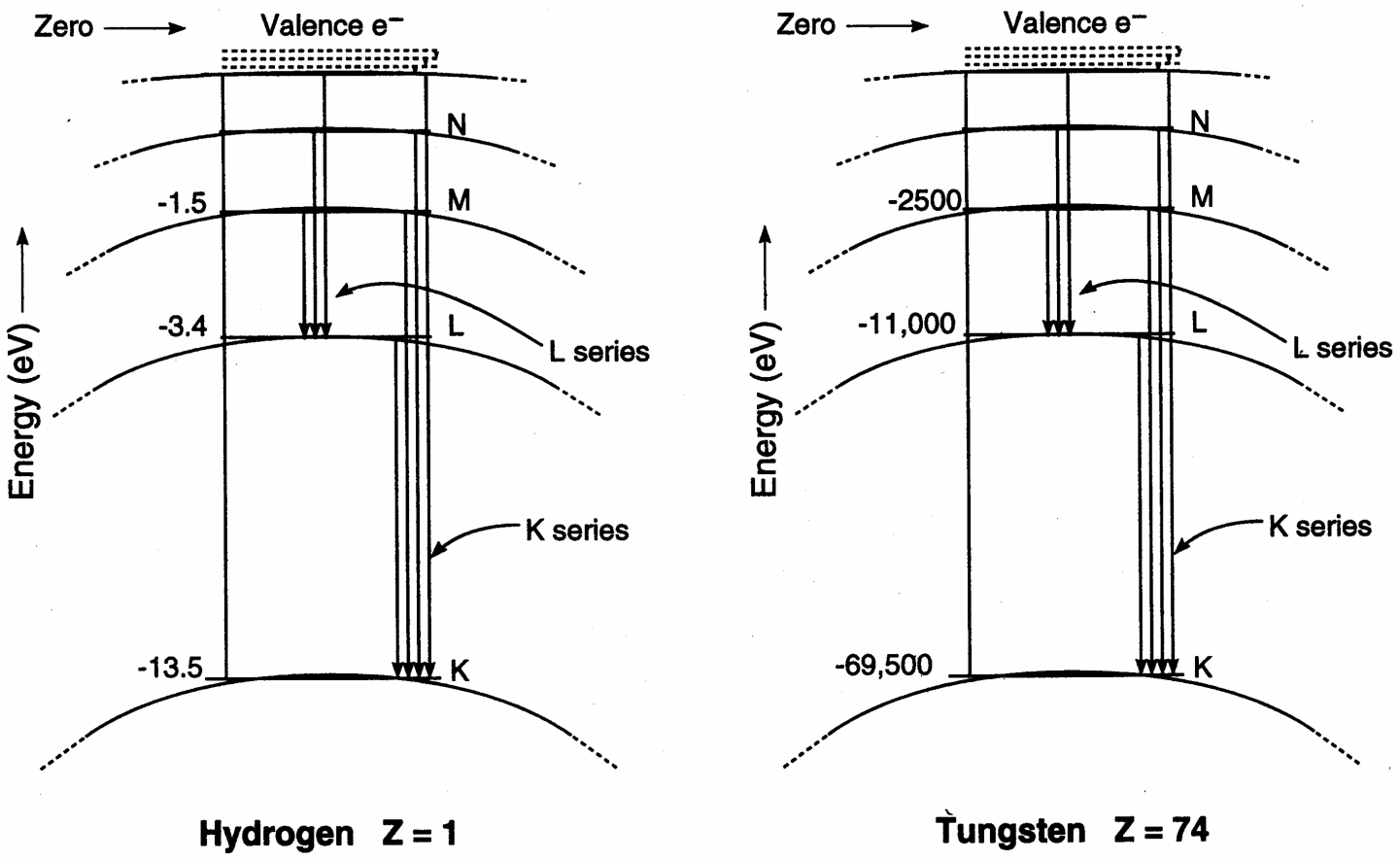
The Unfiltered Bremsstrahlung Spectrum
Thick Target X-ray Formation

We can model target as a series of thin targets. Electrons successively loses energy as they moves deeper into the target.



Each layer produces a flat energy spectrum with decreasing peak energy level.

X-ray Generation – Characteristic X-rays



Electron binding energy

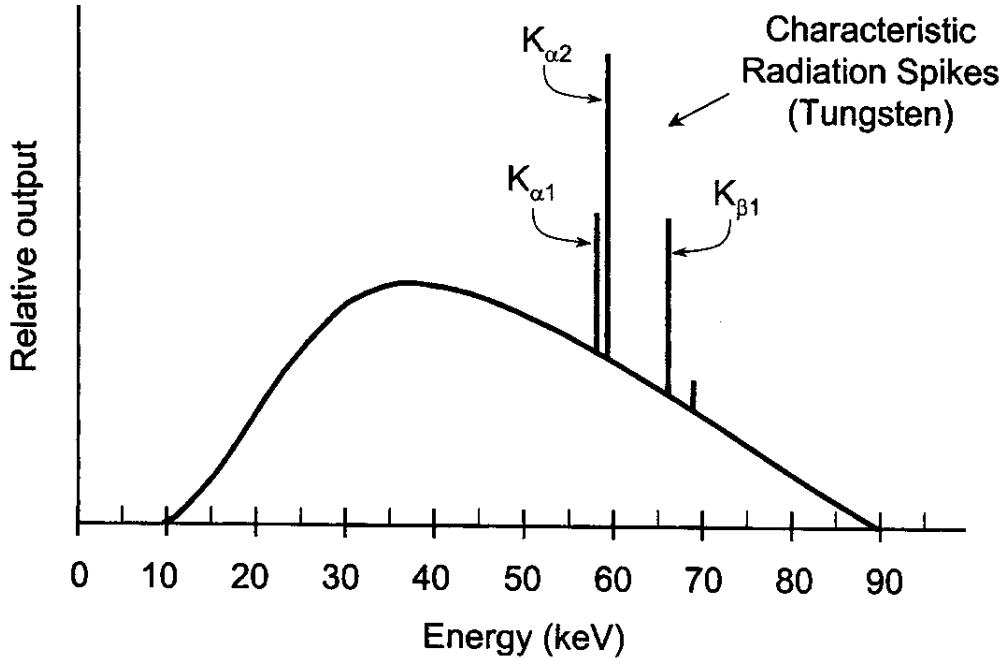
X-ray Generation – Characteristic X-rays

- Within each shell (other than K) there are discrete E orbitals (ℓ = 0, 1, ... , n-1) → characteristic x-ray fine E splitting
- Characteristic x-rays other than those generated through K-shell transitions are unimportant

TABLE 5-2. K-SHELL CHARACTERISTIC X-RAY ENERGIES (keV) OF COMMON X-RAY TUBE TARGET MATERIALS[a]

Shell Transition	Tungsten	Molybdenum	Rhodium
$K_{\alpha 1}$	59.32	17.48	20.22
$K_{\alpha 2}$	57.98	17.37	20.07
$K_{\beta 1}$	67.24	19.61	22.72

[a]*Note:* Only prominent transitions are listed.



Interactions of Ionizing Radiation with Matter

Reading Material:

Chapter 2 in Radiation Detection and Measurements,

Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Interaction of X-rays and Gamma Rays

- Interactions of X-rays and gamma rays
- Photoelectric absorption and contrast-enhanced X-ray imaging
- Compton scattering
  - How does a Compton camera work?
  - Influence of Compton scattering in X-ray imaging

X-ray and Gamma Ray Interactions

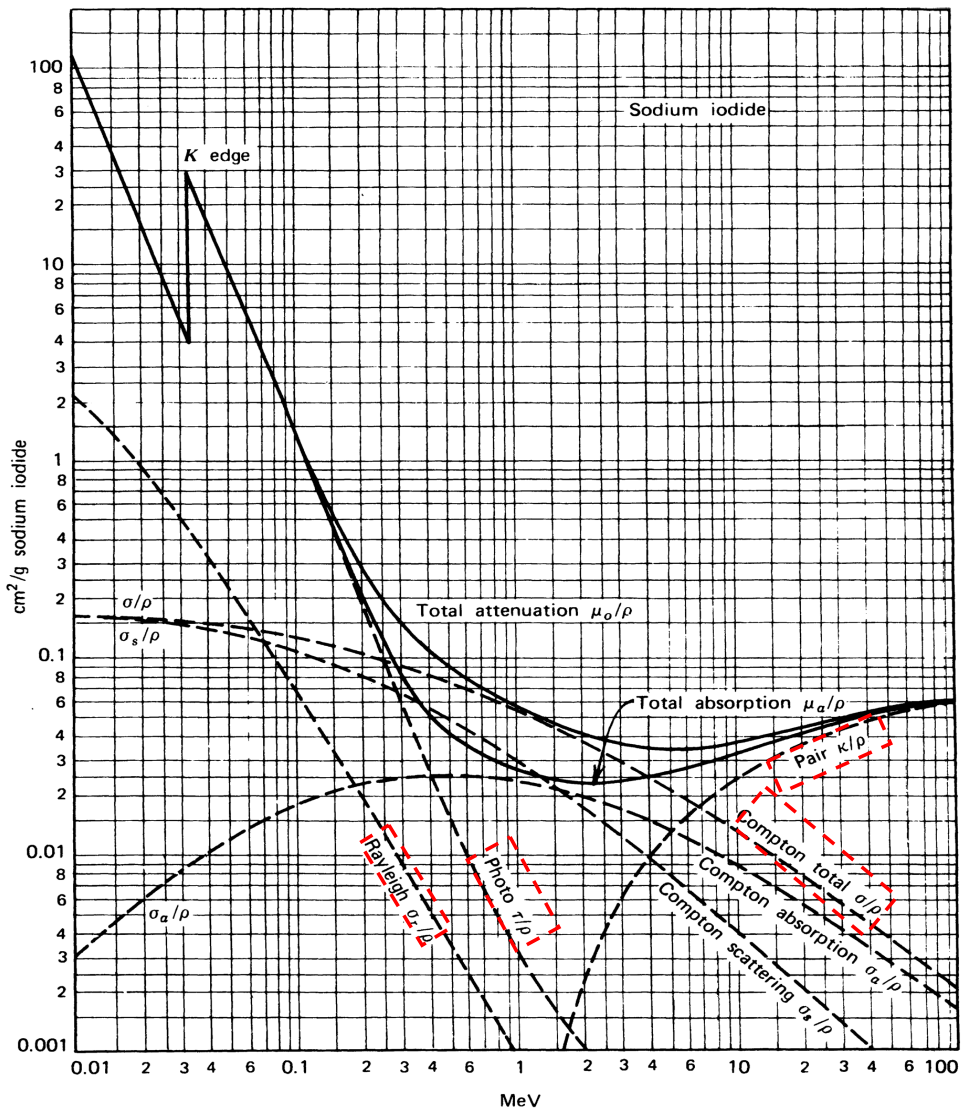


Figure 2.18 Energy dependence of the various gamma–ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

From Page 50, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Photoelectric Effect Cross Section

Probability of photoelectric absorption per atom is

$$\tau \propto \begin{cases} \dfrac{Z^4}{(h\nu)^{3.5}} & low\ energy \\ \dfrac{Z^5}{(h\nu)^{3.5}} & high\ energy \end{cases}$$

- The interaction cross section depends strongly on Z.
- Photoelectric effect is favored at lower photon energies.

Page 49, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Photoelectric Effect (2) – Absorption Edges

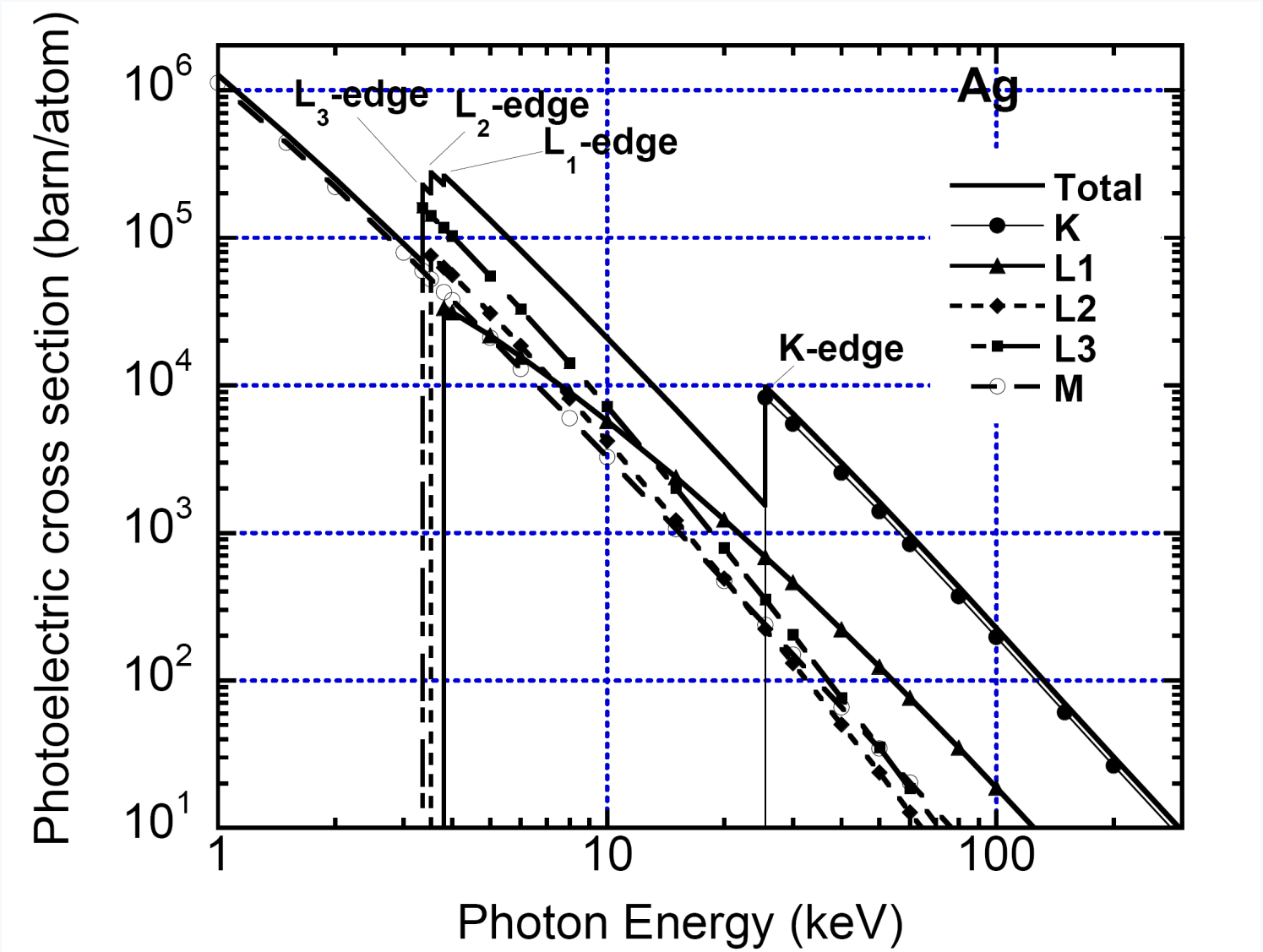
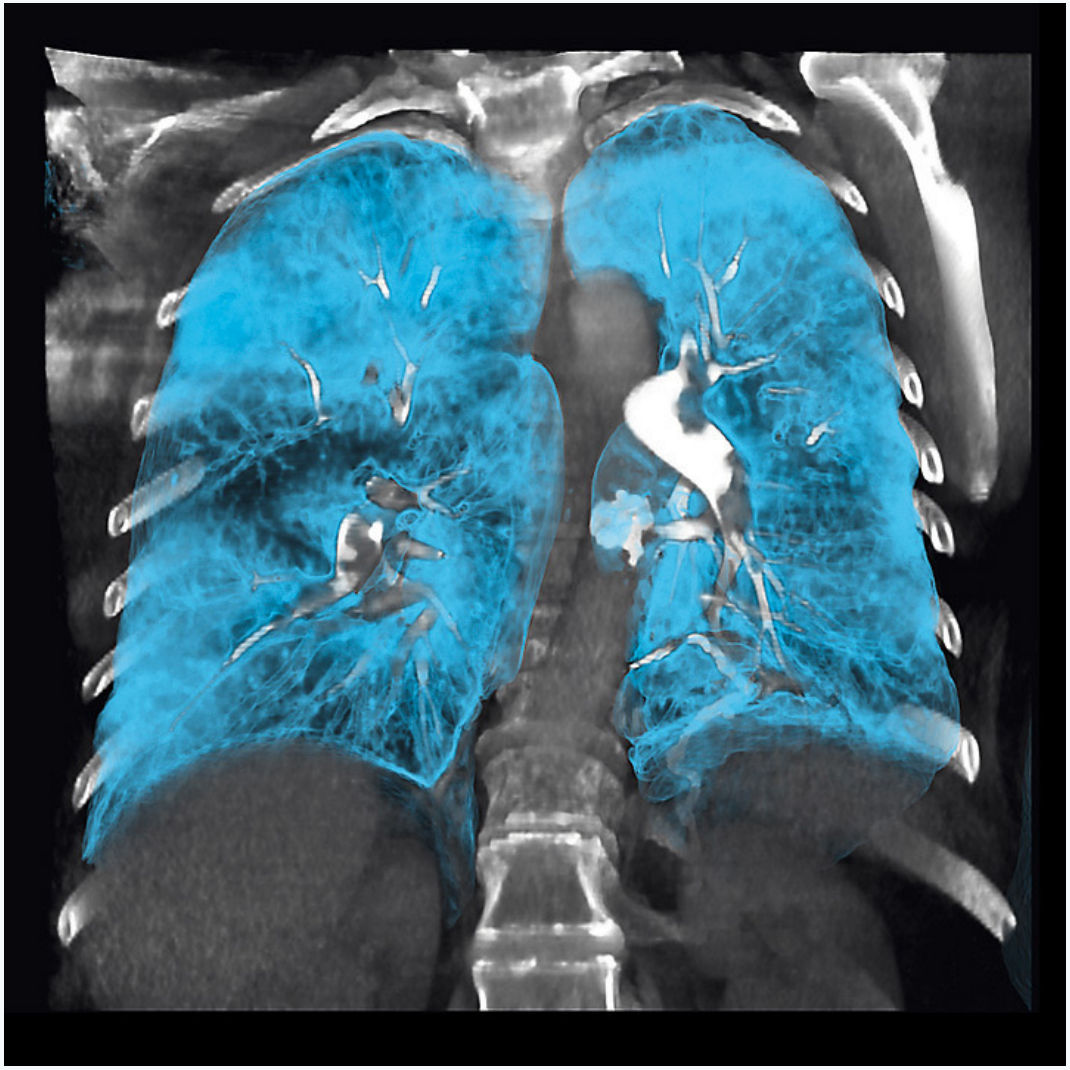


Figure 2: Total and partial atomic photoeffect of Ag.

Photoelectric Effect (2) – Absorption Edges



Contrast enhanced CT image of the lung

www.healthcare.siemens.com

What is Compton Scattering?

The differential scattering cross section ($d\sigma$) – the probability of a photon scattered into a unit solid angle around the scattering angle θ, when passing normally through a layer of material containing one electron per unit area.

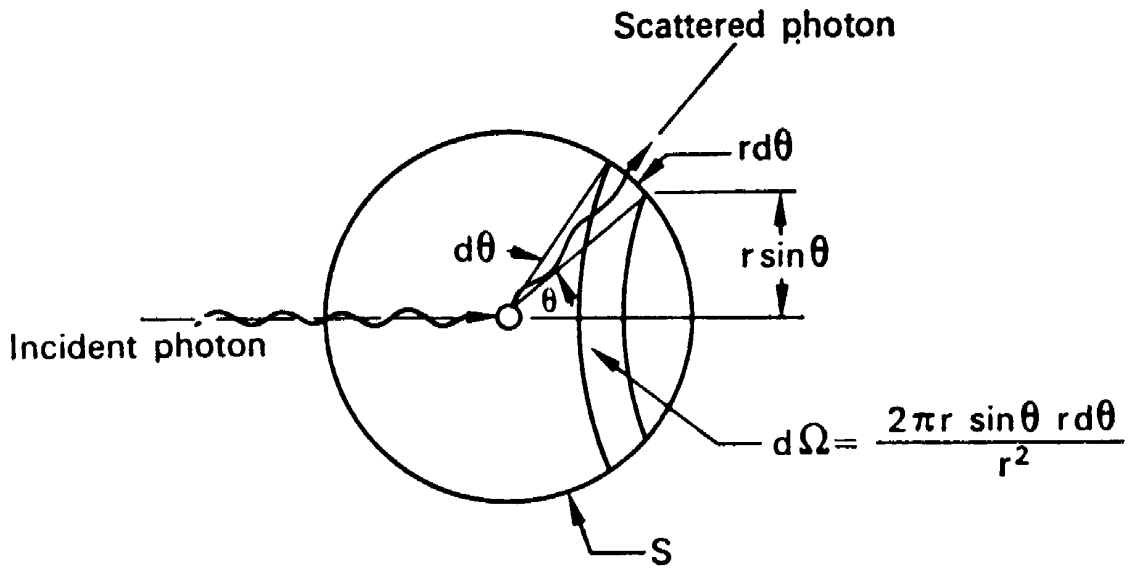


FIG. 5.15. Compton scattering diagram to illustrate differential scattering cross section. *S* is a sphere of unit radius whose center is the scattering electron.

Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

Initial photon energy, v: photon frequency,

h= 6.757704 meter ·kilogram/second, (Planck's constant)

$$h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos(\theta))},$$

mass of electron

Scattering angle

and the photon transfers part of its energy to the electron (assumed to be at rest), which is known as a recoil electron. Its energy is simply

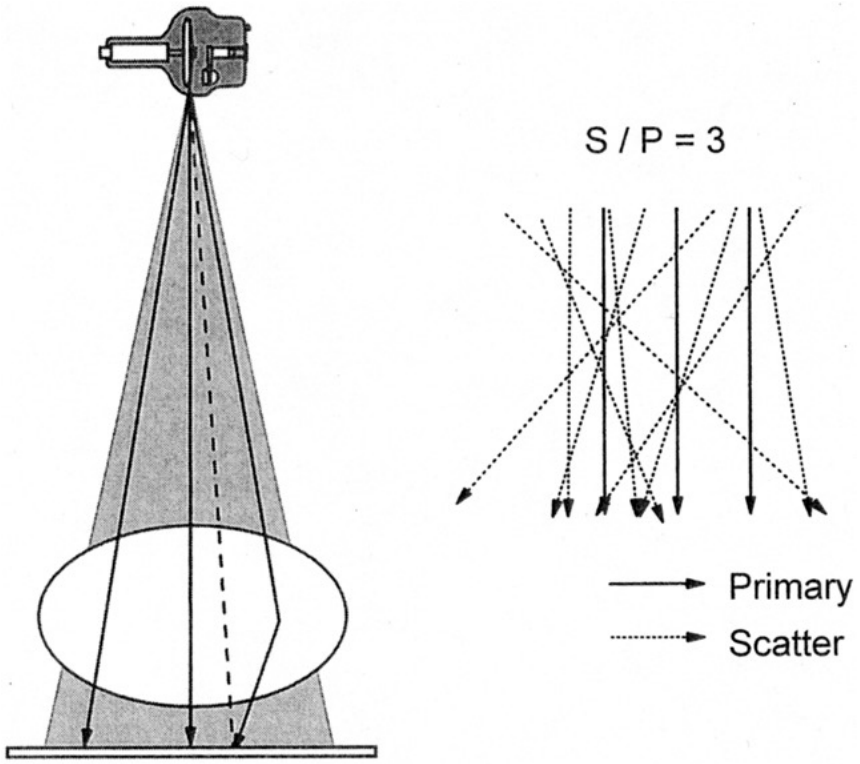
$$E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0 c^2}(1 - \cos(\theta))}$$

The one-to-one relationship between scattering angle and energy loss!!

Reading: Page 51, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Scattered Radiation in X-ray Imaging

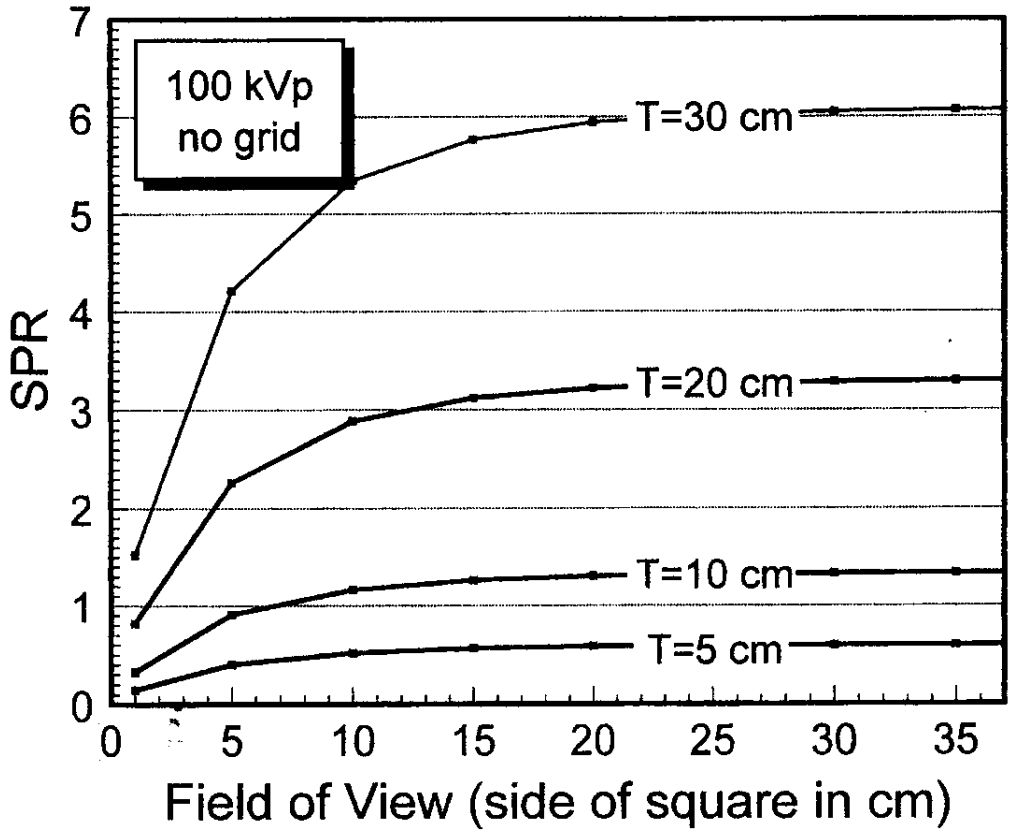
- Most radiographic interactions produce scattered photons
- Scattered photons → violation of the basic principle of projection imaging: mis-information reducing contrast



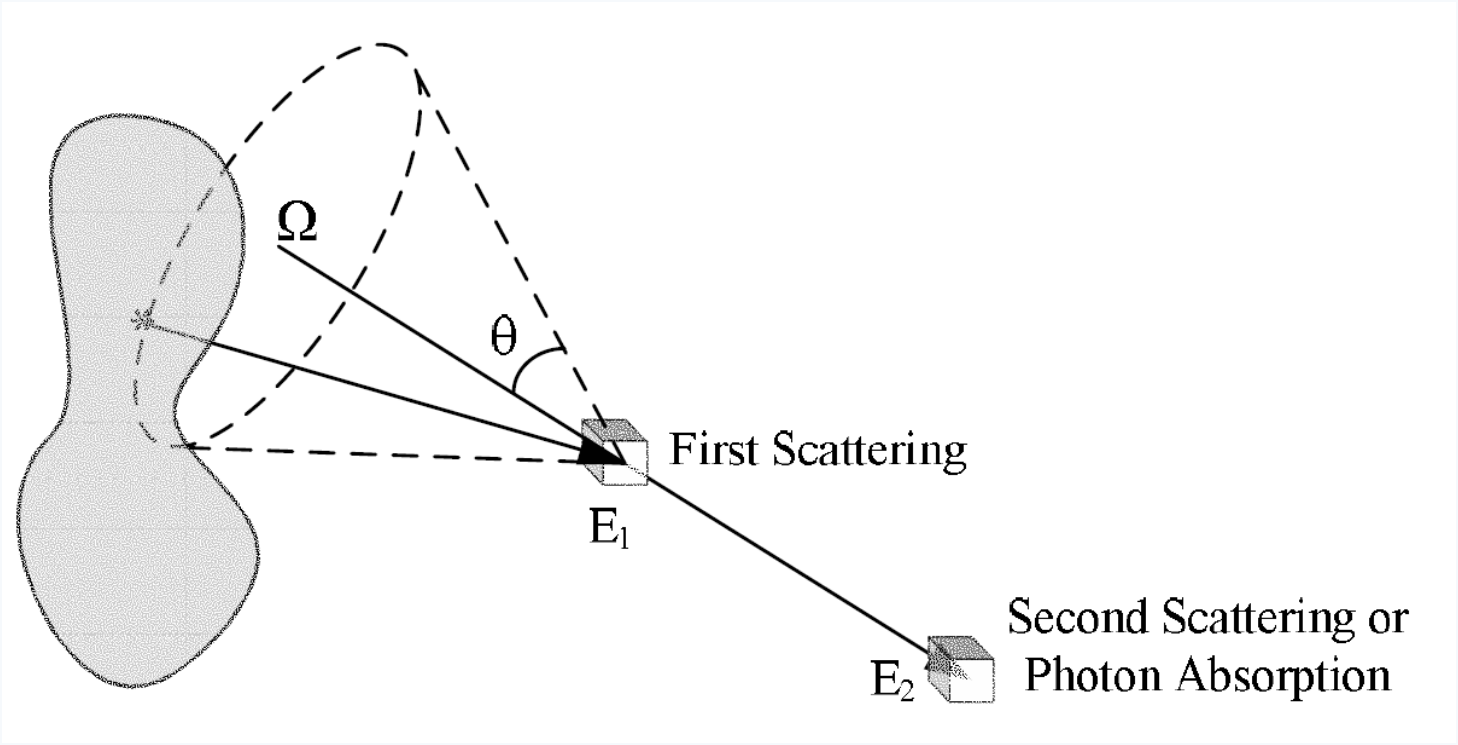
Scattered Radiation

Scatter-to-Primary ratio (S/P)

- Area of collimated x-ray field
- Object thickness
- kVp of x-ray beam

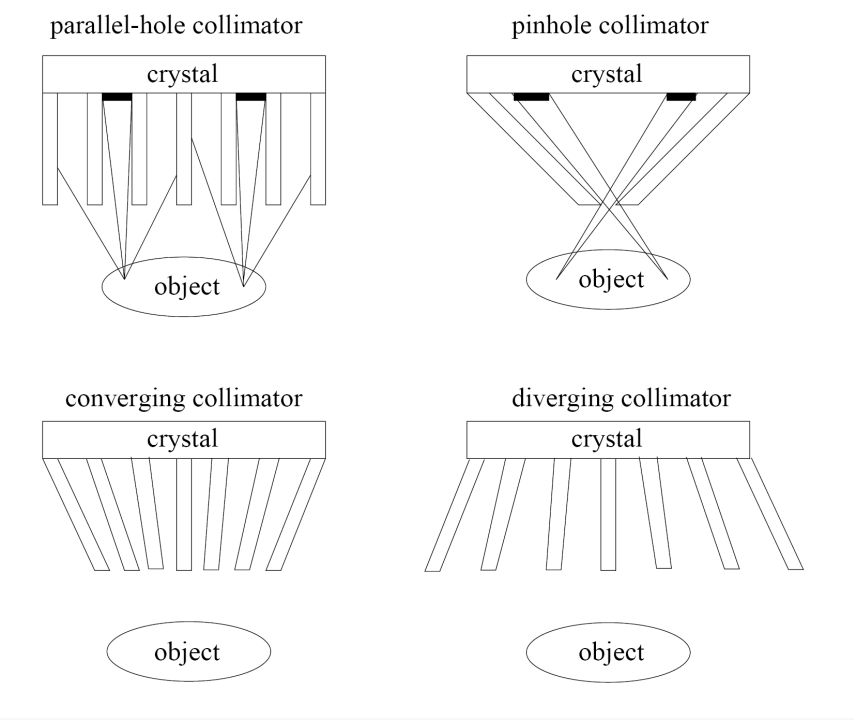


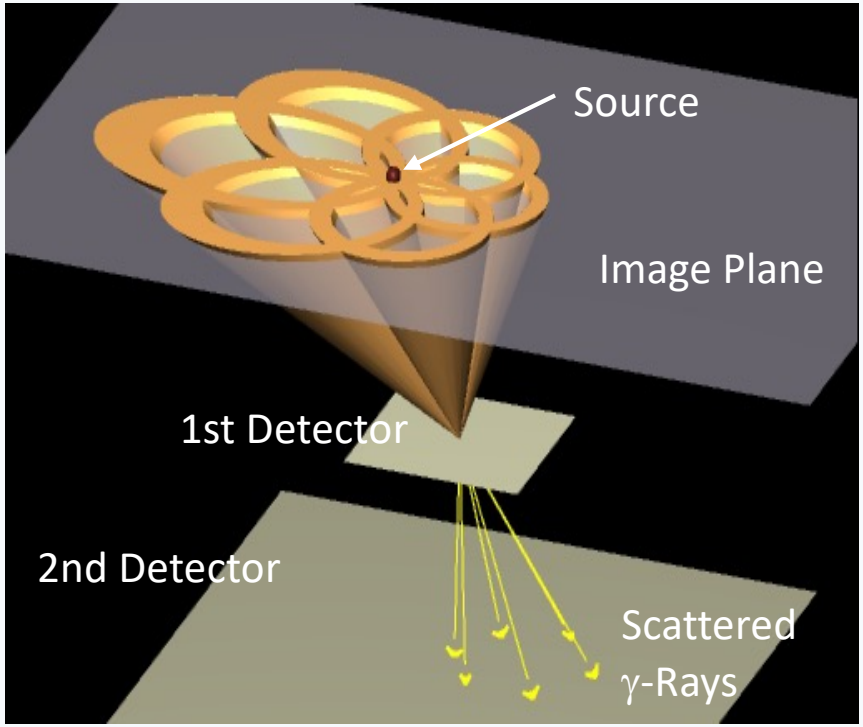
Principle of Compton Imaging



$$\cos\theta = 1 + \frac{m_e c^2}{E_0} - \frac{m_e c^2}{E_0 - E_1}$$

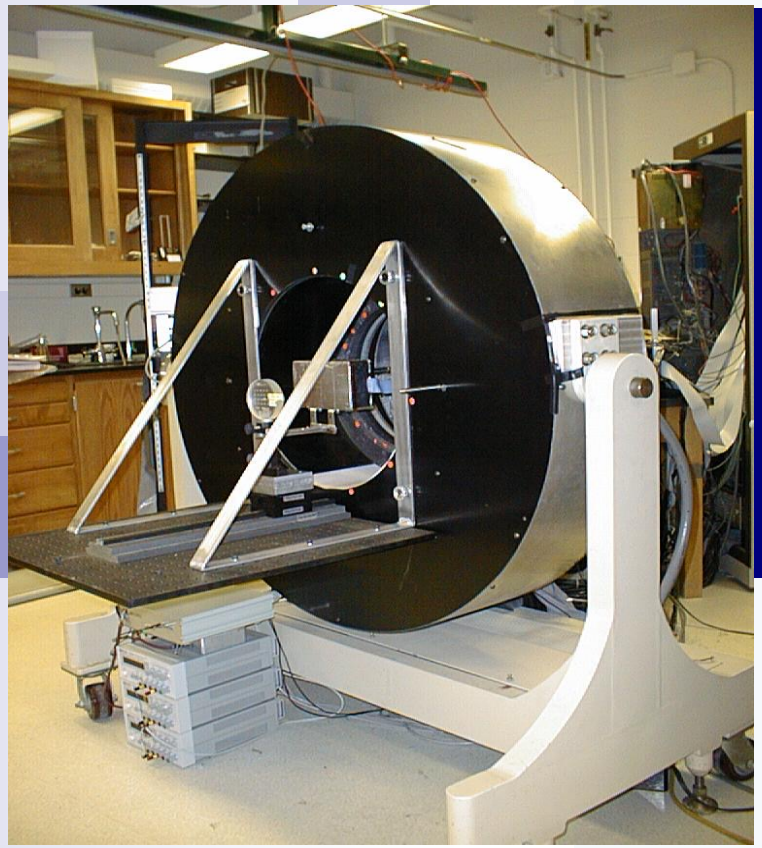
Principle of Compton Imaging

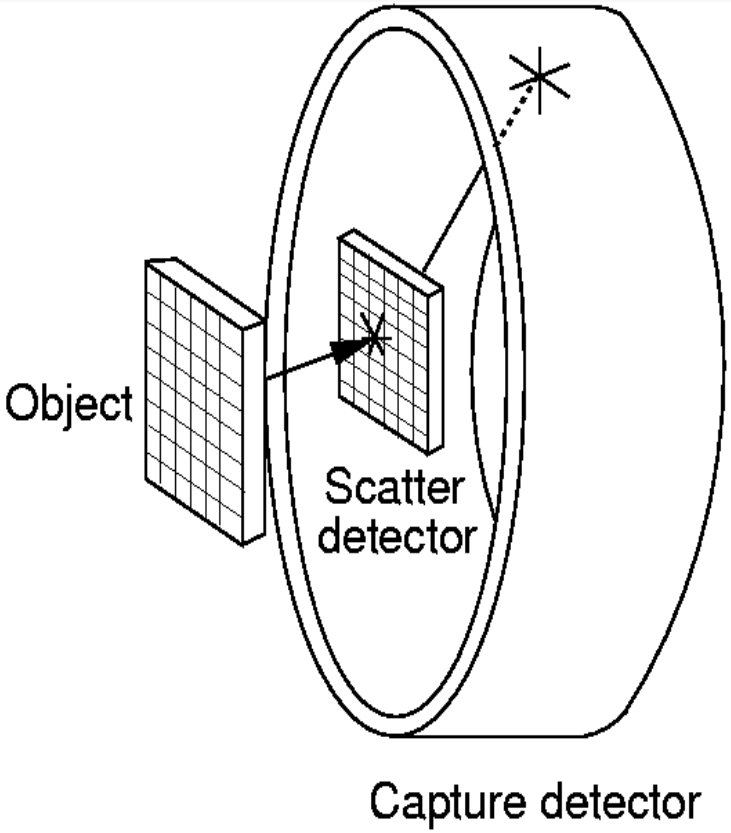




- No need for mechanical collimation – much higher detection efficiency.
- Requires the first detector to have a very good energy resolution.

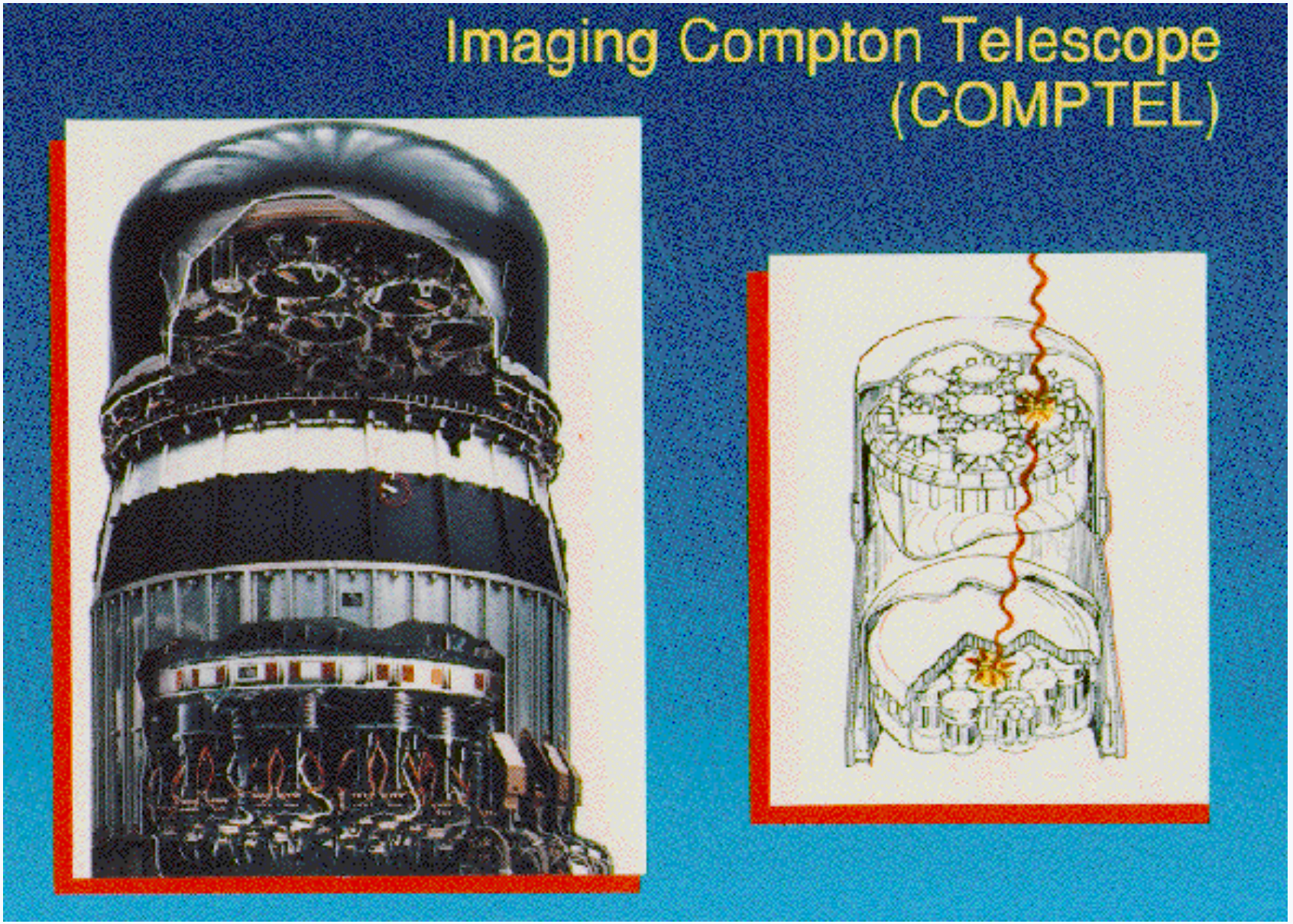
Experimental Compton Camera





Developed by the Detector Physics Group, University of Michigan, lead by Neal Clinthorne and Les. Rogers

Compton Scattering for Imaging?



Compton Scattering for Imaging?

