Homework 3

Due Date: 5 pm on Monday, October 12, 2026

Radon Transform

1. Show that the Radon transform of $f(x - x_0, y - y_0)$ is $g(\ell - x_0 \cos\theta - y_0 \sin\theta, \theta)$.

2. Find the 2-D Radon transform $g(\ell, \theta)$ of $f(x, y) = \exp(-x^2 - y^2)$. (Hint: Use the rotational symmetry of $f(x, y)$ to simply your integration.)

3. In this problem, we want to figure out what is "wrong" with the backprojection summation image given by

$$f_b(x, y) = \int_0^{\pi} g(x \cos\theta + y \sin\theta, \theta) d\theta ,$$

and see how to fix it.

(a) Show that the 2-D Radon transform of the 2-D delta function $\delta(x, y)$ is $\delta(\ell)$. (Hint: Use the projection slice theorem.)

(b) Show that the backprojection summation image of $\delta(x, y)$ is $\dfrac{1}{\sqrt{x^2 + y^2}}$. (Hint: Write (x, y) in polar coordinates (r, ϕ), then use the fact that $\sin\theta \approx \theta$ for small θ and the fact that $\delta(a\ell) = \delta(\ell)/|a|$.)

(c) Show that the backprojection summation image of $\delta(x - x_0, y - y_0)$ is $1/\sqrt{(x - x_0)^2 + (y - y_0)^2}$—that is, that the backprojection summation operator is shift invariant.

(d) Show that $f_b(x, y) = f(x, y) * (1/\sqrt{x^2 + y^2})$.

(e) Describe how one could, in principle, recover $f(x, y)$ from $f_b(x, y)$. Describe any problems you may foresee with your approach.

2-D image reconstruction using the filtered backprojection (FBP) algorithm

Use the attached Matlab codes to study the impact of various sampling conditions and reconstruction parameters (e.g., sampling interval, filter functions, and cut-off frequency) on image quality.