

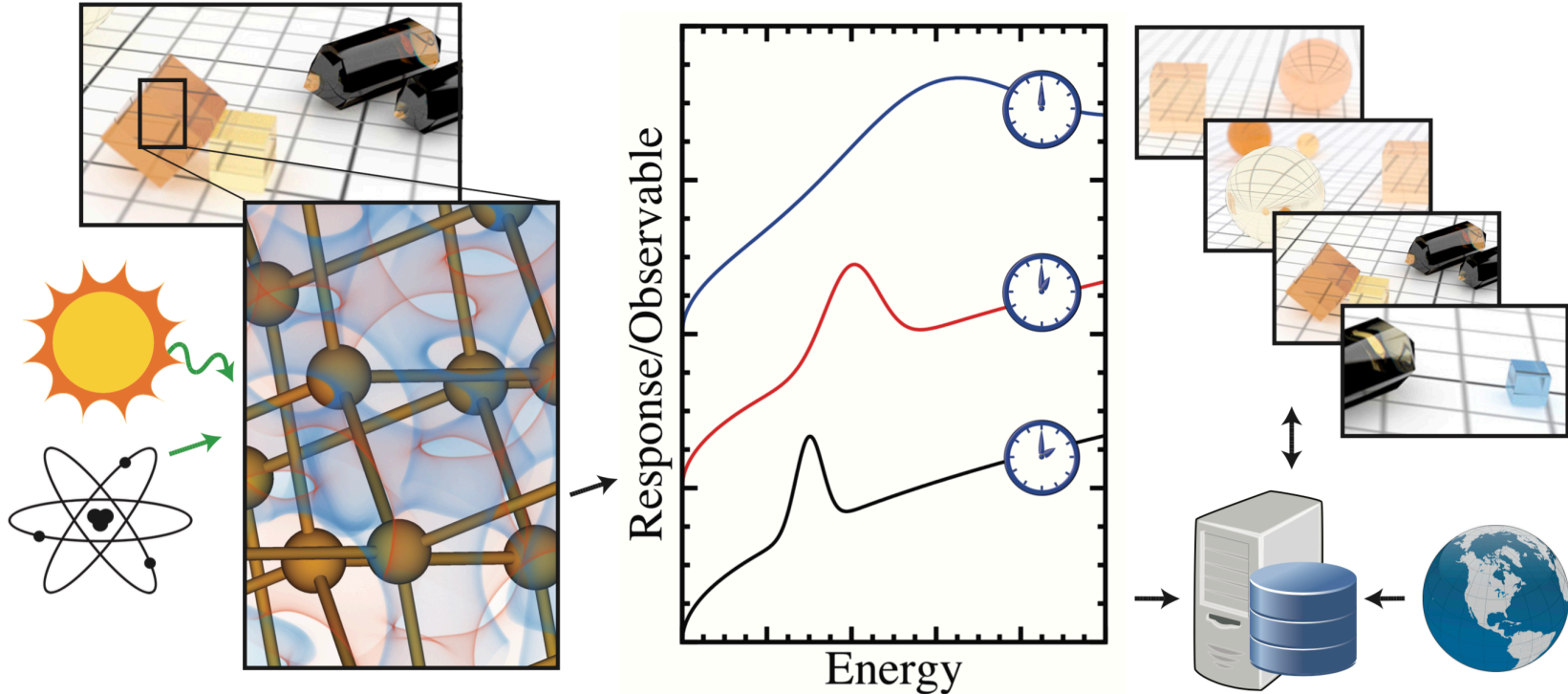
# Why ... Materials selection?



Excited electronic states

Femto-second dynamics

Materials selection



“Pushing The Frontiers Of Modeling Excited Electronic States And Dynamics To Accelerate Materials Engineering And Design”, K. Kang, A. Kononov, C.-W. Lee, J.A. Leveillee, E. Shapera, X. Zhang, A. Schleife, *Comp. Mat. Sci.* **160**, 207–216 (2019); Finalist "Rising Stars in Computational Materials Science"

# Reminders/Introduction: Quantum Mechanics

- Electron described by wave function as basis variable

$$\psi(x, y, z, t)$$

- Square of the wave function describes probability

$$\psi^* \psi$$

- In practice: Solve Schrödinger equation to find wave function

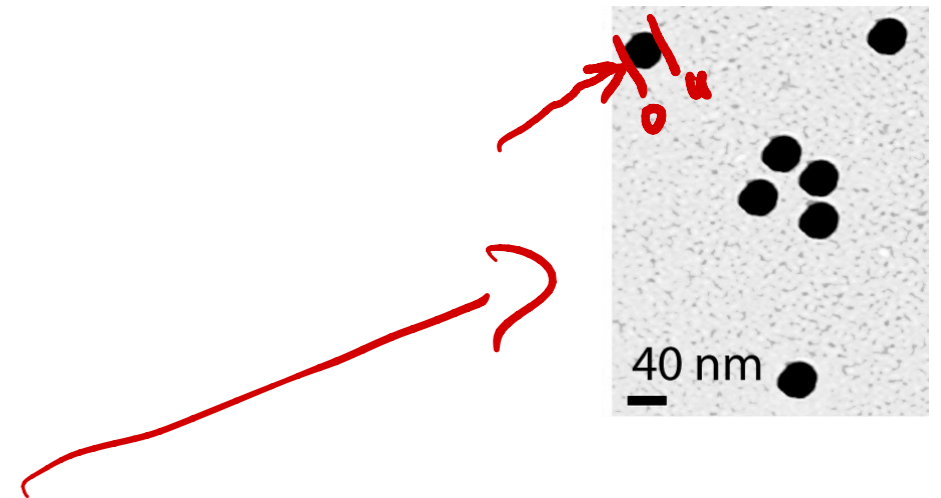
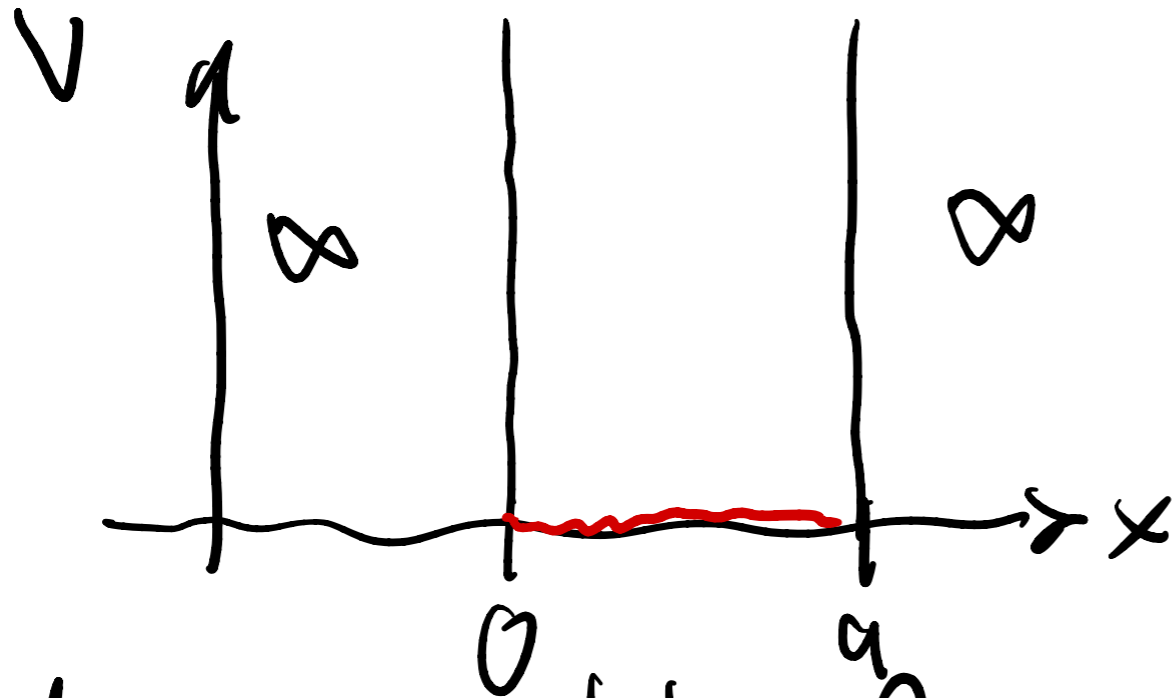
$$\hat{H} \psi = i\hbar \frac{\partial}{\partial t} \psi = E \psi \rightarrow \frac{d^2 \psi(x)}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi(x) = 0$$

$\hat{H} = \hat{T} + V$

$$\frac{d^2 \psi(x)}{dx^2} + k^2 \psi(x) = 0 \quad \leftarrow k^2 = \frac{2mE}{\hbar^2}$$

# Solution of the Schrödinger Equation: Infinite 1D Well

- infinite well potential looks like this:



- outside: wave fun = 0

- inside: free electrons

- from math:  $\psi(x) = A \exp(ikx) + B \exp(-ikx)$

-  $\psi(0) \equiv 0 \rightarrow$  leads to  $B = -A$

$$\psi(x) = 2A i \sin(kx)$$

# Solution of the Schrödinger Equation: Infinite 1D Well

$$\psi(x) = 2Ai \sin(kx) \qquad k^2 \equiv \frac{2m}{\hbar^2} E$$

• still have to determine  $k$  by fulfilling:  $\psi(a) = 0 = 2Ai \sin(ka)$

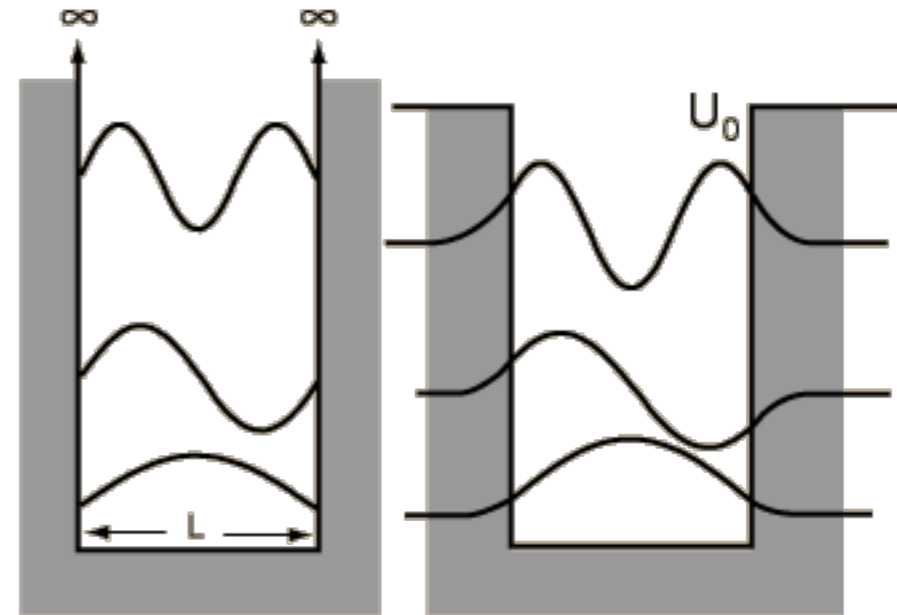
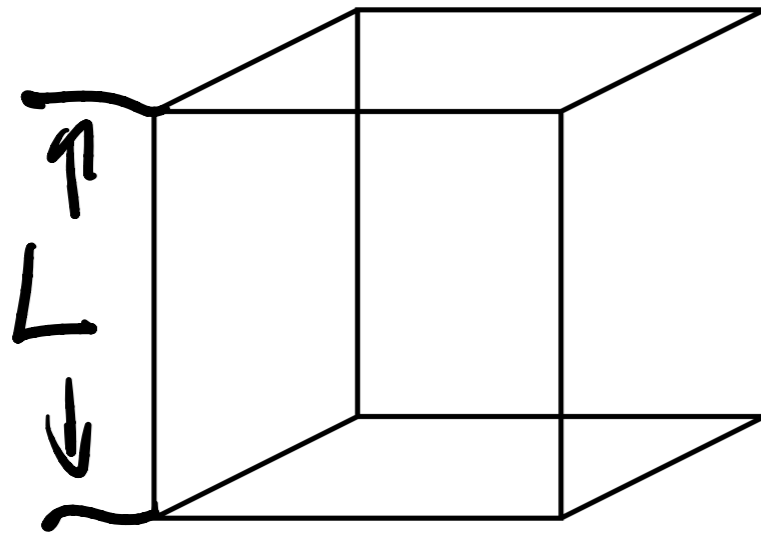
• this requires:  $ka = n\pi \rightarrow k = \frac{n\pi}{a}$

- overall wave function:  $\psi(x) = 2Ai \sin\left(\frac{n\pi x}{a}\right)$

$$E = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

# Solids: Free-electron gas model

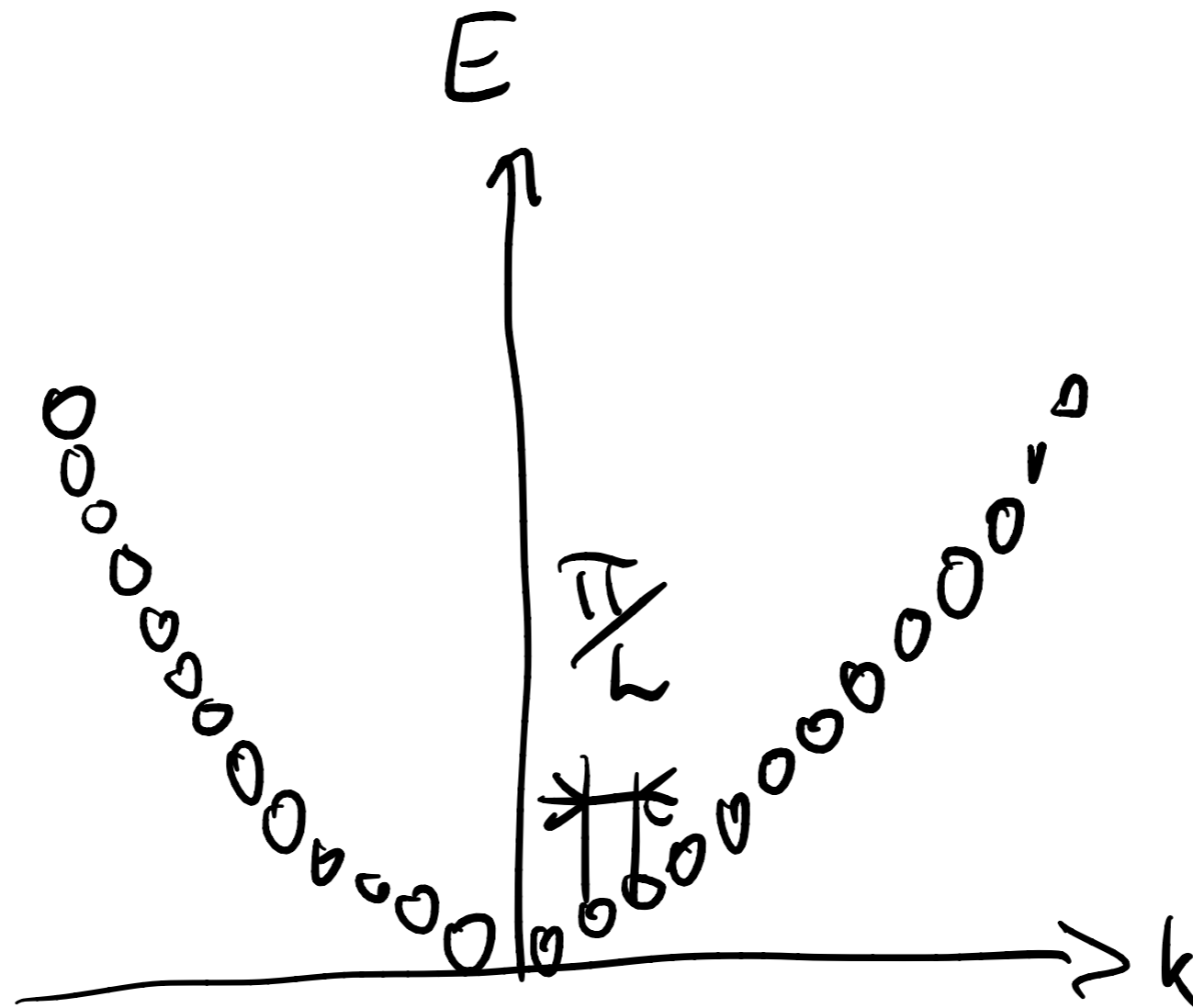
- Free-electron gas model of a solid looks like:



- found for 1D infinite well:  $E = \frac{\hbar^2 k^2}{2m}$   $k^2 = \frac{\pi^2}{L^2}$
- in 2D:  $E = \frac{\hbar^2}{2m} (k_x^2 + k_y^2)$
- in 3D:  $E = \frac{\hbar^2}{2m} (k_x^2 + k_y^2 + k_z^2)$

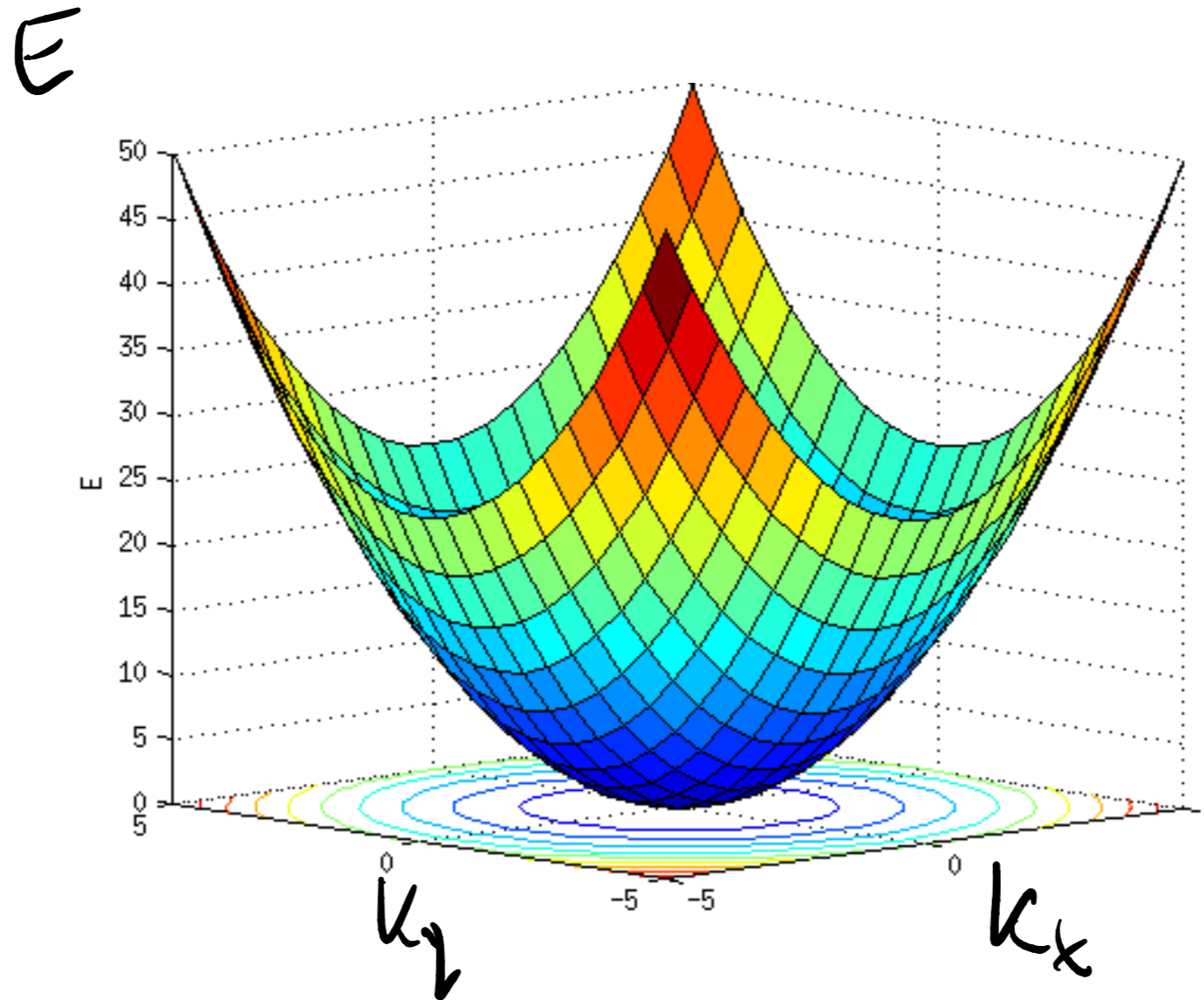
# Solids: Free-electron gas model

- energy-momentum relationship (1D):



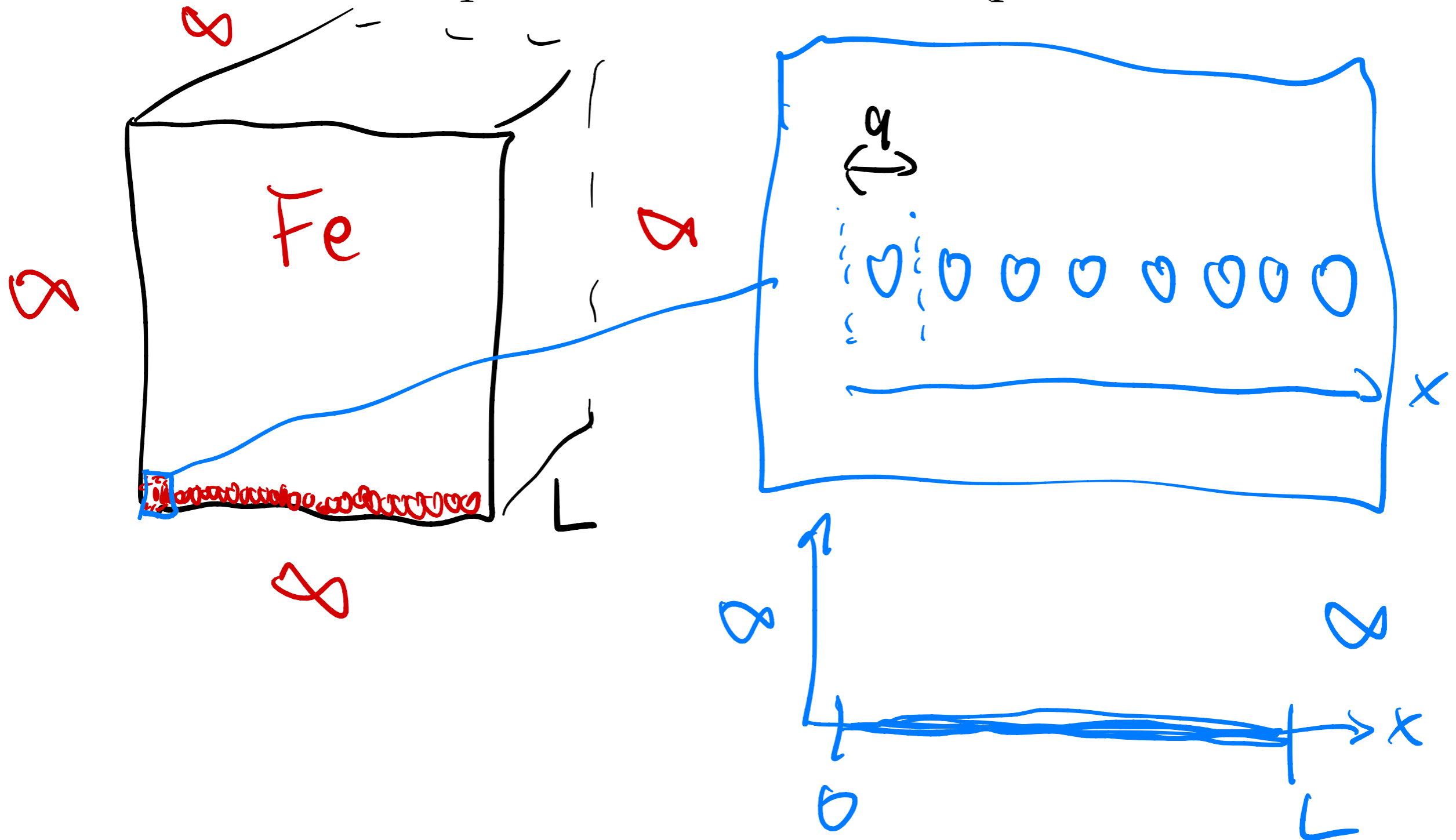
# Solids: Free-electron gas model

- energy-momentum relationship (2D):



# Towards a Band Theory of Solids: Periodic Potential

- we assume: all atoms sit on equilibrium positions
- this leads to a *periodic* lattice of atoms (potential for electrons)



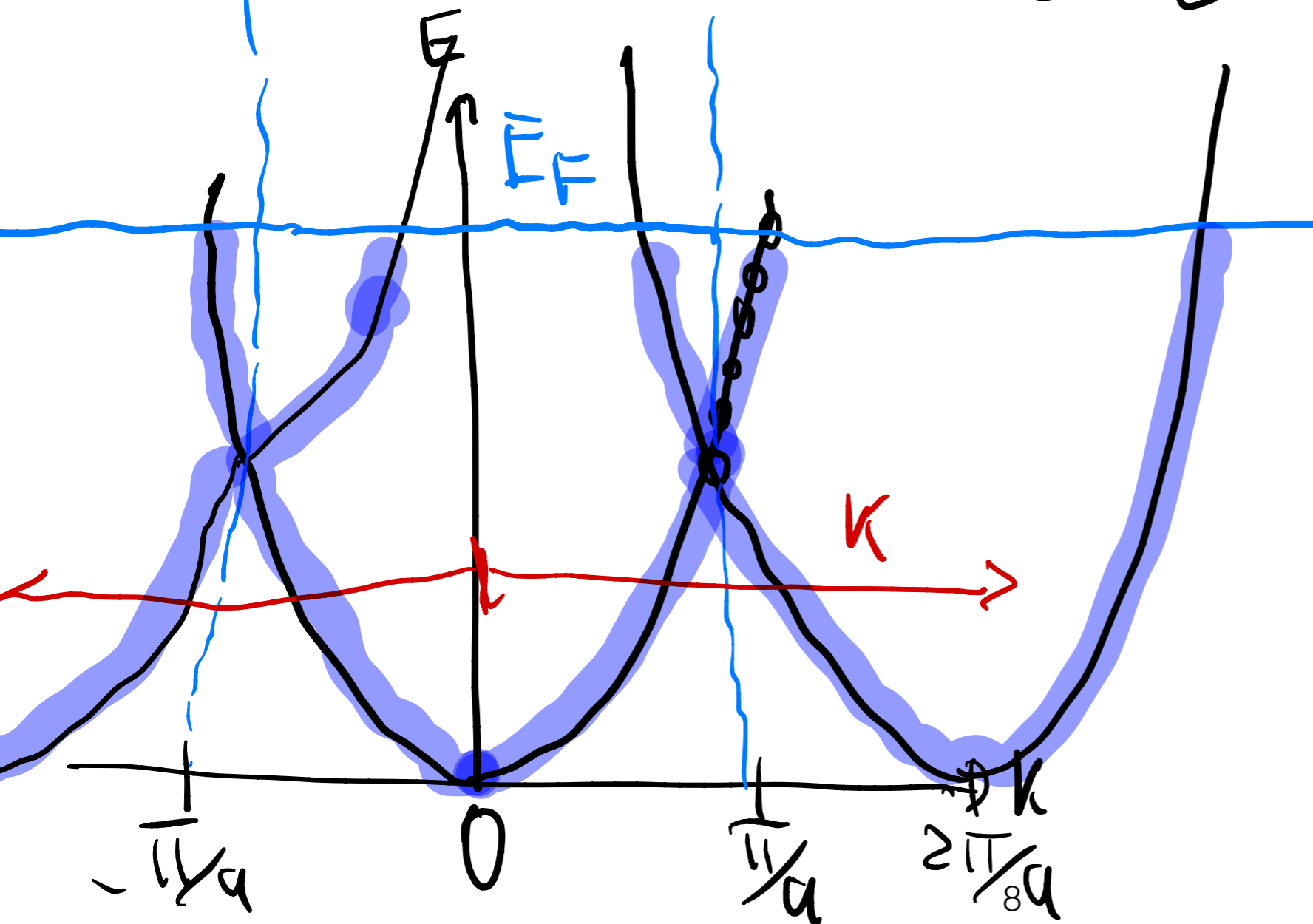


# Towards a Band Theory of Solids: Periodic Potential

- we assume: all atoms sit on equilibrium positions
- this leads to a *periodic* lattice of atoms (potential for electrons)

• Bloch theorem (for all periodic potentials)

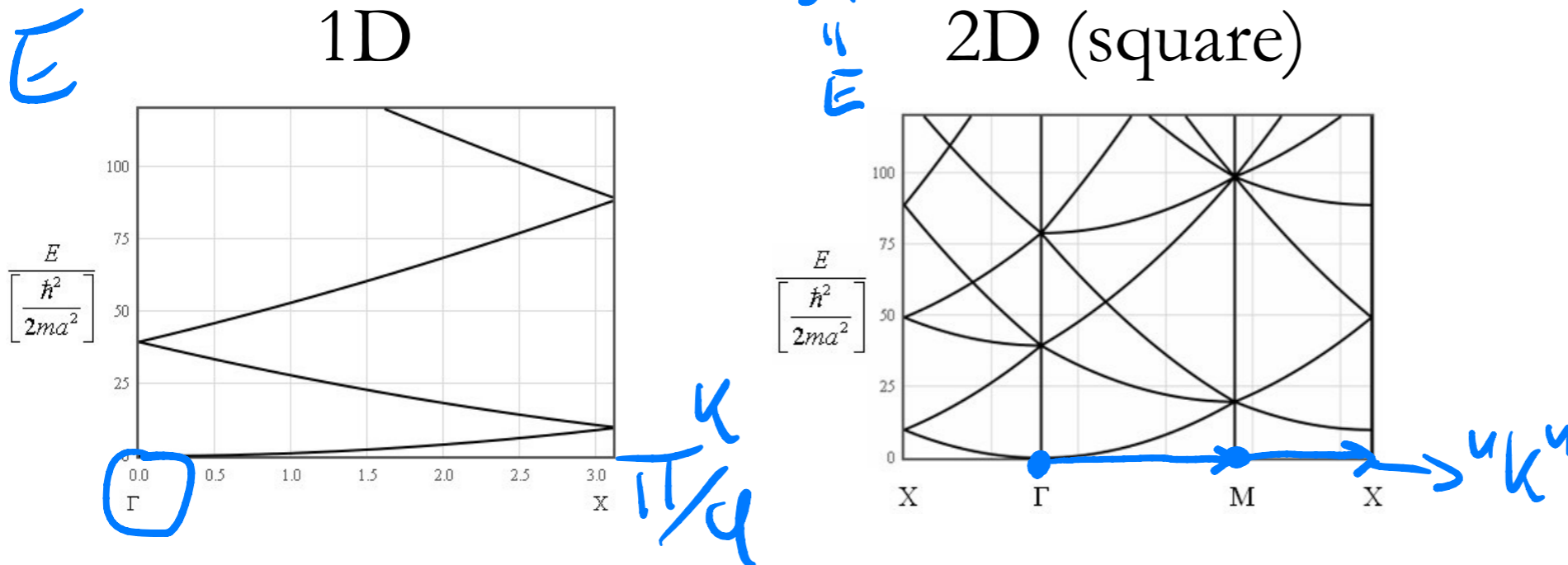
- solution of the free-electron gas:  $E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 (k_{\text{reduced}} + K)^2}{2m}$



$k_{\text{reduced}}$  inside 1st BZ

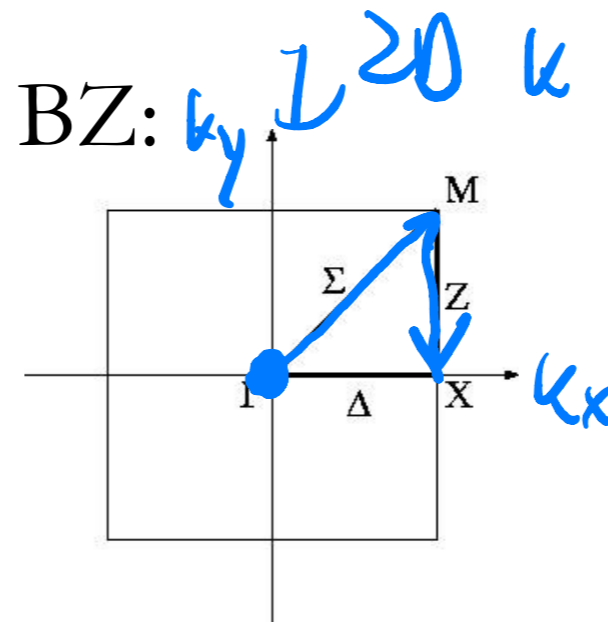
# Towards a Band Theory of Solids: Empty Lattice Approximation

- lattice is periodic, but no atoms are present
- plot solutions for different lattice types: don't show periodicity



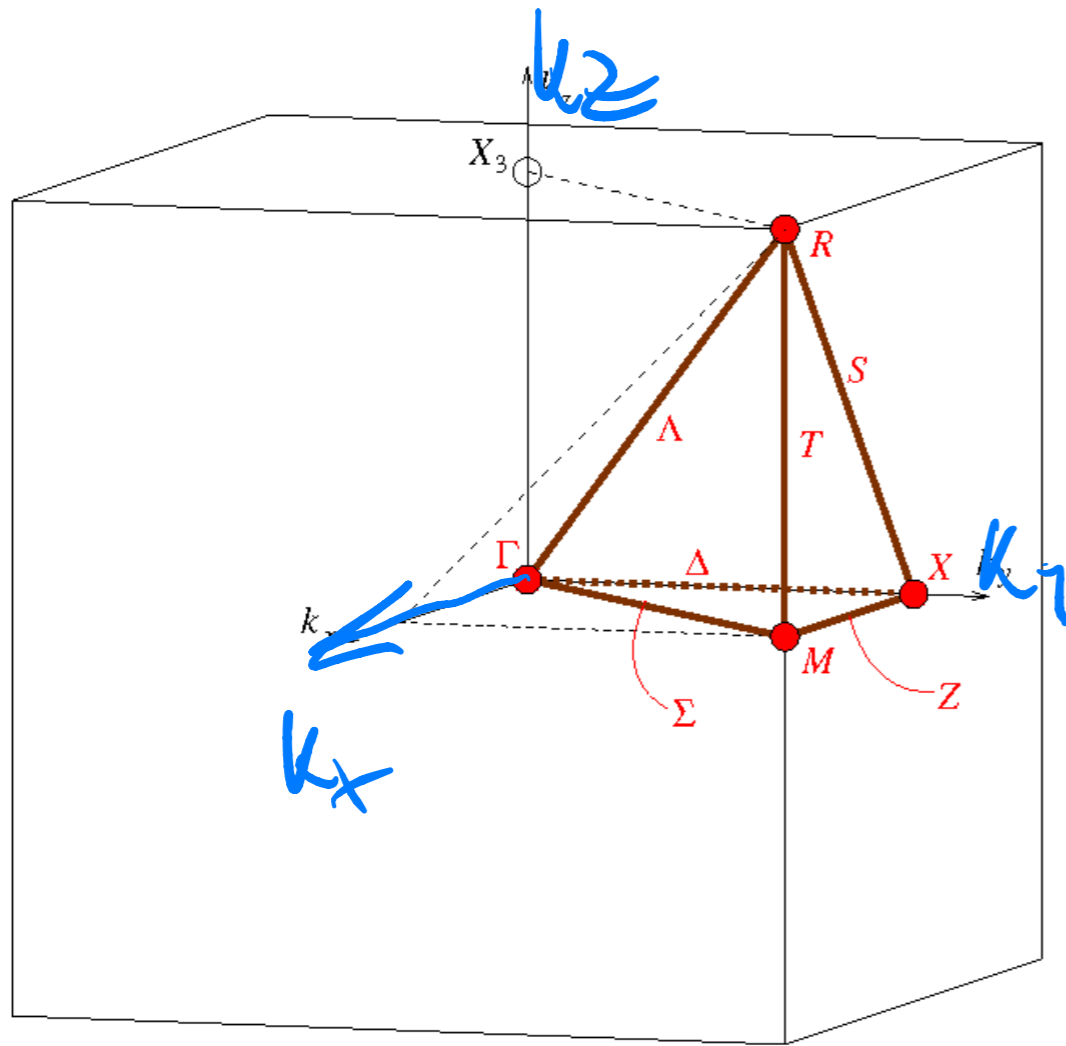
(c) <http://lamp.tu-graz.ac.at/~hadley/ss1/empty/empty.php>

$$E(\mathbf{k}) = \hbar^2 \frac{\mathbf{k}^2}{2m} = \hbar^2 \frac{(\mathbf{k}_{\text{reduced}} + \mathbf{K})^2}{2m}$$

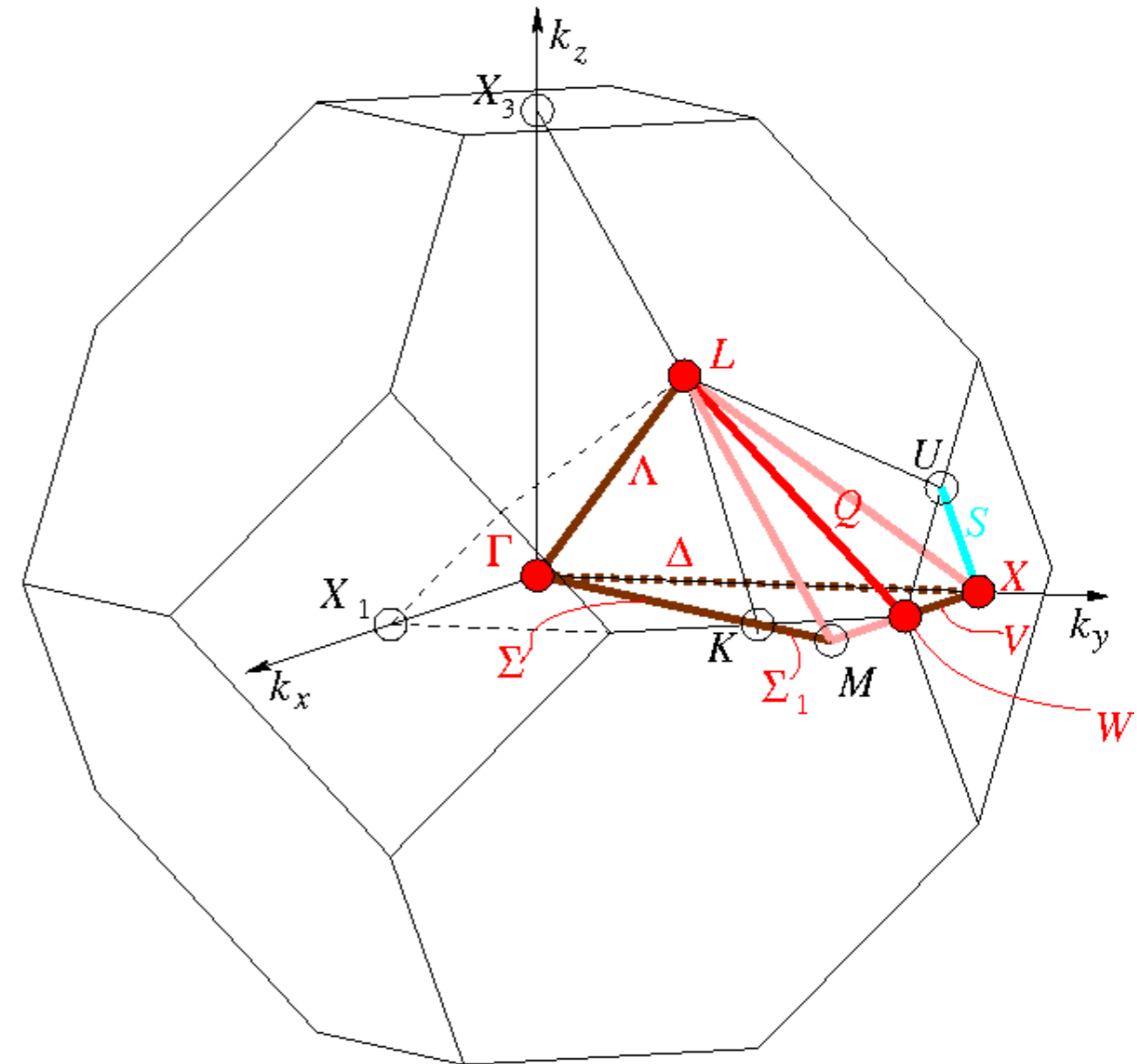


# Towards a Band Theory of Solids: Reciprocal Lattice

- Wigner-Seitz cell of reciprocal lattice: Brillouin zone



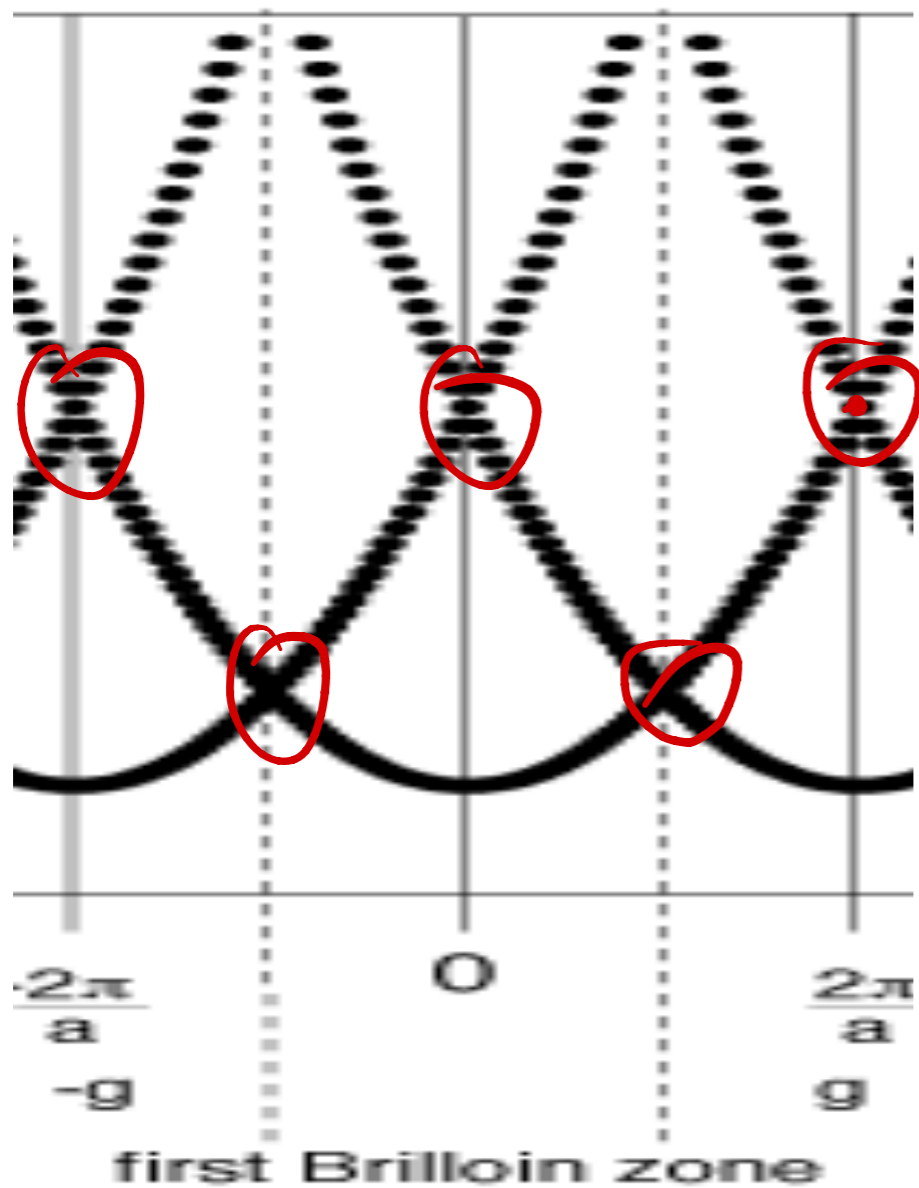
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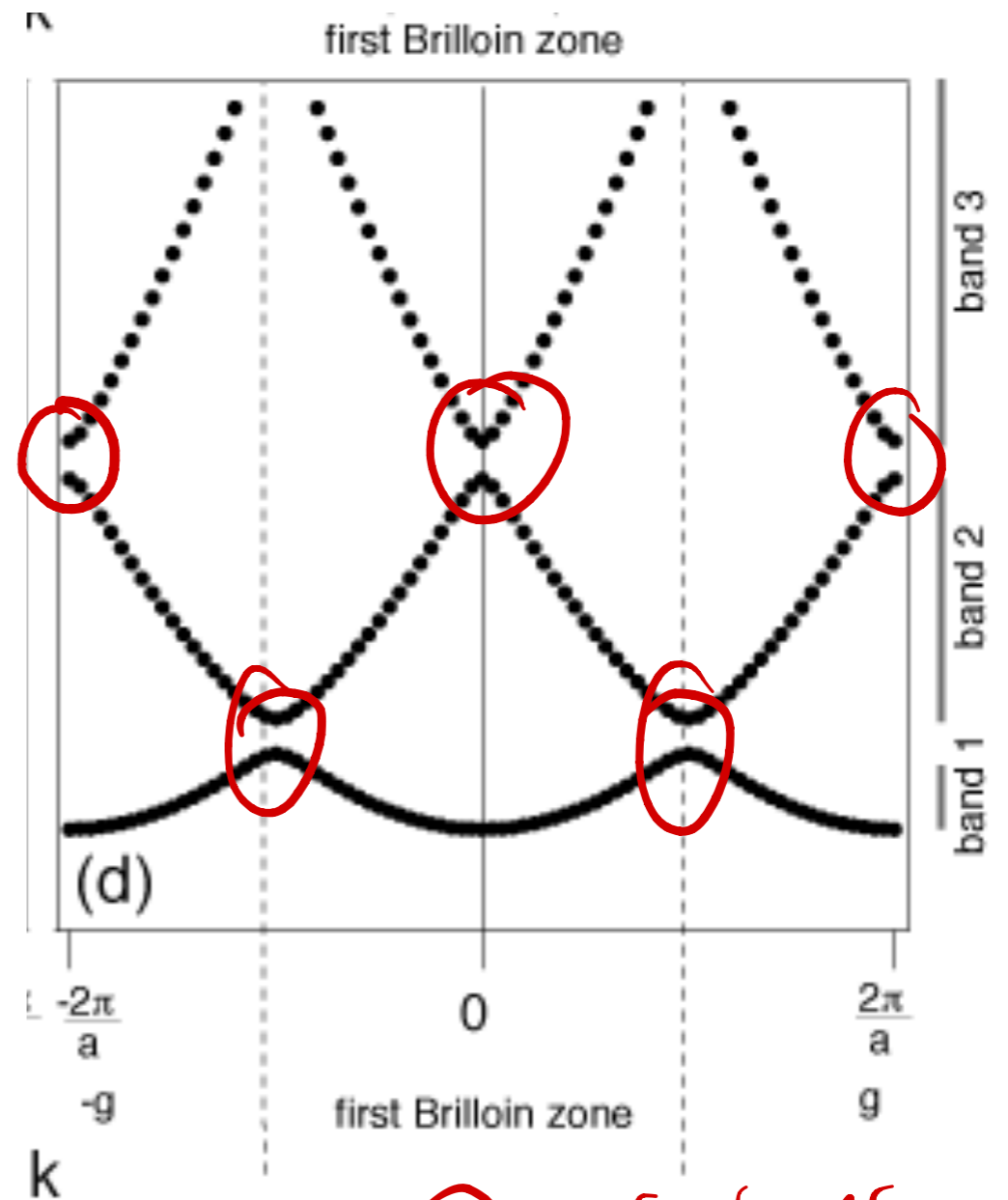
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<http://www.cryst.ehu.es>

# Towards a Band Theory of Solids: Nearly Free Electrons

- these results allow us to draw the following energy diagram for nearly free electrons



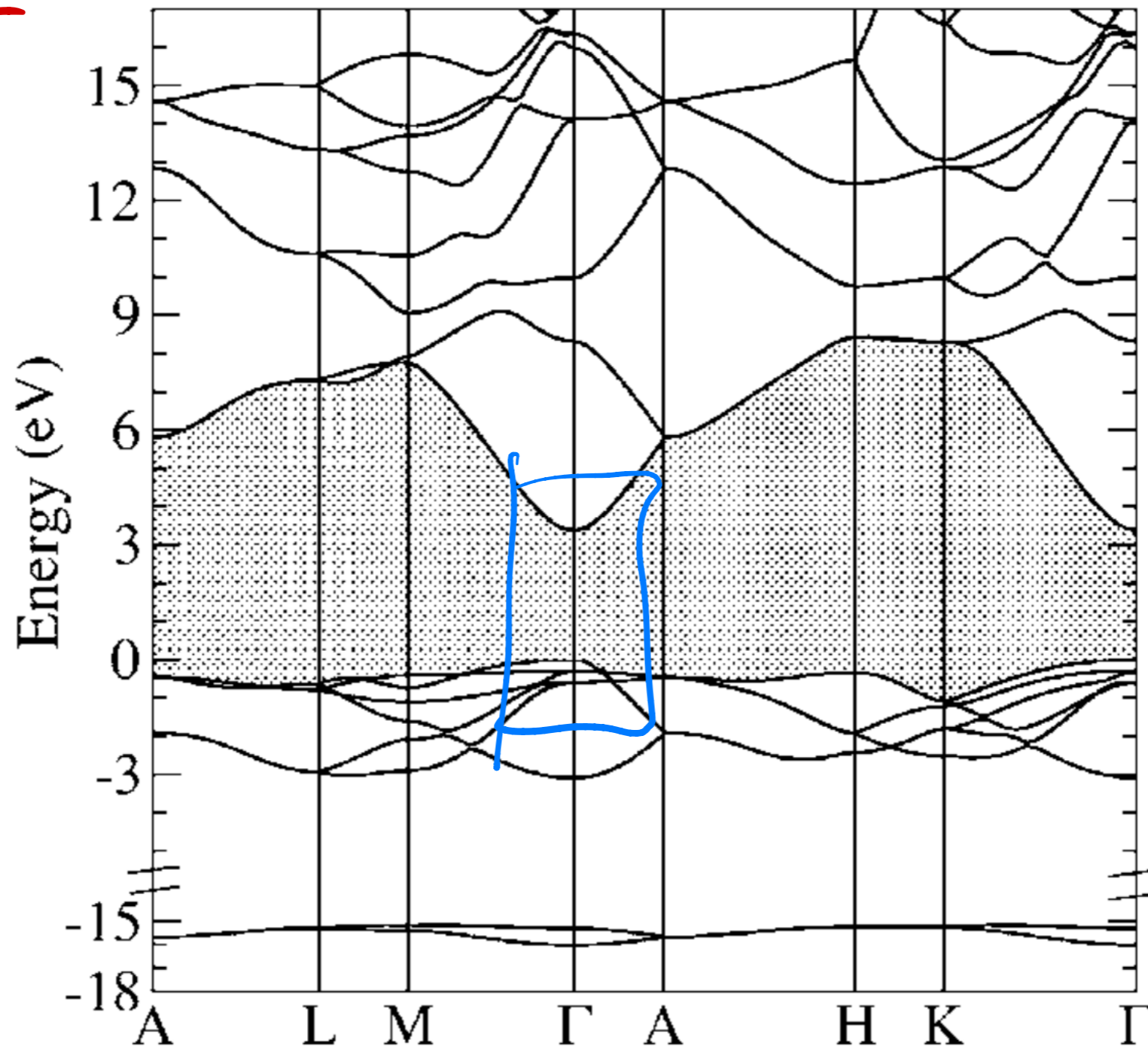
$$V = 0$$



$$V \neq 0 \quad (\text{atoms present})$$

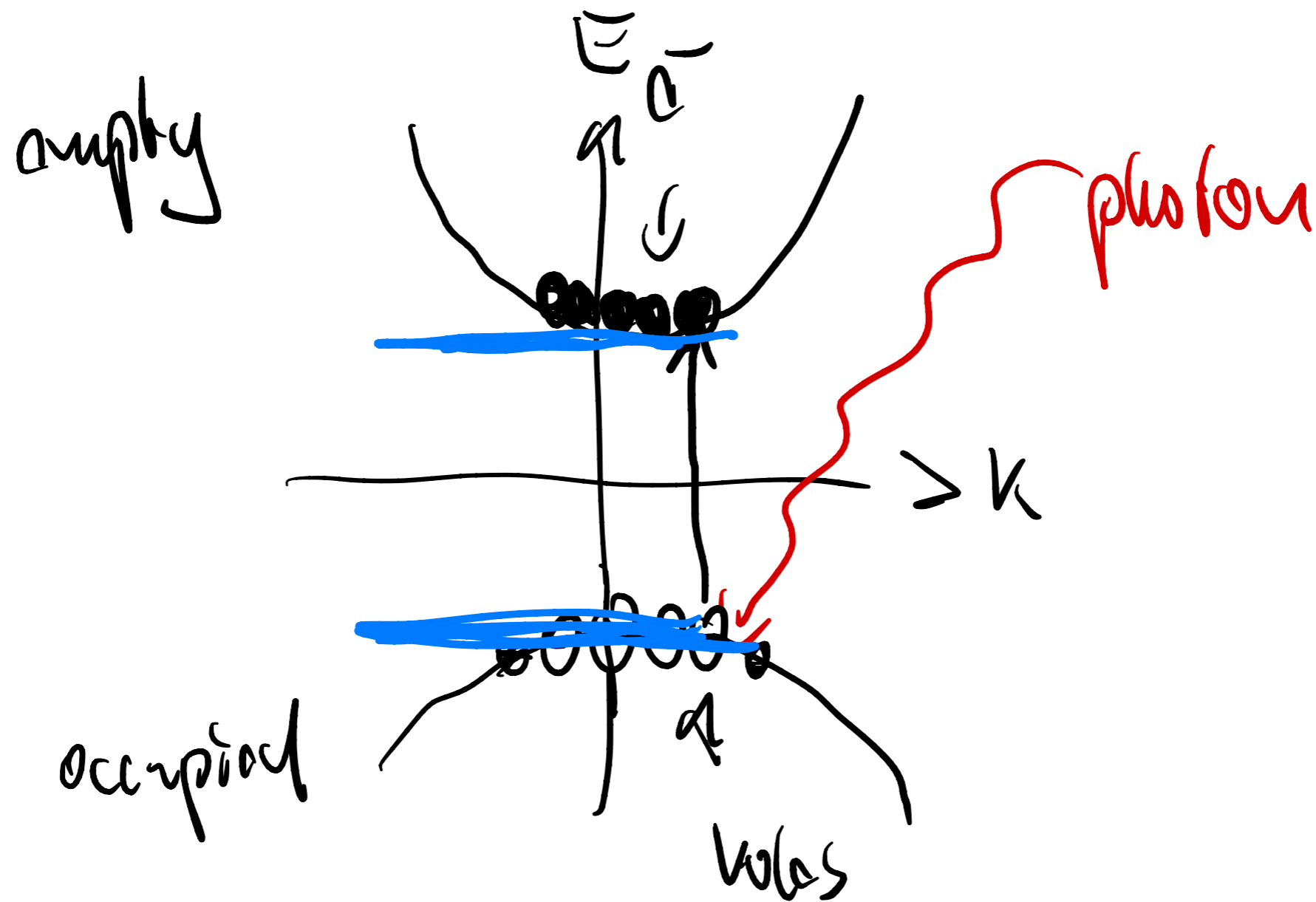
# Band Structure for a real material

E



$\rightarrow K$

# Semiconductors: Conductivity and Absorption

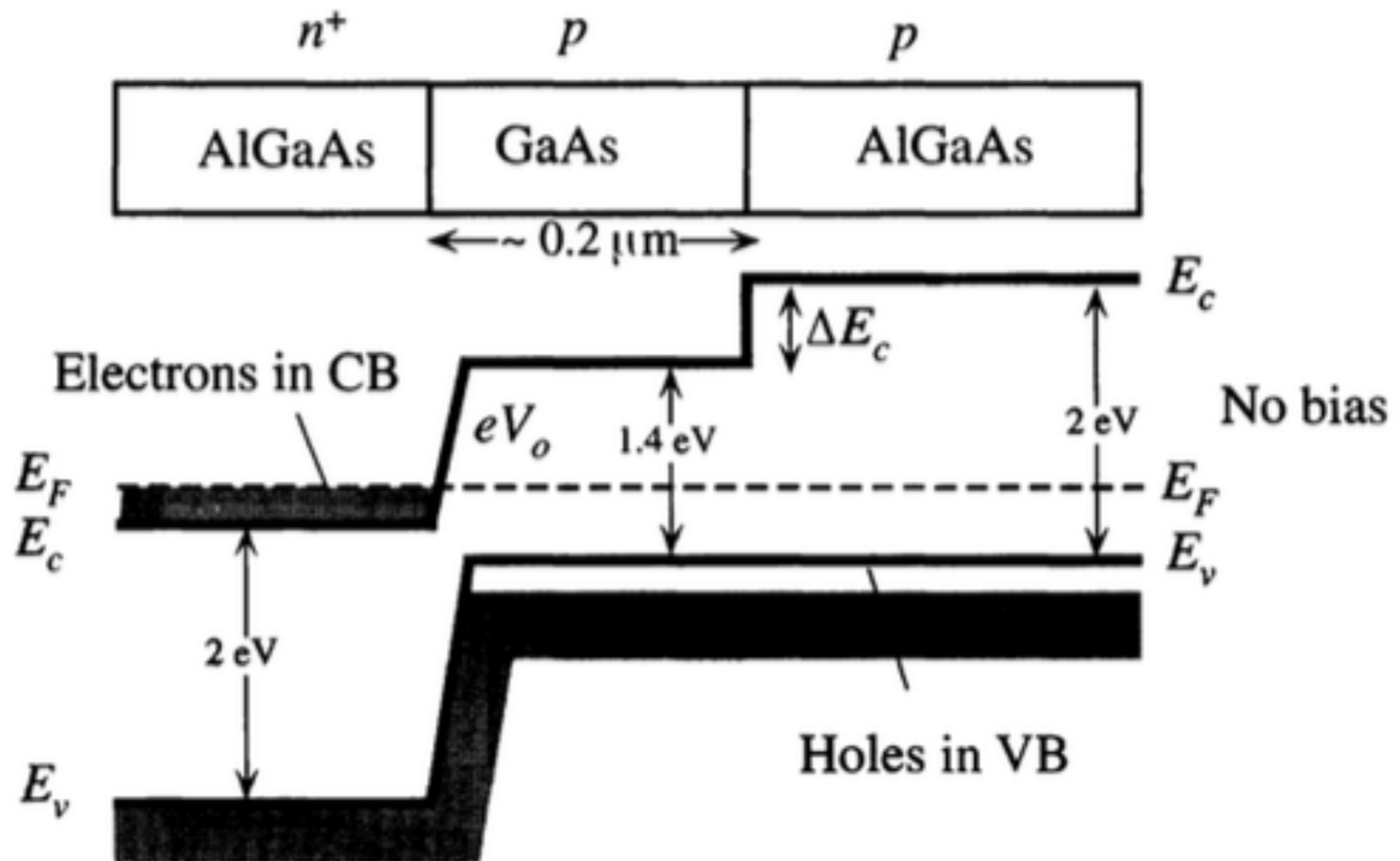


# p-n junction: Energy Band diagram

- before contact:

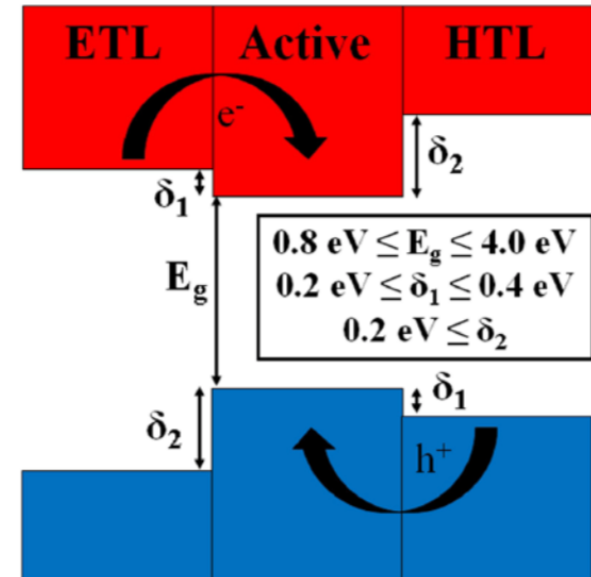
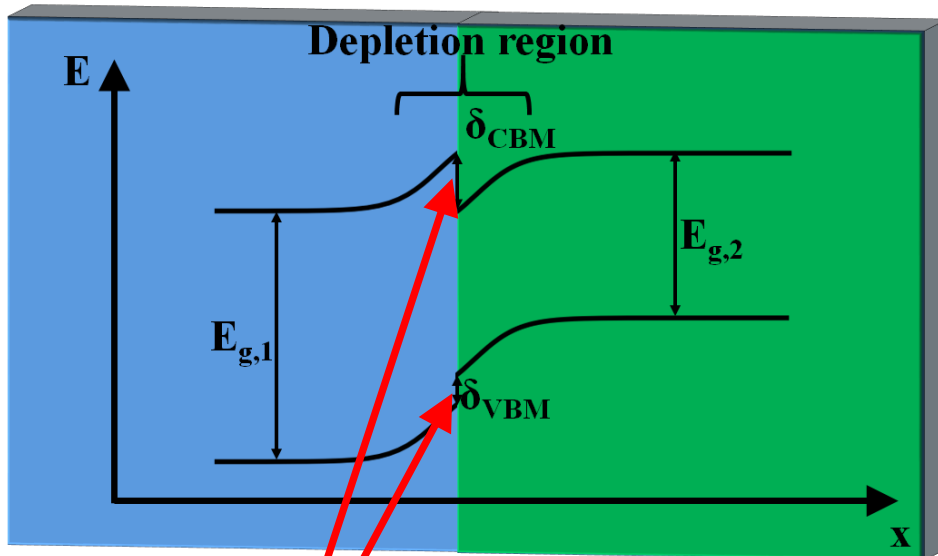
- after contact:

# Heterojunction LED:





# Materials Selection for Semiconductor Heterojunctions



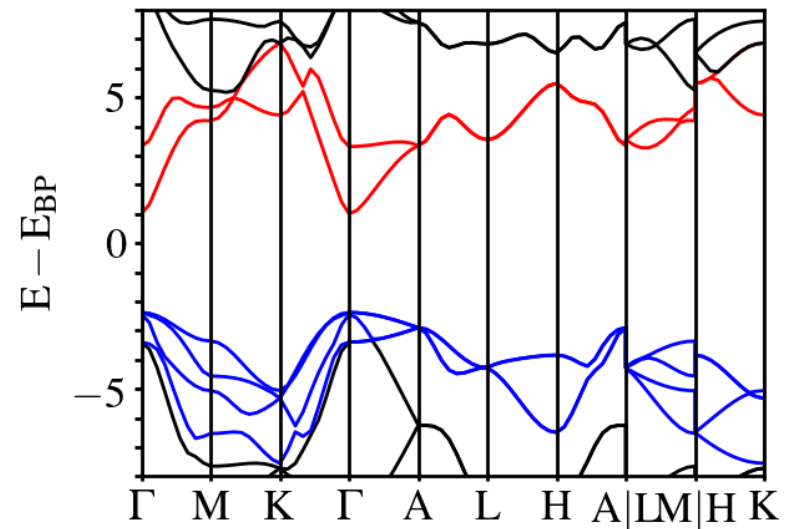
Energy discontinuities at interface for band offsets

# Heterojunctions: Branch-point energy



- Experimentally and computationally difficult to determine how bands bend and align at interface between materials
- Calculate alignment with branch point energy from online electronic structure data in existing databases

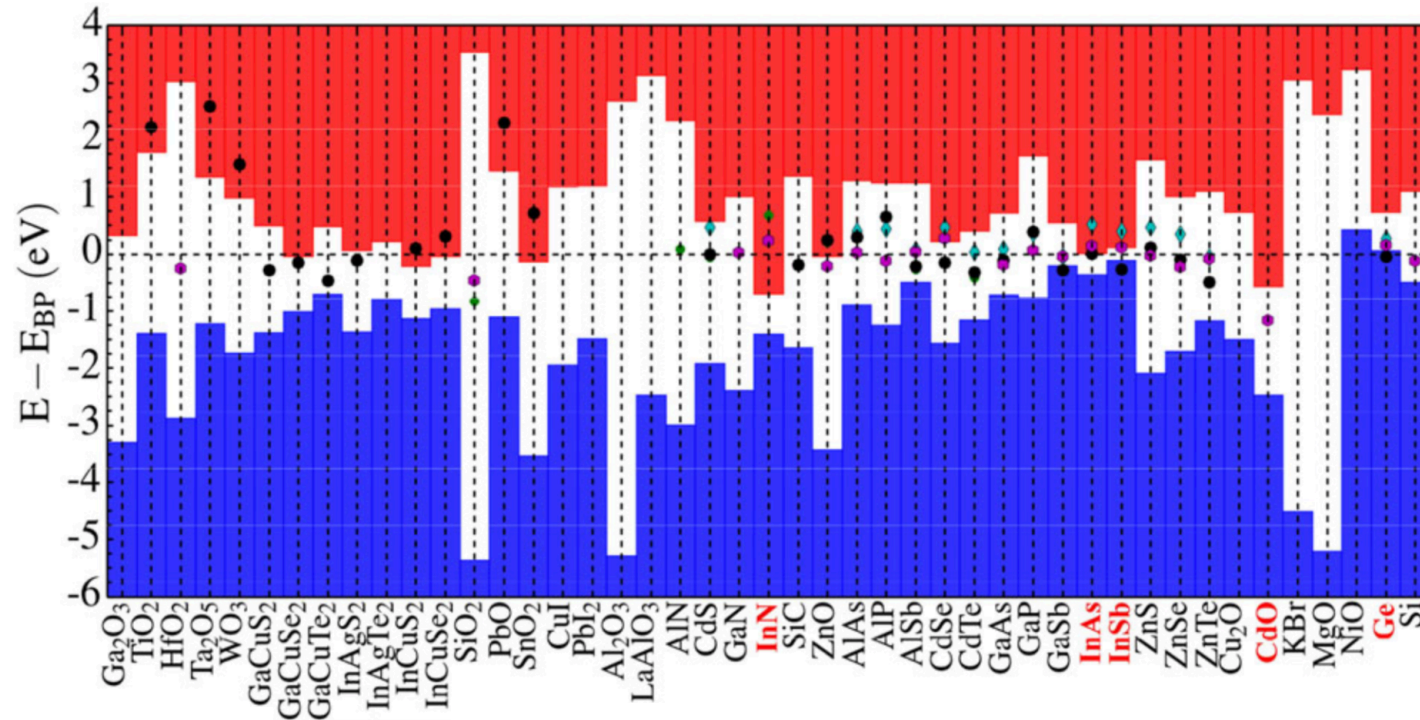
$$E_{BP} = \frac{1}{2N_k} \left( \frac{1}{N_c} \sum_{c_i}^{N_c} \varepsilon_{c_i}^{QP}(\mathbf{k}) + \frac{1}{N_v} \sum_{v_i}^{N_v} \varepsilon_{v_i}^{QP}(\mathbf{k}) \right)$$



- Band gap corrected with two-tiered scheme
  - Use experimental values if available
  - Linear band-gap correction (Curtarolo) otherwise:  $E_g^{\text{corr}} = 1.348E_g^{\text{DFT}} + 0.913 \text{ eV}$
- Use Brus equation for nanocrystals:  $\Delta E_g = \frac{\hbar^2 \pi^2}{2R^2} \left( \frac{1}{m_e^*} + \frac{1}{m_h^*} \right)$

E. Shapera and A. Schleife; Adv. Theor. and Simulations 1, 1800075 (2018)

# Heterojunctions: Branch-point energy



- Visual comparison of band alignment to different experimental (purple, black) and theoretical techniques (green, cyan), 45 semiconductors
- Comparing to Exp. for 21 materials: Mean absolute error = 0.19 eV
- Vacuum-level alignment for 17 materials: MAE = 0.28 eV (us: 0.12 eV)
- Hydrogen-level alignment: MAE = 0.32 eV (us: 0.31 eV)

E. Shapera and A. Schleife; Adv. Theor. and Simulations 1, 1800075 (2018)



+ Code + Text

✓ RAM  
Disk

Editing

This notebook plots the band alignment of materials using the data from Materials Project.

```
import math
import numpy as np
import matplotlib.pyplot as plt
import bisect
import itertools
import copy
import operator
from matplotlib.ticker import AutoMinorLocator
import matplotlib as mpl
mpl.rcParams['axes.linewidth'] = 2.0
csfont = {'fontname': 'Times New Roman'}
from google.colab import drive
drive.mount('/content/drive')
```

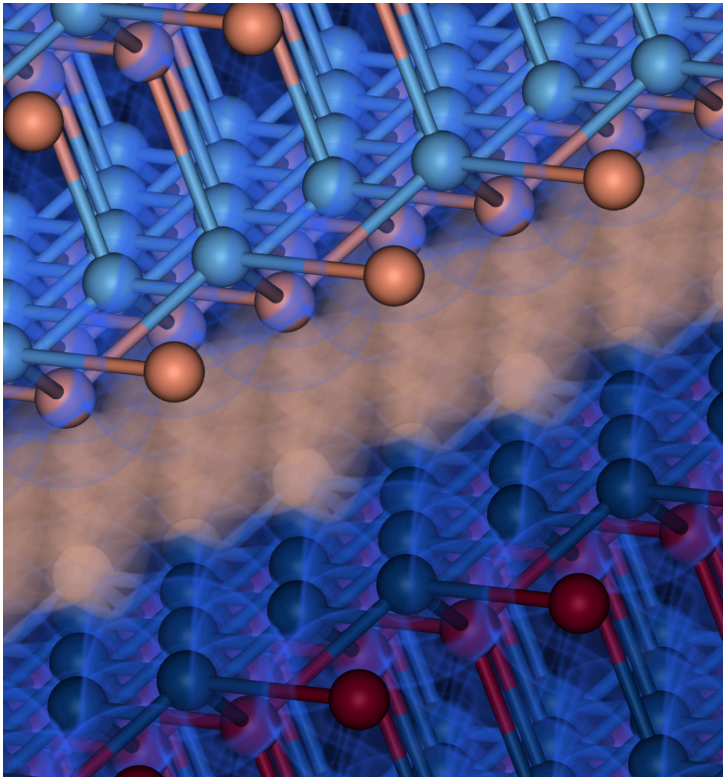
```
↳ Drive already mounted at /content/drive; to attempt to forcibly remount, call drive.mount("/content/drive", force_remount=True).
```

In the next cell, specify the MPIDs for the desired materials. Order is not important, they will be sorted automatically.

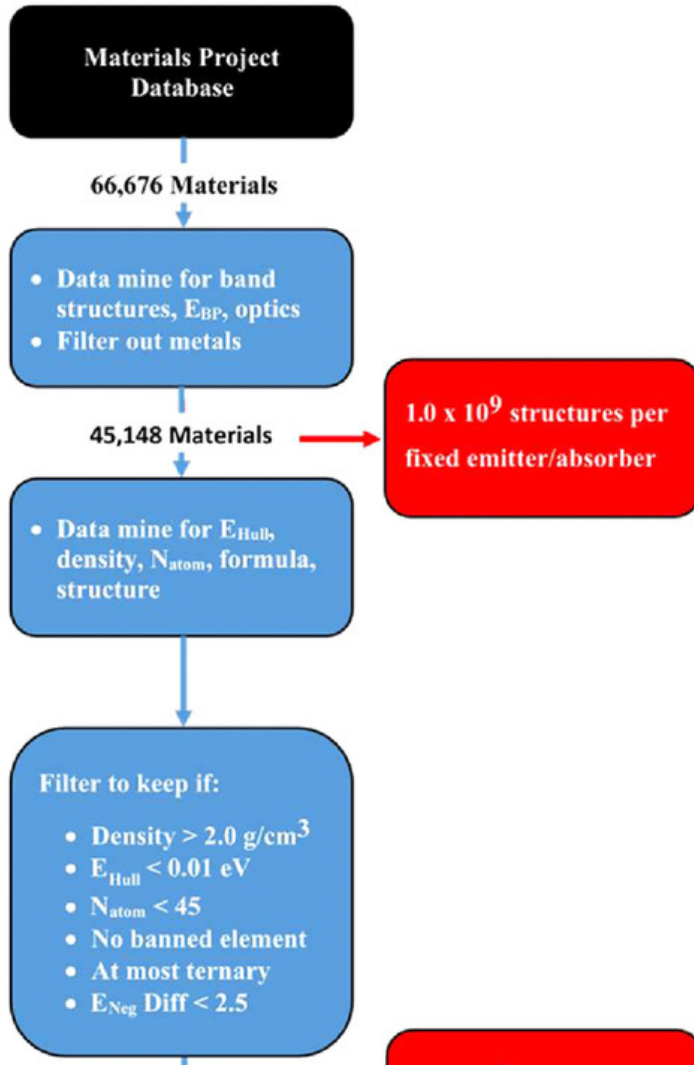
```
[ ] #IDsToPlot=[886,554278,352,10390,19342,5238,4840,3839,19833,22386,22736,22811,546794,19921,856,570136,22883,1143,2920,
#           661,672,804,22205,7631,2133,2172,1550,2624,2691,406,2534,2490,1156,20305,20012,10695,1190,2176,361,1132,
#           23251,1265,715434,32,149]
IDsToPlot=[886,554278,352,10390]
```

The next slide is the main section.

# Semiconductor heterojunctions: Results



# Semiconductor Heterojunctions: Materials selection



- Data-mine Materials Project database for electronic, crystal properties
- Filtering based on experimental criteria

E. Shapera and A. Schleife; Adv. Theor. and Simulations (2018)



In the next cell enter your API key for Materials Project.

```
In [ ]: API_Key=
mp=MPRester(API_Key)
```

The following cell collects the used MPIDs in Materials project and writes to file.

```
In [ ]: IDfilename='idlist.txt'
fID=open(IDfilename, 'w')

data = mp.query(criteria={}, properties=["task_id"])
AllIDs=[]
for i in data:
    AllIDs.append(int(i['task_id'].split('-')[1]))
IDList=list(set(AllIDs))
IDList.sort()
for i in IDList:
    fID.write(str(i)+'\n')
fID.close()
print('Done')
```

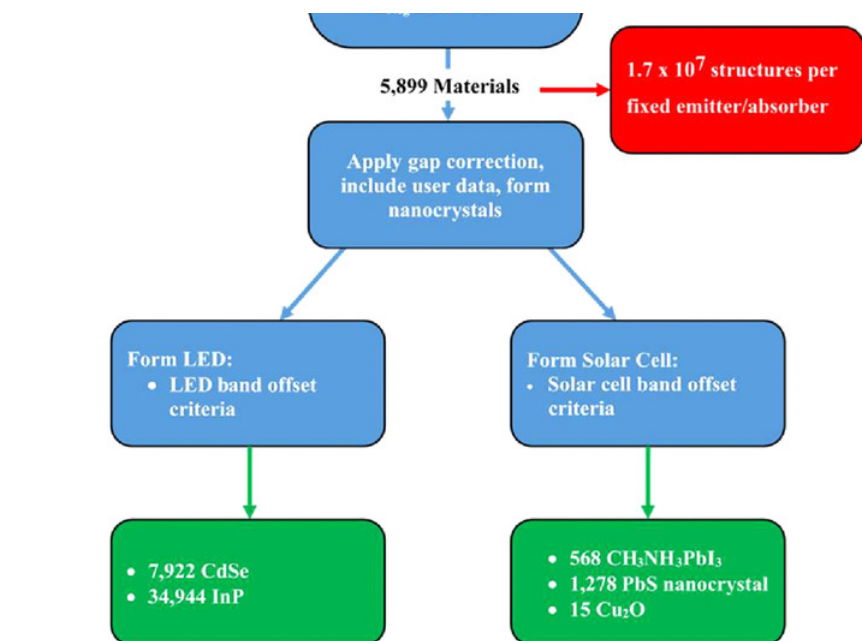
The next cell sets up the output files.

```
In [ ]: filename2='CNData.txt'
g=open(filename2, 'w')
g.write('#MP_Number'+'\t'+ 'CNL'+'\t'+ 'EG'+'\t'+ 'EFermi'+'\t'+ 'ValenceMax'+'\t'+ 'CondMin'+'\n')

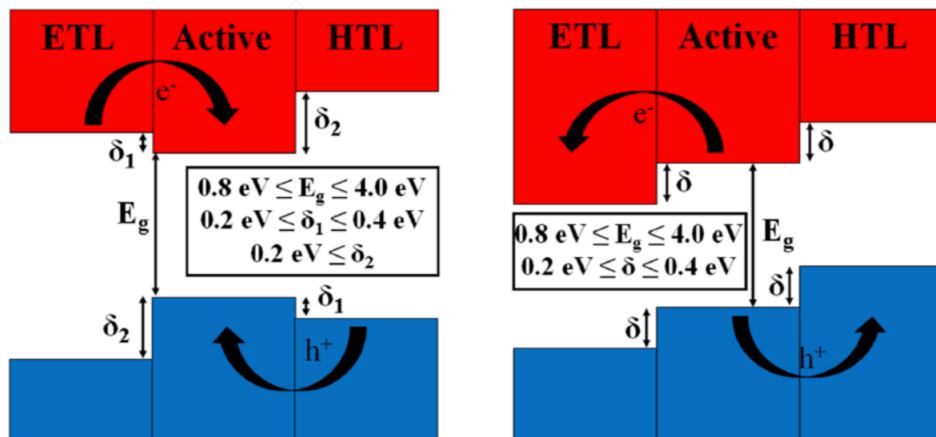
filename3='OpticalProperties.txt'
h=open(filename3, 'w')
h.write('#MP_Number'+'\t'+ 'Eps10'+'\t'+ 'Meff_El'+'\t'+ 'Meff_hole'+'\t'+ 'ExcitonEb'+'\t'+ 'EdgeJDOS'+'\n')
```

The next cell is collects. This will take several hours.

# Semiconductor Heterojunctions: Materials selection



- Impose band offset requirements, determined by application
- Layered structure of semiconductors, electron transport layer, active layer, and hole transport layer
- Rank by figure of merit: Drude model conductivity
- Published as jupyter notebook



E. Shapera and A. Schleife; Adv. Theor. and Simulations (2018)



This is the notebook for designing semiconductor heterostructures based on Materials Project data.

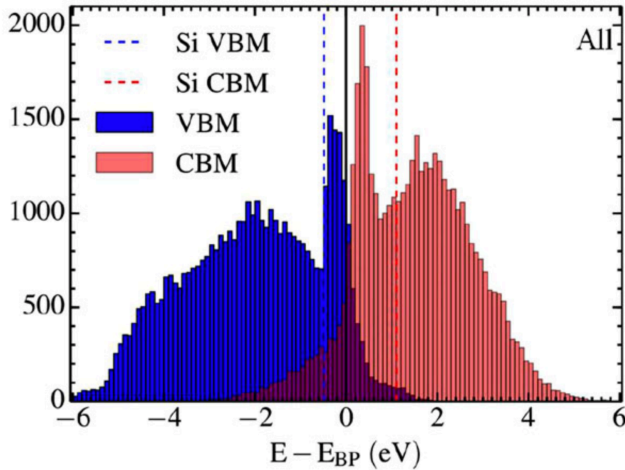
```
In [1]: import numpy as np
import copy
import scipy.constants as cons
import os
import bisect
import re
import itertools
#from sklearn import svm
#from sklearn.model_selection import cross_validate as cross_validation
from shutil import copyfile
import collections
import random
import sys
```

The next section performs preprocessing of data.

```
In [2]: def CostFunction(CostName):
CostVal=0.0
ENegC=[]
for k in range(0, len(CostName), 2):
    ElementIndex=Elements.index(CostName[k])
    CostVal+=float(CostName[k+1])/Abundances[ElementIndex]

    OrderedElementIndex=OrderedElements.index(CostName[k])
    ENegC.append(Electronegativity[OrderedElementIndex])
ENegDif=max(ENegC)-min(ENegC)
```

# Semiconductor Heterojunctions



- More than 500 materials available for any branch-point energy between 0 and 3.5 eV below the CBM as well as 0 and 4.3 eV above the VBM
- Many candidates! Explore?

Example	Binary	Ternary
LED (CdSe)	264	7922
LED (InP)	1764	34 944
Solar Cell (CH <sub>3</sub> NH <sub>3</sub> PbI <sub>3</sub> )	144	568
Solar Cell (PbS nanoparticle R = 5 nm)	212	1278
Solar Cell (Cu <sub>2</sub> O HTL)	4	15

- Found materials that are actually used in practice!
- Found new candidates! Test?
- Problems: Sensitively depends on band-gap correction, branch-point energy, and alignment criteria
- Currently: Fixable by user input

Example	ETL	Active	HTL
LED	Ca <sub>3</sub> N <sub>2</sub> , Mg <sub>3</sub> N <sub>2</sub> , ZnSeO <sub>4</sub>	CdSe	CdS, <sup>[83]</sup> WO <sub>3</sub> , <sup>[84]</sup> MoO <sub>3</sub> , <sup>[85]</sup> SiC <sup>[86]</sup>
LED	ZnSe, <sup>[87]</sup> CuI, NaBiS <sub>2</sub> <sup>[88]</sup>	InP	GaSe, V <sub>2</sub> O <sub>5</sub> , <sup>[89]</sup> SrCuO <sub>2</sub> <sup>[90]</sup>
Solar cell	WO <sub>3</sub> , In <sub>2</sub> S <sub>3</sub> <sup>[91]</sup>	CH <sub>3</sub> NH <sub>3</sub> PbI <sub>3</sub>	Mn <sub>3</sub> O <sub>4</sub> , Cr <sub>2</sub> O <sub>3</sub> <sup>[92]</sup>
Solar cell	PdS, Fe <sub>3</sub> Si <sup>[93]</sup>	PbS, R = 5 nm	MnP, Cr <sub>3</sub> S <sub>4</sub>
Solar cell	Ca <sub>2</sub> Cu <sub>2</sub> O <sub>5</sub>	CdS <sup>[94]</sup>	Cu <sub>2</sub> O

E. Shapera and A. Schleife; Adv. Theor. and Simulations 1, 1800075 (2018)

# Tasks:

- Plot existing data in colab (using colab sheet/files)
- Generate new colab sheet:
  - Connect to MP and download band structures for a few materials
  - Correct the band gap (optional)
  - Compute the branch-point energy
  - Include in your plot