J. Biomechanics Vol. 12, pp. 893–904.

0021-9290/79/1201-0893 $02.00/0

THE INFLUENCE OF TRACK COMPLIANCE ON RUNNING

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Abstract – A model of running is proposed in which the leg is represented as a rack-and-pinion element in series with a damped spring. The rack-and-pinion element emphasizes the role of descending commands, while the damped spring represents the dynamic properties of muscles and the position and the rate sensitivity of reflexes. This model is used to predict separately the effect of track compliance on step length and ground contact time. The predictions are compared with experiments in which athletes ran over tracks of controlled spring stiffness. A sharp spike in foot force up to 5 times body weight was found on hard surfaces, but this spike disappeared as the athletes ran on soft experimental tracks. Both ground contact time and step length increased on very compliant surfaces, leading to moderately reduced running speeds, but a range of track stiffness was discovered which actually enhances speed.

INTRODUCTION

Running is essentially a series of collisions with the ground. As the animal strikes the surface, its muscles contract and ultimately reverse the downward velocity of the body. Intuition argues that a surface of suitably large compliance is bound to change running performance. Running on a diving springboard slows a man down considerably, while running on a trampoline is all but impossible. Our goal in this paper will be to find an analytic expression for the change in the runner's speed, step length and foot contact time as a function of the track stiffness, and to compare these predictions with experiment.

The simplification that the muscles of locomotion and their reflexes act essentially as springs is lent support by recent developments in the study of neural motor control. Reflexes, however, require some time to act – anyone who has unexpectedly stepped off a curb will recall the sharp jolt which results when the antigravity muscles of the leg are not prepared for the impact. Melville Jones and Watt (1971) have shown that approximately 102 msec are required for reflex activity from the otolith apparatus to activate the antigravity muscles in man, so that unexpected falls of less than about 5.0 cm are unaccompanied by reflex accommodation. Even the simple stretch reflex requires a substantial portion of the running step cycle. The latency of EMG changes associated with automatic responses to a change in limb load are found to be in the range of 79 msec for elbow flexion in man (Crago *et al.*, 1976) and near 25 msec for soleus muscles in decerebrate cats (Nichols and Houk, 1976). Since the supported period in human running is typically 100 msec, neither reflexes of vestibular nor stretch origin can be expected to participate in the first quarter of the stance phase, and therefore the antigravity muscles of the leg must be principally under the control of command signals from higher motor centers during this time. In the later portion of the stance phase, however, the stretch reflex can be expected to make important modifications of the efferent activity of α-motorneurons. Houk (1976) has argued that muscle stiffness, rather than muscle length, is the property which is regulated by the stretch reflex. He points out that a competition between length-related excitation contributed by muscle spindle receptors and force-related inhibition contributed by Golgi tendon

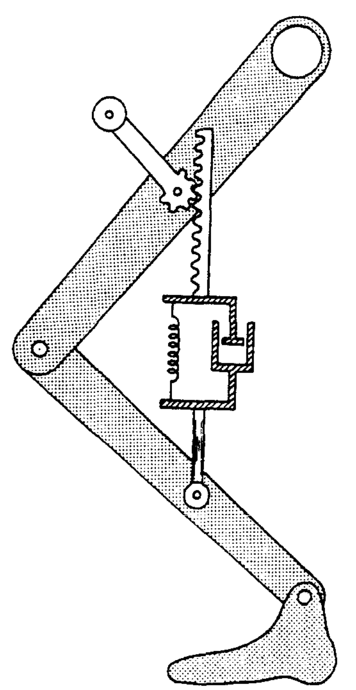

Fig. 1. Schematic representing the separate role of descending commands (rack-and-pinion) and muscle properties plus local reflexes (damped spring). The motion of the rack and pinion element determines the influence of track stiffness on step length. The runner's mass and the damped spring determine the influence of track stiffness on ground contact time.

* *Received* 17 *July* 1978.

organs could result in the ratio of muscle force to length being regulated, rather than either one exclusively. Support for this view comes from ramp stretches of the soleus muscle in decerebrate cats (Nichols and Houk, 1976). These studies show how reflex action can compensate for stretch-induced reductions in muscle force, thus preserving a linear force–length relation in a stretched muscle which would otherwise show acute nonlinearity. Houk suggests that the action of a muscle (or a pair of muscles) about a joint might reasonably be represented as a rack and pinion in series with a spring.

A modification of this scheme is shown in Fig. 1. Movement commands would crank the rack-and-pinion to a new set point for the joint angle, but force disturbances from the outside would deflect the limb by an amount dictated by the damped spring. The dashpot in parallel with the spring is not specifically mentioned in Houk's model, but is necessary to include the rate sensitivity of the stretch receptors and other feedback elements when both muscle force and length are changing rapidly.

Representation of the leg and its musculature as a linear damped spring has already proved successful in describing an exercise in which the subject jumps onto a force platform, falling on the balls of the feet without flexing the knees, and with the ankles forcefully extended (Cavagna, 1970). From the resultant damped oscillation in vertical force (frequency about 3.5 Hz), Cavagna (1970) calculated the effective spring stiffness and damping constant of the extensors of the ankle. The oscillations were always underdamped, with a damping ratio of about 0.2.

In subsequent sections, the function of the damped spring in Fig. 1 is separated from the function of the rack-and-pinion. First, under the assumption that the rack-and-pinion is locked, we treat the vertical motion of the runner as an underdamped mass–spring system, and calculate the time required to rebound from the track as a function of track compliance. The assumption that the rack-and-pinion is locked emphasizes the local control of muscle stiffness at the segmental level during the middle and late portions of the stance phase of limb motion. Later, we assume that the damped spring is locked, and geometric considerations are applied to the rack-and-pinion element to calculate the effect of the track compliance on the man's step length. This assumption emphasizes the pre-programmed, non-reflex control of limb position during the early extension phase, before and just after the foot touches the ground. Finally we obtain a prediction for the top running speed as a function of track compliance. Observations of subjects running on experimental tracks of various stiffness are presented for comparison. Although the calculations show that the man is severely slowed down when the track stiffness is less than his own spring stiffness, there exists an intermediate range of track stiffnesses where his speed is either unaffected by the track or somewhat enhanced.

METHODS

Experiments

Experimental board track. A single-lane running surface 26.25 m in length was constructed of 1.9 cm plywood boards. Each board was 40.6 cm long in the running direction by 121.9 cm wide. The boards were screwed to 4.4 × 8.9 cm rails which served as supports, as shown in Fig. 2. The spring stiffness of the track could be altered by moving the supporting rails closer or farther apart. A typical load–deflection calibration, obtained by applying 0.22 kN weights to a 12.7 cm circular aluminum plate representing the foot, is also shown in Fig. 2(b). The time required for the runner to pass between two transverse light beams 8.20 m apart provided a measure of the runner's speed. The force applied to the track by the runner's foot was measured by a Kissler 9261A force plate, which was linear ±0.5% over a force range of 0–2.0 kN, and had a natural frequency when loaded with a 70 kg man above 200 Hz. A small 60.9 cm square panel of 0.95 cm phenolic resin board supported at either end by 2.54 cm square pine rails rested upon the force plate, as shown. The separation of the 2.54 cm rails was adjusted until the load–deflection curve of the phenolic board matched that of the track to within 2.0%. In this way, the runner was presented with a level track surface of uniform compliance, and the vertical foot force could be measured as he struck the phenolic board. Each subject ran down the center of the track, to ensure that he experienced the compliance measured by the load–deflection calibration. A 16 mm ciné camera, operating at approximately 60 frames per sec, provided a photographic record. A clock in the field of view of the camera was used to calibrate the camera speed.

A total of 8 subjects, all males between the ages of 21 and 34 yr, participated in the experiments (Table 1). The subjects were told to run at a uniform speed. They alternated runs on the track with runs on the concrete surface beside the track. Each subject ran at a variety of speeds, including his top speed. All runners wore conventional running shoes with thin, flat soles.

Pillow track. In order to determine the effect of a very soft surface, the board track was replaced by a 10.9 m long sequence of foam-rubber pillows, each measuring 1.22 m wide by 0.91 m high by 2.74 m long. The runner's speed, step length and ground contact time on each stride were determined by film analysis. The load–deflection curve for the pillow track is shown in Fig. 2(c). There was a large hysteresis, resulting in a different stiffness for loading and unloading at a particular force level.

For the purpose of subsequent calculations, the spring stiffness of the pillows was evaluated at two different force levels. This was done by obtaining the local slope of the force–deflection curve at the 0.8 kN level (1.0g), corresponding to foot forces of the order of the runner's body weight, and at the 1.34 kN level (1.67g), corresponding to the mean foot forces to be

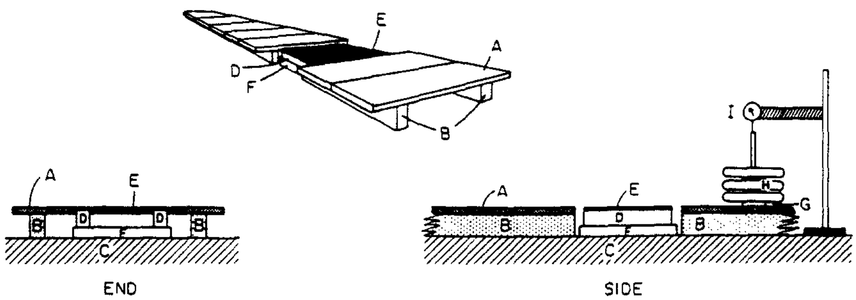

Fig. 2. (a) Three views of the experimental wooden track. A—plywood running surface, B, D–spruce supporting rails, C–concrete floor, E—phenolic resin board, F—force platform, G—aluminum plate representing the foot, H—weights, I—displacement gauge.

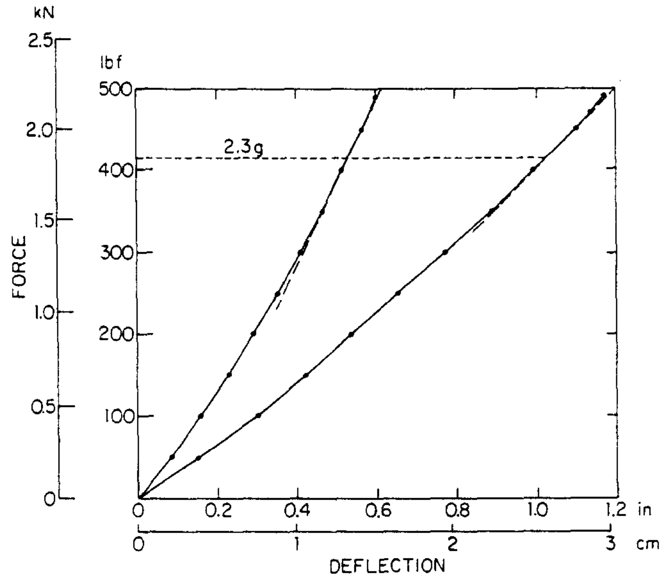

Fig. 2. (b) Force–deflection curves for two configurations of the experimental wooden track. Tangents fit to the 2.3g level give k_t = 13,333 lbf/ft (195 kN/m) and 6857 lbf/ft (100 kN/m).

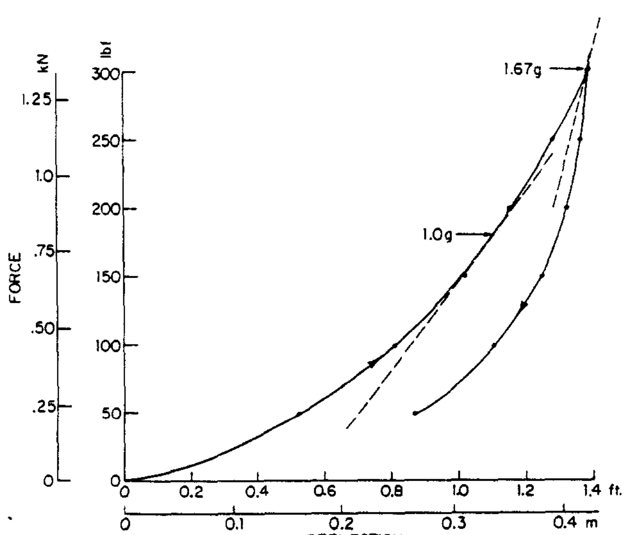

Fig. 2. (c) Force–deflection curves for the foam rubber pillow track, showing 1.0g and 1.67g tangents, which give k_t = 320 lbf/ft (4.67 kN/m) and 985 lbf/ft (14.4 kN/m).

Table 1. Experimental subjects

Subject	Weight (lbf)	Weight (kN)	Height (m)	Leg length, l (m)	Step length, L_o (m)	Hard surface contact time, t_o (sec)	L_o/l	$\left[\frac{(\pi/t_o)^2 l}{(1-\zeta^2)g}\right]$	Runner's spring stiffness $k_m = \frac{m_m \pi^2}{t_o^2}\Big/(1-\zeta^2)$ (lbf/ft)	Runner's spring stiffness (kN/m)
M.F.	180	0.800	1.93	1.09	0.896	0.108	0.83	135.1	6781	98.9
N.H.	175	0.778	1.91	1.00	0.814	0.100	0.81	144.3	7683	112.1
T.M.	175	0.778	1.91	1.01	0.890	0.136	0.88	78.8	4084	59.6
J.J.	180	0.800	1.83	0.978	0.878	0.112	0.89	112.5	6474	94.5
P.G.	190	0.845	1.93	1.05	0.896	0.120	0.86	105.2	5796	84.6
J.C.	160	0.712	1.79	0.969	0.859	0.109	0.89	117.7	5919	86.4
S.R.	160	0.712	1.78	0.960	0.890	0.122	0.93	93.07	4725	68.9
G.L.	150	0.667	1.78	0.960	0.878	0.131	0.92	80.72	3842	56.1

expected during the foot contact time on this compliant surface. The 1.0g and 1.67g pillow stiffnesses were 4.67 kN/m and 14.38 kN/m, respectively.

Each subject generally was tested at between 5 and 8 different running speeds on each of the four track surfaces (concrete, board track at 195 kN/m, board track at 100 kN/m and pillow track). There were exceptions, as in the case of the pillow track, where only 4 subjects participated. As explained in the next sections, foot contact time t_c was included in the tabulations (for Fig. 8) only at the highest running speed of each runner on each surface (27 points). By contrast; each step length determination required a straight-line fitting process like that shown in Fig. 6. Therefore each of the 27 step length points (Fig. 7) represents 5 or more individual runs.

Theoretical considerations

Foot contact time. As a general principle, cushioning works to decrease the forces between colliding bodies by increasing the time of the collision. Joggers know that they are less prone to ligament injuries and shinsplits when they run on somewhat compliant surfaces such as turf, as opposed to city pavements. Typical stiffnesses of some running surfaces are shown in Table 2.

In Fig. 3, a one-dimensional model of the runner and the track is shown which ignores motion in the forward direction and considers only the vertical component. In this model, we have fixed the rack-and-pinion-element from Fig. 1 in a single position, thus emphasizing the role of muscle reflex stiffness. The mass m_m is the man's mass, and k_m is the lumped spring stiffness of the muscles and reflexes acting to extend his hip, knee and ankle. The effective mass of the track surface (the magnitude of an equivalent mass concentrated at a point) is m_t, and the spring stiffness of the track (the inverse of its compliance) is shown as k_t. In the figure, all the masses and springs are attached, so that only that half-cycle of the motion for positive downward displacements of the man (x_m) and track (x_t) has any correspondence with physical reality. When x_m and x_t are negative, the man's foot would, in the actual situation, be separated from the track surface and would therefore not interact with it. Although the permanent connection of the man to the track is fictitious, it makes the mathematics convenient and corresponds approximately to the real situation during the contact portion of the stepping cycle.

Track mass. Let us ignore the man's damping for the moment, and consider the undamped vibration of the man and the track. The natural frequency ω_n(rad/sec) of the lowest mode of vibration, in which the two masses move downward in phase, is given by Den Hartog (1956):

$$\omega_n^2 = \frac{(m_t + m_m)k_m}{2m_t m_m} + \frac{k_t}{2m_t} - \frac{\sqrt{[m_t k_m + m_m(k_t + k_m)]^2 - 4m_t m_m k_t k_m}}{2m_t m_m}. \quad (1)$$

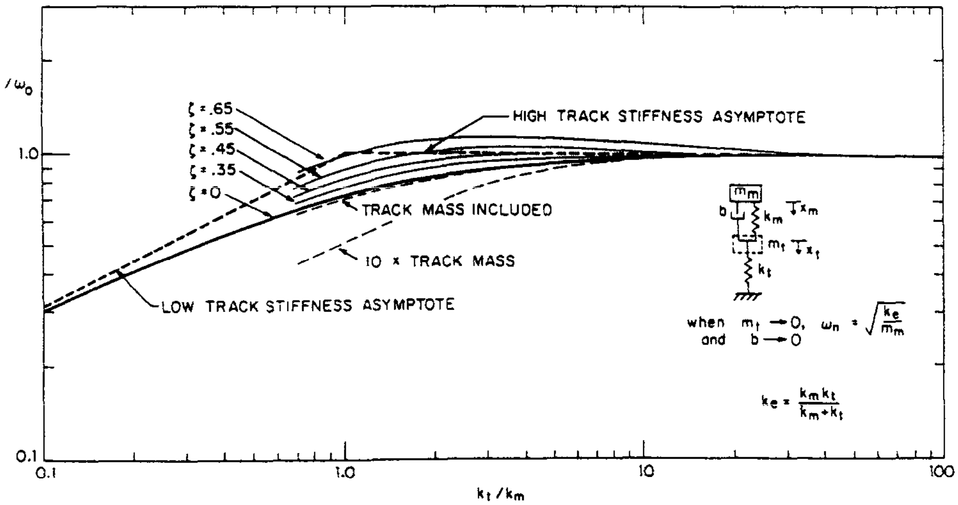

Fig. 3. Normalized natural frequency vs normalized track stiffness. The inset shows the damped two-mass, two-spring system. Heavy line shows zero damping, zero track mass.

Table 2. Stiffness of running surfaces

Material	Stiffness (lbf/ft)	(kN/m)
concrete, asphalt	300,000 +	4376
packed cinders	200,000	2918
board tracks	60,000	875
experimental wooden track	13,333	195
experimental wooden track	6857	100
pillow track at 1.67g	985	14.4

In the rigid-track limit, $k_t/k_m \to \infty$, $m_t/m_m \to \infty$, and the above expression becomes $\omega_o^2 = k_m/m_m$. In the remainder of the paper, the subscript *o* will denote the rigid-track limit. In the limit as the track becomes very soft, $\omega_n^2 = k_t/(m_m + m_t)$. The intersection of these two asymptotic behaviors occurs at a track stiffness $k_t^+ = k_m(m_m + m_t)/m_m$, where the natural frequency ω_n^+ is given by

$$\left(\frac{\omega_n^+}{\omega_o}\right)^2 = \frac{m_t + m_m}{m_t} - \frac{\sqrt{m_m^2 + m_t m_m}}{m_t}. \qquad (2)$$

A broken line showing the influence of track mass on frequency is shown in Fig. 3. Assuming a conventional wooden track construction in which a 1.9 cm × 1.22 m × 2.44 m plywood panel reinforced by 4.4 × 8.9 cm stringers is the running surface, the effective mass of the track is 21.0 kg, which makes $m_t/m_m = 0.25$ for an 87 kg runner. In this calculation, the effective mass of the track is obtained by Rayleigh's method, assuming a sinusoidal two-dimensional mode shape (Timoshenko, 1937). Under these circumstances, the result is that the effective mass is half the total mass of the panel.

The solution shown in Fig. 3 including the track mass is seen to be not very different from the solution for the low track mass limit, $\omega_n^2 = k_t k_m/m_m(k_t + k_m)$, plotted as a heavy solid line just above it. For comparison, the solution when the track mass is increased by a factor of 10 is also shown.

Influence of the man's damping. Since the track mass encountered in practice has so little effect on ω_n, we consider it no further. Taking $m_t = 0$, we investigate the combined effect of the force–velocity relation in the man's muscles and the velocity feedback in the man's stretch reflexes, represented here by the dashpot element, *b*, shown in the schematic drawing in Fig. 3. From the solution presented in Appendix A, the normalized frequency ω_n/ω_o is plotted as a function of dimensionless track stiffness k_t/k_m for 4 choices of the damping ratio $\zeta = b/(2\sqrt{m_m k_m})$. Notice that the damped curves lie above the undamped ones, a consequence of the fact that the dashpot element tends to stiffen the man's impedance in this normalized comparison. The parameter k_m required for this calculation was determined for each ζ from $k_m = m_m\omega_o^2/(1-\zeta^2)$, where m_m and $t_o = \pi/\omega_o$ are the mass and hard surface contact time appropriate for subject M.F. These curves will later be compared with experimental results.

Step length. Two sequences of stick figures, obtained by analysis of the ciné films, are shown in Fig. 4. Each figure was drawn by connecting points locating the major limb joints. The topmost point locates the position of the ear. A remarkable observation is that the trajectory of the ear, and therefore of the otolith apparatus sensing head acceleration, is relatively level, whether the subject runs on the pillows or on the hard surface. Both lower extremities and a single upper extremity are shown in the stick figures. When the subject runs on the pillows, as shown at the bottom, his stance foot sinks into the foam rubber, but the swing foot always remains above the undeflected pillow surface. The extended leg encounters the pillow surface in a position when hip flexion is greater than is the case for running on a hard surface. The step length on the pillow surface is consequently greater.

This observation may be used to construct a model for the influence of track compliance on step length. In the schematic diagram of Fig. 5(a), the leg, length *l*, is shown with the knee fully extended at the

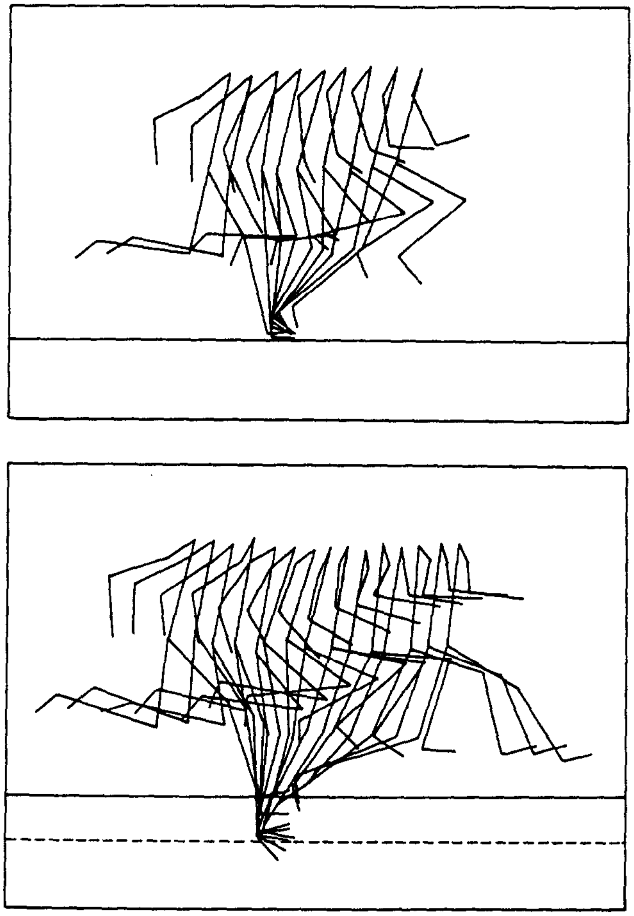

Fig. 4. Stick figures of subject M.F. running, from films. Top: hard surface; bottom: pillow track. Solid line shows undeflected surface of pillow track; broken line shows mean deflection of pillows over an entire step cycle. Only those figures for which the foot was in contact with the surface are drawn. The framing speed was 59 frames/sec.

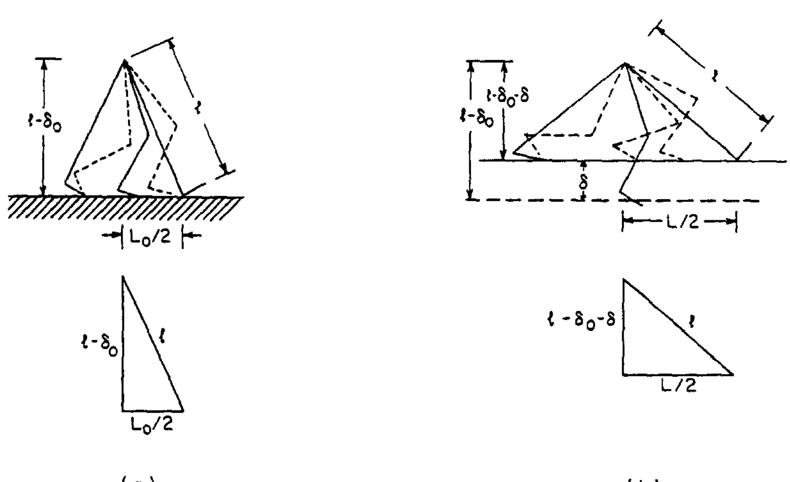

Fig. 5. Schematic of a step on (a) hard surface and (b) pillow track. Solid line shows the stance leg, broken line shows the swing leg moving forward. Because the foot descends a distance δ into the pillows, the step length on the pillow track is necessarily greater.

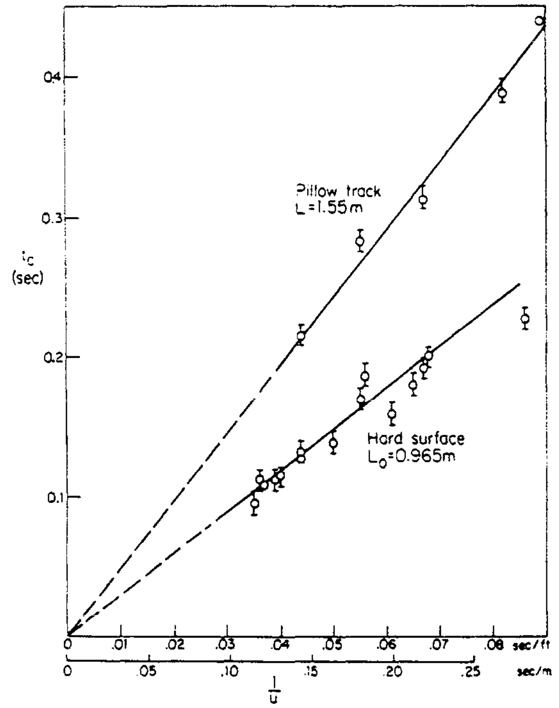

Fig. 6. Ground contact time t_c vs inverse running speed $1/u$, for runner M.F. The straight lines through the origin show that an individual's step length is constant, independent of the running speed, on a particular surface. Step length is greater on the pillow track than on the hard surface. Error bars show maximum uncertainty due to film reading.

moment of contact with the hard surface. It is also shown in mid-stance, when the knee is flexed, and at the end of the stance phase, just before the toe is lifted. In mid-stance, the length of the leg is only $l-\delta_o$, where the shortening δ_o is assumed to be a constant length, independent of running speed, achieved by the "rack-and-pinion" higher postural controllers for the purpose of maintaining the body (and therefore the ear) on an approximately level trajectory. Notice that this assumption effectively fixes the length of the damped spring in Fig. 1 as if the spring stiffness k_m were now taken to be infinite. Since $\delta_o = 9.6$ cm for subject M.F. running on the hard surface, but the maximum deflection of his "spring" would be expected to be only 1.86 cm, this assumption appears to be justified. The important point is that the base of the triangle shown in the lower part of the figure is longer, and thus the step length L is longer on the pillow surface (Fig. 5b). The distance δ is the mean deflection of the pillow surface throughout a complete stride, including the aerial phases. If the man were not running at all, but merely standing quietly on the pillows, he would be standing in a well of depth $\delta = m_m g/k_t$, where k_t in this instance is the pillow stiffness measured at the 1.0g force level. The broken line in Fig. 5(b), representing the mean deflection of the pillow track, plays the same role as the solid line in Fig. 5(a): all details of the step are arbitrarily presumed to be the same, including the distance from the broken line to the hip, $l-\delta_o$. Only the hip flexion angle at which the heel contacts the track is different on the pillows, leading to the longer step length. Applying the Pythagorean theorem to the triangle in Fig. 5(b),

$$L = 2\sqrt{l^2 - (l - \delta_o - \delta)^2}. \tag{3}$$

The constant δ_o may be written in terms of the step length on the hard surface, L_o,

$$\delta_o = l - \sqrt{l^2 - L_o^2/4}. \tag{4}$$

(a)

(b)

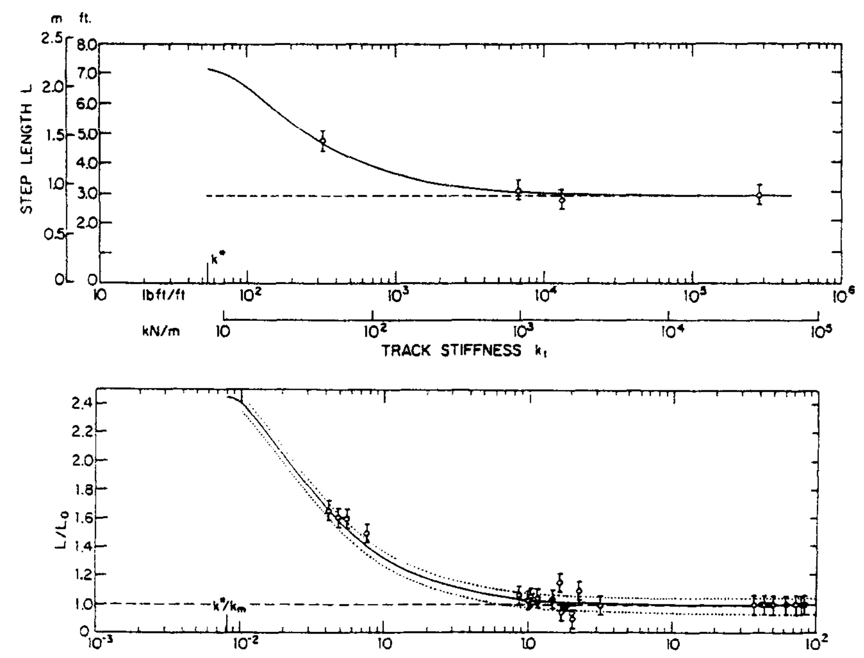

Fig. 7. Step length vs track stiffness. The solid line shows the theoretical prediction. (a) Subject M.F. alone. (b) Dimensionless plot showing all 8 subjects.

Combining equations (4) and (3), with $\delta = m_m g/k_t$,

$$L = 2\sqrt{l^2 - [(l^2 - L_o^2/4)^{1/2} - m_m g/k_t]^2}. \quad (5)$$

Equation (5) is plotted in Fig. 7(a), assuming a 0.8 kN man with a leg length $l = 1.09$ m and a hard-surface step length $L_o = 0.896$ m (appropriate for subject M.F.). When the expression in the square bracket is zero, the step length has reached its maximum, namely twice the leg length. Thus running on surfaces whose stiffness is less than $k_t^* = m_m g/\sqrt{l^2 - L_o^2/4}$ would not be possible, according to this model, since the hips would have descended below the surface of the pillows.

RESULTS

Dimensionless plotting

Since the results are presented on dimensionless axes, we have included a short justification for the validity of this procedure in Appendix B. Basically, the method is required because we wish to compare the performances of several runners on the same figure. If the dimensional axes were retained, the performance of a single runner could be compared to a single line especially computed for that runner (for example, Fig. 7a for subject M.F.), but a complete presentation of the results would require as many figures as there were runners.

Man's spring determined by foot contact time

In plotting each data point on a typical dimensionless graph (e.g. Fig. 7b), it was first necessary to know the man's spring stiffness k_m. This, in general, is a function of the man's effort, and increases as he runs faster. In Figs. 3, 7, 8 and 9, we compare only the maximum running performance as a function of track stiffness and therefore $k_m = m_m\omega_o^2/(1-\zeta^2)$ where $\omega_o = \pi/t_o$, t_o is the time the foot is in contact with the ground while running at maximum effort on the hardest surface, and ζ is the damping ratio (assumed to be 0.55 for each runner, as explained below).

Foot contact time vs track stiffness

In Fig. 8, foot contact time t_c/t_o is plotted against track stiffness k_t/k_m. The theoretical line represents a damping ratio for the man of $\zeta = 0.55$. This damping ratio was chosen among the four shown in Fig. 3, on the basis of its satisfactory fit to the experimental points shown in Fig. 8. The linearized spring stiffness of the pillows was taken as the 1.67g stiffness, 14.4 kN/m, since the pillows acted with this stiffness during most of the time the runner was in contact with the track, when foot forces were in the range of 1.67 times body weight. We shall return to this point later, with an explanation of how the figure 1.67g was determined.

The error bars for each point show the estimated maximum uncertainty in reading the films and force records, which was generally less than $\pm 7.0\%$. The dotted lines on either side of the theoretical line are displaced by one standard deviation σ, where σ is estimated from the root of the mean of squared residuals:

$$\sigma \simeq [\hat{\sigma}^2]^{1/2} = \left[\frac{1}{n-2}\sum_{i=1}^{n}[y_i(x) - y(x)]^2\right]^{1/2}. \quad (6)$$

Here $y_i(x)$ is the measured value and $y(x)$ is the computed value of a parameter at a given x (Meyer, 1975).

Step length independent of running speed on a given surface

In their comprehensive study of human gait, Cavagna *et al.* (1976) noticed that the step length, the distance a man travels while one foot is in contact with the ground, is a constant value for a given individual runner on a hard surface, independent of his running speed. We were able to corroborate this finding, as shown in Fig. 6 for subject M.F. The time in contact with the ground, t_c, is proportional to the inverse of velocity. The slope of this line defines the step length L, which is independent of the speed but very much larger on the pillows than on the hard surface. When L/L_o is plotted against k_t/k_m in Fig. 7, a very good agreement is

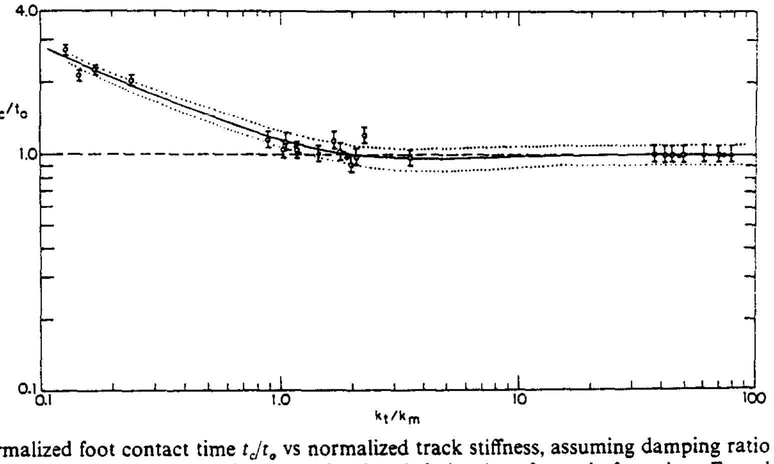

Fig. 8. Normalized foot contact time t_c/t_o vs normalized track stiffness, assuming damping ratio $\zeta = 0.55$. Open circles show data produced by film analysis; closed circles show force platform data. Error bars show limits of uncertainty due to film and oscilloscope reading; dotted lines are one standard deviation above and below the theoretical line.

found with the theoretical line, with a standard deviation of 0.045.

Note that the spring stiffness used for the pillows is now the 1.0g stiffness, 4.67 kN/m, because the deflection δ in equation (3) must correspond to the distance a man would sink down if he were merely standing at rest on the (linearized) pillows. Recall that the step length theory was derived entirely on the basis of geometrical considerations, and did not involve the man's spring stiffness. It is therefore consistent with the observed fact that the step length on a particular surface is independent of running speed.

Foot force

The average vertical force applied to the ground by the foot during a step is equal to the runner's mass times his mean vertical acceleration,

$$\bar{F} = m_m g + 2m_m v/t_c, \quad (7)$$

where v is the downward vertical velocity at the moment of contact. In our experiments, we measured v by integrating the force over the duration of t_c, and found no significant difference in v for a given subject on a hard, as opposed to a compliant, surface. Thus v is taken to be a constant, found for a particular runner from the area under the force–time curve,

$$v = \frac{\int_o^{t_c} (F - m_m g)\,dt}{2m_m}. \quad (8)$$

Taking a representative $v = 0.732$ m/sec for a 0.8 kN subject, and using values for t_c obtained from Fig. 8 in the case where the damping ratio $\zeta = 0.55$, a dimensionless $\bar{F}/\bar{F}_o$ vs k_t/k_m curve may be plotted (Fig. 9). This theoretical line agrees reasonably well with the force-plate data points, and shows that no appreciable change in the mean levels of foot force can be expected until the track stiffness is significantly less than the stiffness of the man. Note that it was not possible to measure foot force during the pillow running experiments, but the prediction would be that average force was lowered to 0.71 times its hard-surface value, or about 1.67 times body weight.

Representative force signatures, traced from the oscilloscope photographs for subject J.C., are shown in the lower portion of Fig. 9. On the hard surface, the initial contact of the foot with the ground produced a spike in foot force which often exceeded 5 times body weight. This spike was either absent or very much attenuated when the same subject ran on a compliant track. We suspect this dramatic reduction of foot force at initial contact is the reason that all subjects reported a subjective impression of increased running comfort on the compliant surfaces relative to the hard surfaces.

Running speed

Having obtained predictions for the ground contact time t_c and step length L, we may put these together to obtain the running speed, $u = L/t_c$. A consequence of the fact that L and t_c are nearly constant in the intermediate range of track stiffness is that running speed should not be significantly affected until k_t/k_m drops below 1.0. At low track stiffness, foot contact time t_c increases, but so does step length, so the runner is not slowed down as much as contact time alone would predict. For example, running on the pillows increased t_c by an average factor of 2.3, but the runner's speed was not halved. Instead, since step length increased by a factor of 1.6, the runner's speed was preserved at 70% of its hard-surface value.

DISCUSSION

Limitations of the analysis

It is important to review the assumptions made at various points throughout this paper, and to understand how they limit the analysis. We began by representing the antigravity muscles and their reflexes

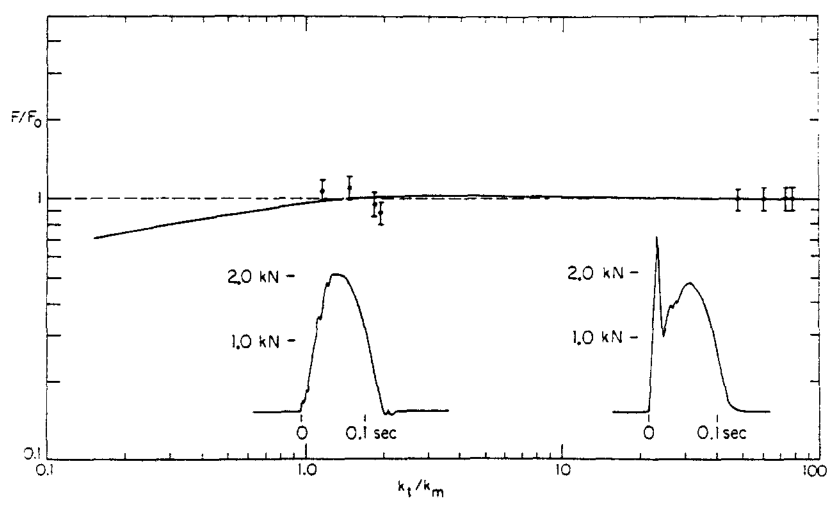

Fig. 9. Normalized average foot force vs normalized track stiffness. Solid line shows theory, solid points show average force platform results for each of four subjects. Insets show how the initial force transient experienced on hard surfaces is abolished on the experimental wooden track.

by the simple mechanical system shown in Fig. 1, and used the dynamic characteristics of the spring and dashpot to calculate the influence of track stiffness on ground contact time. Later, we used the conceptual model of the rack-and-pinion element, ignoring the damped spring, to calculate the influence of track stiffness on step length. We acknowledged the intrinsic nonlinearity of the force–length characteristic of stretched muscle, but claimed, following Houk, that reflex compensation acts to restore linearity. In another paper (Greene and McMahon, 1979), we have measured the short-range spring stiffness of the muscular reflexes of the leg, as a function of both knee angle and total force, and find that the effective spring stiffness of the leg varies by a factor of 2 over the knee angles encountered in running, but most of the variation occurs in the first 15 degrees of knee flexion. Remarkably, the spring stiffness at a constant knee angle is found to be no more than 25% greater as the subject carries loads up to twice body weight on his shoulders. Thus, as long as the knee angle θ is kept within the range $15° \leq \theta \leq 45°$, as it commonly is during running, our assumption of one single spring stiffness for the leg throughout the step cycle is reasonably valid.

Another simplification involves the pillows: we have assumed that their load–deflection curve is linear, whereas Fig. 2 shows that it is most distinctly nonlinear. We have also neglected damping in the pillows, which is probably not entirely justified. In addition, we have neglected the horizontal compliance of the pillows.

We dealt with the nonlinearities of the pillows by assigning the 1.0g stiffness its proper role in determining step length, while we assumed that the 1.67g stiffness determines foot contact time. Since we found that most of the subjects applied a sustained vertical force of about 2.4 times body weight to the hard-surface track, according to Fig. 9, $\bar{F}$ on the pillows should be about 0.7 times this value, or about 1.67 times body weight. Thus our measurements and predictions are consistent with our basic assumptions about foot force on the pillows.

Man's stiffness

The concept of the man's spring stiffness is complicated by the fact that it depends on his effort. Under the assumptions of equation (1), when the man runs twice as fast, his spring stiffness increases by a factor of 4. We have attempted to eliminate the effort dependence of the man's stiffness in this paper by making comparisons of performance on different track stiffnesses only when the man is running at maximal effort.

Man's damping

We employed a damping element in parallel with the man's spring because we knew that (1) isolated muscles obey a Hill force–velocity curve (Hill, 1938), and (2) the muscle spindles return velocity information to the spinal cord. Our decision about how much damping was realistic depended on the curve-fitting procedure shown in Fig. 3. Since the curve representing a damping ratio of $\zeta = 0.55$ provided the best fit through the experimentally determined points for t_c/t_o, we took that value of ζ for subsequent calculations of foot force and running speed.

Our assumption that the damping element is linear is certainly a great oversimplification. Katz (1939) showed 40 years ago that the damping parameter b (Fig. 3) is about 6 times greater for slow lengthening as opposed to slow shortening in isolated muscles. The extent to which this effect is modified by reflex phenomena is unknown.

Could an independent set of experiments, not involving running, be proposed to measure the value of ζ appropriate for running? Cavagna (1970) was able to measure ζ by allowing his subjects to go through several damped cycles of ringing while the muscles of the calf remained in sustained contraction. In running, no such ringing oscillations could ever be observed because the foot remains in contact with the ground for only half a ringing cycle, and the total mechanical energy is the same at the beginning and the end of each supported period. In fact, this is a property of all nonlinear oscillations, that the energy lost in the dissipative mechanism matches the energy added per cycle by the "negative resistance" phenomenon. Thus, only indirect techniques which change the operating characteristics of the oscillator by changing one of its component parts (here we used the track) can serve to analyze the remaining components.

As a final remark it is worth noting that the model of the vertical motion of the runner shown in Fig. 3 can easily be made into a nonlinear oscillator. Suppose that when both the man and the track are descending, and therefore when the leg is being flexed by the man's downward momentum, the damping constant of the dashpot, b, is positive, as was assumed in the body of the paper. As an additional feature, suppose that when the trajectory of the center of mass x_m reaches its lowest point, b suddenly switches sign, and provides negative damping for the next half-cycle. The sudden change in the sign of b requires a sudden advance in the phase of $x_m - x_t$ with respect to x_m, and this requires a step change in the length of x_t. The essential result is that, by postulating a damping which switches sign at mid-stride (as if it were determined by joint receptors), we may generate an oscillatory motion whose amplitude does not decay with time, and yet whose period is the same as the simple system with linear damping discussed in the body of the paper.

SUMMARY AND CONCLUSIONS

Beginning with a model of the antigravity muscles and reflexes which assumes that they have an automatic, or reactive, component which makes them behave like a damped linear spring, and this is in series with a purposeful component which behaves like an externally controlled rack-and-pinion, we have derived ground contact time, step length, foot force and running speed as functions of track compliance. These predictions are compared with the results of experiments in which subjects ran alternately on a compliant and a hard surface, and the agreement is generally good.

Very compliant tracks, which have a spring stiffness much less than the man's stiffness, are responsible for a marked penalty in the runner's performance. For example, when a man runs on a track which is 0.15 times his own stiffness, his running speed is reduced to 0.70 times the speed he could run on a hard surface.

On tracks of intermediate compliance, the analytical model predicts a slight speed enhancement, due to a decrease in foot contact time and an increase in step length, by comparison with running on a hard surface. Another important advantage of such tracks of intermediate compliance is the marked attenuation of the early peak in foot force, which can reach 5.0 times body weight in running on a hard surface.

A permanent indoor track having a stiffness about three times the man's stiffness has recently been completed in the new indoor athletic facility at Harvard University. Experience to date indicates that good runners are able to better their usual times in the mile by about 5 sec on this track. This represents a speed enhancement of 2%, in good agreement with the theoretical prediction. The runners also report that this track is particularly comfortable to run on, and is apparently responsible for a very low rate of running injuries.

Acknowledgements – This work was supported in part by the Harvard University Planning Office and by the Division of Applied Sciences, Harvard University, Cambridge, Massachusetts.

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NOMENCLATURE

- b — linear dashpot damping constant of man, N.sec.m^{-1}
- $\bar{F}$ — average vertical force during a step
- k_m — $m_m\omega_0^2/(1-\zeta^2)$ = stiffness of man's muscles and reflexes acting to extend hip, knee and ankle, N/m
- k_t — spring stiffness of track (= 1/compliance), N/m
- k_t^* — $m_m g(l^2 - L_0^2/4)^{-1/2}$ = lowest possible track stiffness for running, N/m
- L — step length; distance moved during foot contact, m
- L_0 — step length on infinitely hard surface
- m_m — mass of the man, kg
- m_t — effective mass of the track, evaluated by Rayleigh method
- t_c — foot contact time on any track, sec
- t_0 — π/ω_0 = foot contact time on infinitely hard surface
- u — L/t_c = running speed
- v — downward vertical velocity at moment of contact, m/sec
- x_m — downward displacement of the man
- x_t — downward displacement of the track
- δ — mean deflection of pillow surface in a stride, m
- δ_0 — shortening of the leg at mid-stance, m
- ζ — $b/(2\sqrt{m_m k_m})$ = damping ratio of man
- l — fully extended leg length, acetabulum to heel, m
- ω_n — natural frequency of man and track in lowest mode of vibration, rad/sec

Subscripts

- m — man
- o — rigid-track limit
- t — track.

APPENDIX A

Calculation of Natural Frequency

Assume the track mass $m_t = 0$ in the schematic drawing in Fig. 3. Summing the forces acting on the track to zero,

$$(x_m - x_t)k_m + (\dot{x}_m - \dot{x}_t)b - x_t k_t = 0. \quad \text{(A-1)}$$

Summing the forces acting on the man,

$$m_m \ddot{x}_m = -(x_m - x_t)k_m - (\dot{x}_m - \dot{x}_t)b. \quad \text{(A-2)}$$

The frequency of the lowest mode of vibration, where the track and the man move down together, may be found by assuming a solution of the form

$$x_m = e^{i\omega t} \quad \text{(A-3)}$$

$$x_t = A\,e^{i\omega t}, \quad \text{(A-4)}$$

where A is a complex constant. Substituting equations (A-3) and (A-4) into (A-1) and (A-2)

$$(1-A)k_m + i\omega(1-A)b - Ak_t = 0 \quad \text{(A-5)}$$

$$(1-A)k_m + i\omega(1-A)b - m_m\omega^2 = 0. \quad \text{(A-6)}$$

Subtracting equation (A-6) from equation (A-5) gives

$$A = \frac{m_m\omega^2}{k_t}. \quad \text{(A-7)}$$

Substituting equation (A-7) into equation (A-5),

$$\left(1 - \frac{m_m\omega^2}{k_t}\right)k_m + i\omega b\left(1 - \frac{m_m\omega^2}{k_t}\right) - m_m\omega^2 = 0. \quad \text{(A-8)}$$

Collecting terms in ω,

$$\omega^3[im_m b] + \omega^2[m_m(k_t + k_m)] - \omega i k_t b - k_t k_m = 0. \quad \text{(A-9)}$$

This cubic equation was solved numerically to obtain both the real and imaginary parts of ω as a function of the parameters b, k_t, k_m and m_m. The real part of ω is called ω_n and plotted in Fig. 3 for four choices of the damping ratio $\zeta = b/(2\sqrt{m_m k_m})$.

APPENDIX B

Dimensionless Plotting

The techniques of dimensional analysis allow great simplification and reduction of labor in experimental problems where a large number of variables appear (Bridgeman, 1931). In this paper, two such problems have been discussed, the determination of step time t_c and step length L as a function of track stiffness k_t and other variables. Let us consider the dimensional analysis of each problem separately.

(a) *Foot contact time*

Assume that a functional relationship of the following form exists:

$$f(t_c, t_o, m_m, \zeta, k_t, k_m) = 0,$$

where

t_c = foot contact time, sec
t_o = contact time on hard surface, sec
m_m = runner's mass, kg
ζ = runner's damping ratio, dimensionless
k_t = track stiffness, N/m
k_m = man's stiffness, N/m.

One of these variables, the man's damping ratio, is already dimensionless. From the remaining five variables, the two dimensionless products t_c/t_o and k_t/k_m can be formed. A third dimensionless product using m_m, k_m and ζ may also be formed, so that the assumed form of the equation becomes:

$$\phi\left(\frac{t_c}{t_o}, \frac{k_t}{k_m}, \zeta, \frac{k_m t_o^2}{\pi^2 m_m(1-\zeta^2)}\right) = 0.$$

The form of the last dimensionless group was chosen in such a way that its value is unity when applied to any of the runners, according to the definition of k_m assumed in the paper. Thus the functional relationship between t_c/t_o and k_t/k_m may be determined theoretically or experimentally, and applied to any runner.

(b) *Step length*

In the calculation of step length, the runner's spring stiffness and damping were excluded from the problem, but his leg length and weight were assumed to be important (his weight determines the average deflection of the track surface over one stride cycle).

$$f(L, L_o, l, m_m, g, k_t) = 0,$$

where

L = step length, m
L_o = step length on hard surface, m
l = leg length, m
$m_m g$ = runner's weight, N
k_t = track stiffness, N/m.

The dimensionless form of the equation becomes

$$\phi\left(\frac{L}{L_o}, \frac{lk_t}{mg}, \frac{L_o}{l}\right) = 0.$$

The third dimensionless group, L_o/l, is assumed to be a constant for all runners. The validity of this assumption is reasonably good, as shown in Table 1.

Since k_m is assumed to be a constant, we may write the second group in the form:

$$\frac{lk_t}{mg} = \frac{k_t}{k_m}\left[\frac{(\pi/t_o)^2 l}{(1-\zeta^2)g}\right].$$

If the term in square brackets is the same number for all runners, then a functional relationship may be found between L/L_o and k_t/k_m, as was done in Fig. 7. In fact, this term is evaluated for each of the runners in Table 1. It is not particularly constant, but is greater for the faster runners. The variation in this term explains some of the spread of the data points in Fig. 7(b) and shows why comparisons retaining the dimensions (Fig. 7a) may be preferred in this case.