ECE 580 **SPRING 2019**

Correspondence # 20 April 4, 2019

ASSIGNMENT 7

Reading Assignment: Text: Chapter 6

Recommended Reading: Curtain & Pritchard: pp. 65-67, and Chapter 12; Balakrishnan: pp. 62-80, and Chapter 2.

Advance Reading: Text: Chapter 7

Problems (to be handed in): Due Date: **Thursday, April 11.**

To be returned to Mr. Sayin (TA), by 11 am.

51. Let X be a Banach space, and Q a linear bounded operator mapping X into itself. **Show** that if the norm of Q is less than 1, the operator $(I-Q)$ is invertible, and its inverse admits the infinite series representation

$$(I-Q)^{-1} = \sum_{n=0}^{\infty} Q^n$$

[This is known as the *Neumann* series.]

Hint : Use the Banach Contraction Mapping Theorem and the Proposition on page 147 of the text.

52. Let X be a Banach space, and $\|\cdot\|_1$ and $\|\cdot\|_2$ be two different norms on it. Suppose that there exists a constant α such that $\|x\|_1 \le \alpha\,\|x\|_2$ for all $x \in X$. **Show** that there exists another constant, β, such that $\|x\|_2 \le \beta\,\|x\|_1$ for all $x \in X$.

Hint : Use the Banach Inverse Theorem on page 149 of the text.

53. Let $X = L_p[0,2]$, $1 < p < \infty$, $Y = L_q[0,2]$, $\frac{1}{p} + \frac{1}{q} = 1$. Let $A \in B(X,Y)$ be defined by

$$A(x)(t) = \int_0^2 K(t,s)\,x(s)\,ds$$

where

$$\int_0^2 \int_0^2 |K(t,s)|^q\,dt\,ds \;<\; \infty$$

Find A^*, the adjoint of A.

54. Let x and y be two scalar second-order random variables on a common probability space $(\Omega, \mathcal{F}, \mathcal{P})$, admitting a joint probability density function (with respect to the Lebesgue measure) $f_{x,y}(\xi,\eta)$. Let the marginals for y and x be defined by

$$\int_{-\infty}^{\infty} f_{x,y}(\xi,\eta)\,d\xi =: f_2(\eta)\;; \qquad \int_{-\infty}^{\infty} f_{x,y}(\xi,\eta)\,d\eta =: f_1(\xi)\;.$$

Let H_i be the space of all Lebesgue-measurable mappings γ, with finite norm, where the norm is defined as follows:

$$\|\gamma\|_i := \left\{ \int_{-\infty}^{\infty} |\gamma(s)|^2 f_i(s)\, ds \right\}^{1/2}, \quad i = 1, 2.$$

Introduce a transformation $A : H_1 \to H_2$ by

$$A(\gamma)(\eta) = \int_{-\infty}^{\infty} \gamma(\xi)[f_{x,y}(\xi, \eta)/f_2(\eta)]\, d\xi\ .$$

i) **Show** that A is linear and bounded.

ii) **Find** the norm of A.

iii) **Obtain** an expression for the adjoint of A, if the inner product on H_i is defined by

$$(\gamma, \beta)_i := \int_{-\infty}^{\infty} \gamma(\xi)\beta(\xi) f_i(\xi)\, d\xi, \quad i = 1, 2.$$

55. Let H and G be two real Hilbert spaces, and $A : H \to G$ be a linear bounded operator. Let A^* denote the adjoint of A. **Prove or disprove** the following two statements:

i) "The operator norm of A^*A is equal to the square of the operator norm of A."

ii) "The operator norms of A^*A and AA^* are the same."

56. Read section 6.11 of the text on "Pseudoinverse Operators" (also discussed in class), and solve the following problem: **Find** the pseudoinverse of the operator A on $\mathbf{R}^3$ defined by the matrix representation:

$$\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

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