ASSIGNMENT 6

Reading Assignment: Correspondence # 14. Text: Chapters 5.
Advance Reading: Text: Chapter 6.

Recommended Reading: Curtain & Pritchard: pp. 65-67, and Chapter 12;
Balakrishnan: pp. 62-80, and Chapter 2.

Problems (to be handed in): Due Date: **Thursday, April 4.**

The first three problems below are based on the material of Correspondence # 14.

46. **Find** a continuous function x_o that minimizes the functional

$$\int_0^2 |x(t)|^3 \, \mathrm{d}t$$

subject to the constraint

$$\int_0^2 t^2 \, x(t) \, \mathrm{d}t = 2 \, .$$

Identify the space(s) in which you work, and also indicate whether the solution you obtained is **unique** or not.

47. We wish to find a function x, satisfying the pointwise bound $|x(t)| \leq 1$ (that is, $\|x\|_\infty \leq 1$), and integral constraints

$$\int_0^T x(t) \, \mathrm{d}t = -3 \qquad \text{and} \qquad \int_0^T t \, x(t) \, \mathrm{d}t = 0$$

for the smallest possible $T > 0$.
(Hence, you have to find the optimum value of T, say T_o, and a corresponding $x(t)$ on $[0, T_o]$.)
Hint : Show that the problem can be reformulated as one of first minimizing $\|x\|_\infty$ subject to the given integral constraints and with T fixed (say this solution is x_T), and then finding the smallest value of T such that $\|x_T\|_\infty \leq 1$. Material in Section 5.9 of the Text could be useful here.

48. **Find** two functions x and y that minimize the functional

$$\int_0^1 \left[x^2(t) + y^2(t) \right]^{\frac{1}{2}} \mathrm{d}t$$

while satisfying the equality constraints

$$\int_0^1 (1-t) \, x(t) \, \mathrm{d}t = 1 \qquad \text{and} \qquad \int_0^1 (1-t) \, y(t) \, \mathrm{d}t = \frac{3}{2}$$

Note : Part of the problem/question is to identify the space where the solution should be sought. See also Problem 7, page 138 of the Text.

49. Let X be a Hilbert space, and $\{x_n\}$ be a sequence in X, converging weakly to $x^o \in X$.

i) Show that the convergence is also in the strong sense (that is in norm) if further the sequence of real numbers $\{\|x_n\|\}$ converges to the norm of x^o, $\|x^o\|$.

ii) Again for the original problem, **show** that one can find a subsequence $\{x_{n_k}\}$ such that the sequence of arithmetic means

$$y_m = \frac{1}{m} \sum_{k=1}^{m} x_{n_k} , \quad m = 1, 2, \ldots$$

converges strongly to x^0 (that is, $\|y_m - x^o\| \to 0$).

50. Let X be a real normed linear space, and X^* be its dual.

i) Show that a linear functional f on X is weakly continuous if and only if it is of the form $f(x) =< x, x^* >$, for some $x^* \in X^*$.

ii) Show that a linear functional g on X^* is weak* continuous if and only if it is of the form $g(x^*) =< x, x^* >$, for some $x \in X$.

Hint : Use the result of Problem 45 of the previous set.

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