1 Component Models

Suppose there are N components. M of these are type 1, having distribution $F_1(t)$; N-M are of type 2, having failure distribution $F_2(t)$. Suppose that the system fails at time S when the k$^{\text{th}}$ component (of either kind) fails. Write an equation for the CDF of S, $F_S(t)$.

Solution:
For any particular assignment of types to the N components, the probability that $a \leq M$ of the type 1 components fails and that $b \leq N - M$ type 2 components fails (and the others do not) is

$$q(a, b, t) = F_1(t)^a \cdot (1 - F_1(t))^{M-a} \cdot F_2(t)^b \cdot (1 - F_2(t))^{N-M-b}.$$

Within the collection of type 1 components there are $\binom{M}{a}$ ways of choosing which type 1 components fail, and $\binom{N-M}{b}$ ways of choosing which type 2 components fail. Each of these assignments of failures and successes to components have the same probability, so that for a given assignment of types to components we have

$$q_{joint}(a, b, t) = \Pr\{a \text{ type 1 failures by } t, b \text{ type 2 failures by } t\} = \binom{M}{a} \cdot \binom{N-M}{b} \cdot q(a, b, t).$$

Therefore, for a given k we can write

$$F_S(t) = \Pr\{\text{exactly k components fail}\} = \sum_{m=0}^{\min\{\text{k,M}\}} q_{joint}(m, k-m, t)$$

2 Hazard Rate

Imagine a random variable X which is constructed by selecting from one of two distributions, with probability p of selecting type 1 with pdf $f_1(t)$ and cdf $F_1(t)$, and with probability (1-p) of selecting type 2 with pdf $f_2(t)$ and cdf $F_2(t)$. Give the hazard rate function for X.

Solution:
We first construct the CDF for X

$$F_X(t) = p \cdot Pr(X \leq t|\text{type 1}) + (1-p) \cdot Pr(X \leq t|\text{type 2})$$

which can be re-written as

$$F_X(t) = p \cdot F_1(t) + (1-p) \cdot F_2(t).$$

To get the pdf we differentiate with respect to t and get

$$f_X(t) = p \cdot f_1(t) + (1-p) \cdot f_2(t).$$

The same technique for identifying $\bar{F_X}(t)$ gives us

$$\bar{F_X}(t) = p \cdot \bar{F_1}(t) + (1-p) \cdot \bar{F_2}(t).$$

Which leads straightforwardly to the hazard rate function for X

$$r_x(t) = \frac{p \cdot f_1(t) + (1-p) \cdot f_2(t)}{p \cdot \bar{F_1}(t) + (1-p) \cdot \bar{F_2}(t)}$$

3 Message Passing

A message needs to be delivered originating from point A, and arrive at point B. There are four ways this could be accomplished, and any one of them will suffice. Method 1 is by electric train. For the train to be operational there need to be power, the train needs to be in good repair, and there needs to be an operator available. Method 2 is by car. For the car to work, there needs to be an operational car and an available driver. Method 3 is by marathon runner, for which there must be an available marathon runner. Method 4 is by telegraph. For the telegraph to work, there must be an operational telegraph wire, power and an available telegraph operator. A marathon runner can drive a car, operate a train, and run the telegraph. A non-marathon operator can also drive a car, operate a train, and run the telegraph.

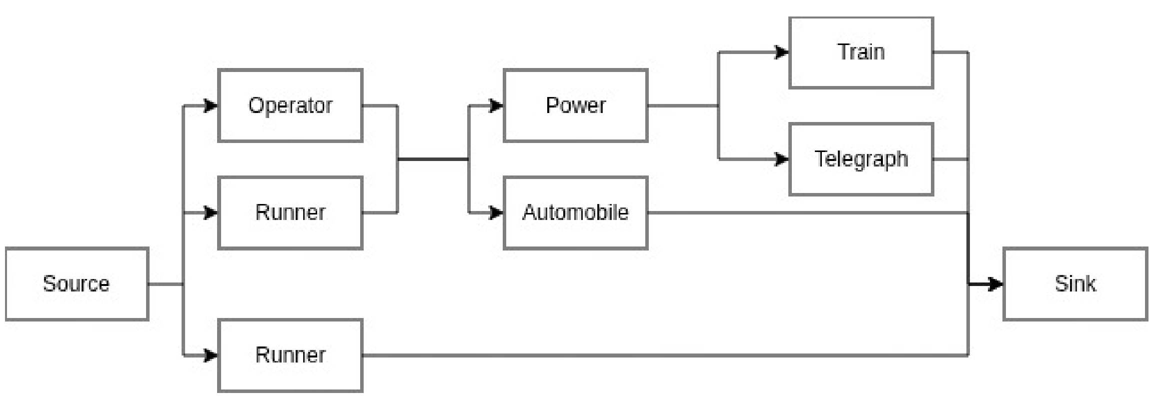
Imagine a system with components

- Electric train
- Power source
- Automobile
- Telegraph Wire
- Marathon runner
- Non-marathon running operator

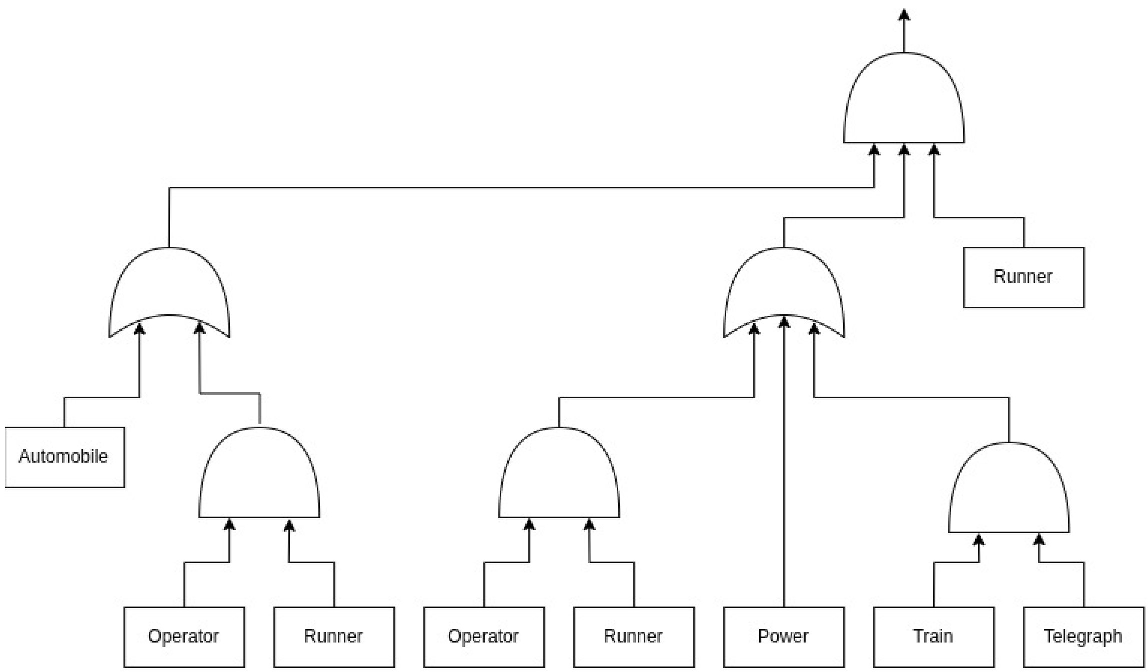
Develop (a) a Reliability Block Diagram describing the ability to deliver the message, and (b) a Fault Tree Diagram describing the failure conditions. Minimize the number of times a given component is represented in the RBD and Fault Tree.

Solution:

a. Reliability Block Diagram



b. Fault Tree


Runner
Automobile
Operator
Runner
Operator
Runner
Power
Train
Telegraph