Lecture 1: Review of Signal Processing

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ECE 537: Speech Processing

Outline

Welcome to ECE 537, Speech Processing!

- This course is about speech production and perception, synthesis and recognition, coding and voice conversion
- At this point, let's talk about the web page: `https://courses.grainger.illinois.edu/ece537/`
- If you're not yet added to the CampusWire or GradeScope pages, please add yourself.

Outline

Dimensions on which Fourier representations differ

- Is the signal's spectrum. . .
  - Continuous in frequency $\Leftrightarrow$ Infinite in time
  - Discrete in frequency $\Leftrightarrow$ Periodic in time
- Is the signal. . .
  - Continuous in time $\Leftrightarrow$ Infinite in frequency
  - Discrete in time $\Leftrightarrow$ Periodic in frequency

Continuous-Time Fourier Transform (CTFT)

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\Omega) e^{j\Omega t} d\Omega$$

$$\text{Parseval: } \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\Omega)|^2 d\Omega$$

Continuous-Time Fourier Series (CTFS)

$x(t)$ is periodic in time ($x(t) = x(t + T_0)$), so it's discrete in frequency.

$$X_k = \frac{1}{T_0}\int_0^{T_0} x(t)e^{-jk\Omega_0 t}dt, \quad \Omega_0 = \frac{2\pi}{T_0}$$

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t}$$

$$\text{Parseval:} \frac{1}{T_0}\int_0^{T_0} |x(t)|^2 dt = \sum_{-\infty}^{\infty} |X_k|^2$$

Discrete-Time Fourier Transform (DTFT)

$x[n]$ is discrete in time, so it's periodic in frequency $(X(\omega) = X(\omega + 2\pi))$.

$$X(\omega) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_0^{2\pi} X(\omega)e^{j\omega n}d\omega$$

$$\text{Parseval: } \sum_{-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_0^{2\pi} |X(\omega)|^2 d\omega$$

Discrete-Time Fourier Series (DTFS)

Periodic in time ($x[n]=x[n+N]$) $\Leftrightarrow$ Discrete in frequency.
Discrete in time $\Leftrightarrow$ Periodic in frequency ($X_k = X_{k+N}$).

$$X_k = \frac{1}{N}\sum_0^{N-1} x[n]e^{-jk\omega_0 n}, \quad \omega_0 = \frac{2\pi}{N}$$

$$x[n] = \sum_0^{N-1} X_k e^{jk\omega_0 n}$$

$$\text{Parseval:} \frac{1}{N}\sum_0^{N-1} |x[n]|^2 = \sum_0^{N-1} |X_k|^2$$

Discrete Fourier Transform (DFT)

$$X[k] = \sum_{0}^{N-1} x[n] e^{-jk\omega_0 n}, \quad \omega_0 = \frac{2\pi}{N}$$

$$x[n] = \frac{1}{N} \sum_{0}^{N-1} X[k] e^{jk\omega_0 n}$$

$$\text{Parseval: } \sum_{0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{0}^{N-1} |X[k]|^2$$

Outline

Types of impulses

Impulses can be:

- Discrete-time
- Discrete-frequency
- Continuous-time
- Continuous-frequency

Discrete-time impulses

$$f[n] = \delta[n - n_1] = \begin{cases} 1 & n = n_1 \\ 0 & \text{otherwise} \end{cases}$$

$$F(\omega) = e^{-j\omega n_1}$$

$$F[k] = e^{-j\frac{2\pi k n_1}{N}}$$

$$F_k = \frac{1}{N} e^{-j\frac{2\pi k n_1}{N}}$$

Discrete-frequency impulses

$$F_k = \delta[k - k_1] = \begin{cases} 1 & k = k_1 \\ 0 & \text{otherwise} \end{cases}$$

$$F[k] = N\delta[k - k_1]$$

$$f(t) = e^{jk\Omega_0 t}$$

$$f[n] = e^{jk\omega_0 n}$$

Continuous-time impulses

A continuous-time impulse $f(t) = \delta(t - t_1)$ is defined by:

$$f(t) = \begin{cases} \infty & t = t_1 \\ 0 & |t - t_1| > 0 \end{cases}$$

$$\int_{t_a}^{t_b} f(t)dt = \begin{cases} 1 & t_a < t_1 < t_b \\ 0 & \text{otherwise} \end{cases}$$

The sampling theorem then gives us, for any signal $x(t)$, that:

$$\int_{t_a}^{t_b} f(t)x(t)dt = x(t_1)$$

Its transform and (if periodic) Fourier series are $F(\Omega) = e^{-j\Omega t_1}$ and $F_k = e^{-jk\Omega_0 t}$.

Continuous-frequency impulses

$$F(\Omega) = 2\pi\delta(\Omega - \Omega_1) \Leftrightarrow f(t) = e^{j\Omega_1 t}$$

$$F(\omega) = 2\pi\delta(\omega - \omega_1) \Leftrightarrow f[n] = e^{j\omega_1 n}$$

Outline

Convolution

- Convolution in time $\Leftrightarrow$ Multiplication in frequency
- Multiplication in time $\Leftrightarrow$ Convolution in frequency
- Multiplication in discrete time $\Leftrightarrow$ Circular convolution in frequency
- Circular convolution in time $\Leftrightarrow$ Multiplication in discrete frequency

Convolution in time $\Leftrightarrow$ Multiplication in frequency

$$y(t) = h(t) * x(t) \Leftrightarrow Y(\Omega) = X(\Omega)H(\Omega)$$

...where...

$$x(t) * h(t) \equiv \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \equiv \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

Multiplication in time ⇔ Convolution in frequency

$$y(t) = w(t)x(t) \Leftrightarrow Y(\Omega) = \frac{1}{2\pi} X(\Omega) * W(\Omega)$$

...where...

$$X(\Omega) * W(\Omega) \equiv \int_{-\infty}^{\infty} X(\theta) W(\Omega - \theta) d\theta \equiv \int_{-\infty}^{\infty} W(\theta) X(\Omega - \theta) d\theta$$

Multiplication in discrete time $\Leftrightarrow$ Circular convolution in frequency

$$y[n] = w[n]x[n] \Leftrightarrow Y(\omega) = \frac{1}{2\pi}X(\omega) * W(\omega) = \frac{1}{2\pi}X(\omega) \circledast W(\omega)$$

...where...

$$X(\omega) * W(\omega) \equiv \int_0^{2\pi} X(\theta)W(\omega-\theta)d\theta$$

$$X(\omega) \circledast W(\omega) \equiv \int_0^{2\pi} X(\theta)W\left(\langle\omega-\theta\rangle_{2\pi}\right)d\theta$$

...where $\langle\omega-\theta\rangle_{2\pi}$ means "$\omega-\theta$ modulo 2π." That modulo operator is obviously not necessary here, because $W(\omega)$ is periodic with a period of 2π, but for time-domain convolution we might need to be more careful...

Circular convolution in time ⇔ Multiplication in discrete frequency

$$y[n] = h[n] \circledast x[n] \Leftrightarrow Y[k] = H[k]X[k]$$

. . . where. . .

$$h[n] \circledast x[n] \equiv \sum_{0}^{N-1} x[m]h\left[\langle n-m \rangle_N\right]$$

Outline

Summary: Basic signal processing facts you need to know

- Discrete in time means periodic in frequency, and vice versa
- Continuous in time means infinite in frequency, and vice versa
- The Fourier transform of an impulse is a complex exponential, and vice versa
- The Fourier transform of multiplication is convolution, and vice versa