

ECE 537 Homework 7

Due: October 23, Urbana time

1. Estimate the front cavity resonant frequencies of /t/ and /k/. In beatboxing (<https://en.wikipedia.org/wiki/Beatboxing>), a /k/ is often used to represent a snare, while a /t/ is often used to represent a high-hat. A snare is a circle of mylar about 30cm across and held unmoving at the rims, while a high-hat is a circle of brass approximately 30cm across and free to move at both ends. What can you conclude about the speed of sound in brass versus mylar?

Solution: A /t/ has a front cavity of about 2.0cm, for a front cavity resonance of about 4400Hz. A /k/ can have a front cavity anywhere between 4.0-6.0cm, for a front cavity resonance in the range 1500-2200Hz. A snare and high-hat are 2d resonators that are clamped on all sides or open on all sides, respectively, thus they are both half-wave resonators; if we model them as if they were 1d resonators of length 30cm, then their first resonant frequencies should be $c/2l = c/0.6$ in both cases. If $c/0.6 = 1500\text{Hz}$ for a snare, then Mylar must have a speed of sound around $c \approx 900\text{m/s}$. If $c/0.6 = 4400\text{Hz}$ for a high-hat, then brass must have a speed of sound around $c \approx 2600\text{m/s}$.

2. In (Fujimura, 1962), the combined pharynx+nasal cavity can be approximately modeled as a tube that has fixed input at the glottis (so that $U(x, \Omega) = U_G(\Omega)$ at $x = 0$), is open at the lips (so that $P(x, \Omega) = 0$ at $x = \ell$), and is uniform in cross-sectional area. The velum is an arbitrary point, x , somewhere between the larynx and the lips. Use a uniform tube model to calculate $B(\Omega) = \Im \{U(x, \Omega)/P(x, \Omega)\}$ at an arbitrary point, x , in a uniform tube acoustic model. Does your result resemble $B_i(\Omega)$ in Fig. 2 of (Fujimura, 1962)?

Solution: The Fourier transform solutions of the 1d acoustic wave equation are:

$$U(x, \Omega) = U_+(\Omega)e^{-jx\Omega/c} - U_-(\Omega)e^{jx\Omega/c}$$

$$P(x, \Omega) = \frac{\rho c}{A} \left(U_+(\Omega)e^{-jx\Omega/c} + U_-(\Omega)e^{jx\Omega/c} \right)$$

The boundary conditions give us:

$$U_+(\Omega) = U_G(\Omega) + U_-(\Omega)$$

$$U_-(\Omega) = -U_+(\Omega)e^{-2j\ell\Omega/c}$$

which combine to

$$U_+(\Omega) = \frac{U_G(\Omega)}{1 + e^{-2j\ell\Omega/c}}$$

$$U_-(\Omega) = -\frac{e^{-2j\ell\Omega/c}U_G(\Omega)}{1 + e^{-2j\ell\Omega/c}}$$

thus

$$U(x, \Omega) = U_G(\Omega) \frac{e^{-jx\Omega/c} + e^{j(x-2\ell)\Omega/c}}{1 + e^{-2j\ell\Omega/c}}$$

$$P(x, \Omega) = \frac{\rho c U_G(\Omega)}{A} \frac{e^{-jx\Omega/c} - e^{j(x-2\ell)\Omega/c}}{1 + e^{-2j\ell\Omega/c}}$$

Dividing the two gives

$$Y(\Omega) = \frac{A}{\rho c} \frac{e^{2j(\ell-x)\Omega/c} + 1}{e^{2j(\ell-x)\Omega/c} - 1}$$

$$= \frac{A}{\rho c} \frac{2 \cos(2(\ell-x)\Omega/c)}{2j \sin(2(\ell-x)\Omega/c)} B(\Omega) = -\frac{A}{\rho c} \cot(2(\ell-x)\Omega/c)$$

Negative cotangent has the shape shown in the figure.

3. Suppose that $\mathbf{z}$ is a multivariate normal random variable (https://en.wikipedia.org/wiki/Multivariate_normal_distribution) with zero mean and unit variance, and $\mathbf{x} = \mathbf{W}\mathbf{z} + \mathbf{t}$ for some non-symmetric matrix $\mathbf{W}$. Prove that the probability distributions $p_X(\mathbf{x})$ and $p_Z(\mathbf{z})$ are related by the flow equation.

Solution: If $\mathbf{x}$ is a linear transform of a Gaussian vector, then it is also Gaussian. Since $\mathbf{z} \sim \mathcal{N}(0, I)$, therefore $\mathbf{x} \sim \mathcal{N}(\mathbf{t}, \mathbf{W}\mathbf{W}^T)$. If they are both N -dimensional, their pdfs are

$$p_Z(\mathbf{z}) = (2\pi)^{-N/2} e^{-\frac{1}{2}\mathbf{z}^T \mathbf{z}}$$

$$p_X(\mathbf{x}) = \frac{(2\pi)^{-N/2}}{\sqrt{|\det(\mathbf{W}\mathbf{W}^T)|}} e^{-\frac{1}{2}(\mathbf{x}-\mathbf{t})^T (\mathbf{W}\mathbf{W}^T)^{-1}(\mathbf{x}-\mathbf{t})} = \frac{(2\pi)^{-N/2}}{|\det(\mathbf{W})|} e^{-\frac{1}{2}(\mathbf{x}-\mathbf{t})^T \mathbf{W}^{-T} \mathbf{W}^{-1}(\mathbf{x}-\mathbf{t})}$$

Writing $\mathbf{z} = \mathbf{f}^{-1}(\mathbf{x}) = \mathbf{W}^{-1}(\mathbf{x} - \mathbf{t})$, the Jacobian and its determinant are

$$\mathbf{J}(\mathbf{f}^{-1}(\mathbf{x})) = \mathbf{W}^{-1}$$

$$|\det(\mathbf{J})| = 1/|\det(\mathbf{W})|$$

Combining, we get that

$$p_X(\mathbf{x}) = (2\pi)^{-N/2} |\det(\mathbf{J})| e^{-\frac{1}{2}\mathbf{f}^{-1}(\mathbf{x})^T \mathbf{f}^{-1}(\mathbf{x})}$$