

ECE 537 Homework 6

Due: October 16, Urbana time

1. Consider a two-tube model of the vocal tract with $\ell_1 = \ell_2 = 8.85\text{cm}$, and assume that $c = 354\text{m/s}$ and $F_s = 4000$ samples/second, so that $\ell_1 F_s / c = \ell_2 F_s / c = 1$ sample. Suppose that the glottis has infinite impedance ($Z_g = \infty$) and the radiation impedance is zero ($Z_R = 0$). As a function of the reflection coefficient $r_1 = \frac{A_2 - A_1}{A_2 + A_1}$, find the system function $H(z) = U_L(z)/U_G(z)$, where $u_L(t) = u_2(\ell_2, t)$ is the volume velocity of air leaving the lips, and $z = e^{j\omega}$ with ω equal to the frequency measured in radians per sample. Note that there are no losses in this vocal tract model, so $H(z)$ should have four poles on the unit circle. Find the frequencies of both formants as functions of r_1 . Demonstrate that your result is sensible as $r_1 \rightarrow 1$ (a good model of /a/) and as $r_1 \rightarrow -1$ (a good model of /i/).
2. Suppose that $s[n]$ is well modeled by $s[n] = v[n] + a_1 s[n-1] + a_2 s[n-2]$ for an impulse train or white noise signal $v[n]$. Estimate the frequency and bandwidth of the first formant, in Hertz, as functions of a_1 , a_2 , and F_s . Assume that $a_1^2 + 4a_2 < 0$ and $|a_2| < 1$.
3. Ignoring their perceptual weighting filter, Rose and Barnwell's self-excited LPC vocoder chose β_3 and T_0 to minimize $\mathcal{E}_{\text{Adaptive}}$, then chose β_2 and k to minimize $\mathcal{E}_{\text{Stochastic}}$, where $\hat{s}[n]$ is synthetic speech from previous frames, $c_k[n]$ is the k^{th} stochastic codevector, and

$$\mathcal{E}_{\text{Adaptive}} = \sum_{n=0}^{N-1} (s[n] - \beta_3 \hat{s}[n - T_0])^2$$

$$\mathcal{E}_{\text{Stochastic}} = \sum_{n=0}^{N-1} (s[n] - \beta_2 h[n] * c_k[n] - \beta_3 \hat{s}[n - T_0])^2$$

Find functions $R_1[m]$, $P_1[m]$, $R_2[m]$, and $P_2[m]$ such that the minimizers of $\mathcal{E}_{\text{Adaptive}}$ and $\mathcal{E}_{\text{Stochastic}}$ can be written as:

$$T_0 = \underset{m}{\operatorname{argmax}} \frac{R_1[m]}{P_1[m]}, \quad \beta_3 = \frac{R_1[T_0]}{P_1[T_0]}$$

$$k = \underset{m}{\operatorname{argmax}} \frac{R_2[m]}{P_2[m]}, \quad \beta_2 = \frac{R_2[k]}{P_2[k]}$$