

ECE 537 Homework 3

Due: September 18, Urbana time

1. The function of Tandem is to eliminate temporal variation caused by cross-products between the harmonics, so that, assuming $w[n]$ to be symmetric in time, the tandem spectrum is well approximated as

$$P_T(\omega) \approx \sum_k |H_k|^2 W^2(\omega - k\omega_0)$$

where, in the notation of our lecture note slides, $H_k = H\left(\frac{2\pi k}{T_0}\right)$ is the vocal tract transfer function evaluated at the k^{th} harmonic frequency. Tandem-STRAIGHT then computes a cumulative spectrum, $C(\omega)$, and a smoothed spectrum, $P_S(\omega)$, as

$$C(\omega) = \int_0^\omega P_T(\lambda) d\lambda$$

$$P_S(\omega) = C\left(\omega + \frac{\omega_0}{2}\right) - C\left(\omega - \frac{\omega_0}{2}\right)$$

From these equations, $P_S(\omega)$ can be written as

$$P_S(\omega) \approx \sum_k |H_k|^2 G^2(\omega - k\omega_0)$$

In terms of $W(\omega)$, what is $G(\omega)$? Why might $P_S(\omega)$ be a better estimate of the spectral envelope than $P_T(\omega)$ (consider especially the case when the window length is $2.5T_0$ or longer).

Solution:

$$G(\omega) = \int_{\omega - \omega_0/2}^{\omega + \omega_0/2} W^2(\theta) d\theta,$$

or equivalently, in terms of the rectangle $R(\omega)$ with height 1 and width ω_0 ,

$$G(\omega) = W^2(\omega) * R(\omega)$$

If the window length is $2.5T_0$ or larger, then its mainlobe will be narrower than ω_0 in frequency, so $P_T(\omega)$ will show considerable peaks at the harmonics and dips in between. Convolving with $R(\omega)$ spreads out the window spectrum in frequency, forcing $P_S(\omega)$ to be an interpolation between neighboring harmonics.

2. (a) The Wikipedia page on viscosity gives this equation for the force-to-velocity relationship:

$$F = \mu A \frac{u}{y}$$

Note that these are not the same F, A, u that were used in the Flanagan and Landgraf article or lecture slides. Here, A is the area of the plate over the surface of which air is moving (i.e., the medial surface of the vocal fold), u is the air particle velocity at the center of the channel (e.g., in meters/second), and y is the half-width of the channel. By re-writing these variables into the Flanagan-Landgraf variables, can you get a viscous resistance equation similar to that declared by Flanagan and Landgraf? You may get a numerical constant that is not quite 12, but you should be able to explain all of the other terms.

Solution: The force is $F = (P_1 - P_2)A_g$. The plate surface area is $A = dl$, the depth times length of the vocal folds. If flow varies linearly as a function of y , then the velocity in the center of the channel is twice its average velocity, thus $u = 2U_g/A_g$. The channel halfwidth is $y = A_g/2l$. Making these substitutions, we get

$$P_1 - P_2 = 8\mu dl^2 \frac{U_g}{A_g^3}$$

- (b) Air pressure is best interpreted as force applied by a fluid on its boundary per unit area of the boundary. If A is cross-sectional area and z is the stream direction, therefore, $-\frac{\partial P}{\partial z}$ can be interpreted as the amount of force per unit volume applied on an infinitesimal block of air at position z . The mass per unit volume is ρ . If velocity is $v = U/A$, then acceleration is

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial t}$$

...but notice that $\frac{\partial z}{\partial t} = v$. Subglottal pressure must accelerate every air molecule from a velocity of $v \approx 0$ in the lungs ($z = 0, P = P_S$) to a velocity of $v = U_g/A_g$ at the narrowest part of the glottis ($z = z_g, P = P_2$). Express Newton's second law of motion in terms of v and P , and integrate it from $z = 0$ to z_g in order to derive the form of the kinetic resistance. You may get a numerical constant that is not quite 0.44, but you should be able to explain all of the other terms.

Solution:

$$f \text{ per unit volume} = (m \text{ per unit volume})a$$

$$-\frac{\partial P}{\partial z} = \rho v \frac{\partial v}{\partial z}$$

Integrating with respect to z gives

$$P_S - P_2 = \frac{1}{2} \left(\frac{U_g}{A_g} \right)^2$$