ECE 537 Homework 3

Due: September 18, Urbana time

1. The function of Tandem is to eliminate temporal variation caused by cross-products between the harmonics, so that, assuming $w[n]$ to be symmetric in time, the tandem spectrum is well approximated as

$$P_T(\omega) \approx \sum_k |H_k|^2 W^2(\omega - k\omega_0)$$

where, in the notation of our lecture note slides, $H_k = H\left(\frac{2\pi k}{T_0}\right)$ is the vocal tract transfer function evaluated at the k^{th} harmonic frequency. Tandem-STRAIGHT then computes a cumulative spectrum, $C(\omega)$, and a smoothed spectrum, $P_S(\omega)$, as

$$C(\omega) = \int_0^\omega P_T(\lambda) d\lambda$$
$$P_s(\omega) = C\left(\omega + \frac{\omega_0}{2}\right) - C\left(\omega - \frac{\omega_0}{2}\right)$$

From these equations, $P_S(\omega)$ can be written as

$$P_S(\omega) \approx \sum_k |H_k|^2 G^2(\omega - k\omega_0)$$

In terms of $W(\omega)$, what is $G(\omega)$? Why might $P_S(\omega)$ be a better estimate of the spectral envelope than $P_T(\omega)$ (consider especially the case when the window length is $2.5T_0$ or longer).

2. (a) The Wikipedia page on viscosity gives this equation for the force-to-velocity relationship:

$$F = \mu A \frac{u}{y}$$

Note that these are not the same F, A, u that were used in the Flanagan and Landgraf article or lecture slides. Here, A is the area of the plate over the surface of which air is moving (i.e., the medial surface of the vocal fold), u is the air particle velocity at the center of the channel (e.g., in meters/second), and y is the half-width of the channel. By re-writing these variables into the Flanagan-Landgraf variables, can you get a viscous resistance equation similar to that declared by Flanagan and Landgraf? You may get a numerical constant that is not quite 12, but you should be able to explain all of the other terms.

(b) Air pressure is best interpreted as force applied by a fluid on its boundary per unit area of the boundary. If A is cross-sectional area and z is the stream direction, therefore, $-\frac{\partial P}{\partial z}$ can be interpreted as the amount of force per unit volume applied on an infinitesimal block of air at position z. The mass per unit volume is ρ. If velocity is $v = U/A$, then acceleration is

$$\frac{\partial v}{\partial t} = \frac{\partial v}{\partial z} \cdot \frac{\partial z}{\partial t}$$

…but notice that $\frac{\partial z}{\partial t} = v$. Subglottal pressure must accelerate every air molecule from a velocity of $v \approx 0$ in the lungs ($z = 0$, $P = P_S$) to a velocity of $v = U_g/A_g$ at the narrowest part of the glottis ($z = z_g$, $P = P_2$). Express Newton's second law of motion in terms of v and P, and integrate it from $z = 0$ to z_g in order to derive the form of the kinetic resistance. You may get a numerical constant that is not quite 0.44, but you should be able to explain all of the other terms.