ECE 537 Homework 1

Due: September 4, Urbana time

Submit your answers on Gradescope by no later than the end of the day on September 4, Urbana time. You may write your answers on this page, or use your own paper. You may submit either typeset or handwritten answers, as long as your answers are legible. Full credit is given if you show your work and your answer is mostly correct. Full credit is not given for an answer provided without any derivation, though the distributed solutions will be in that form.

1. Use Euler's formula to find the CTFT and CTFS of $x(t) = A\cos(\Omega_1 t + \theta)$ and $y(t) = B\sin(\Omega_1 t + \phi)$, and the DTFT, DTFS, and DFT of $x[n] = A\cos(\omega_1 n + \theta)$ and $y[n] = B\sin(\omega_1 n + \phi)$. When computing the CTFS, DTFS, and DFT, assume that Ω_1 and ω_1 are integer multiples of $\Omega_0 = \frac{2\pi}{T_0}$ and $\omega_0 = \frac{2\pi}{N}$, respectively.

Solution:

$$
\begin{aligned}
X(\Omega) &= \pi A\left(e^{j\theta}\delta(\Omega-\Omega_1) + e^{-j\theta}\delta(\Omega+\Omega_1)\right)\\
Y(\Omega) &= -j\pi B\left(e^{j\theta}\delta(\Omega-\Omega_1) - e^{-j\theta}\delta(\Omega+\Omega_1)\right)\\
X_k &= \frac{A}{2}\left(e^{j\theta}\delta\left[k-\frac{\Omega_1}{\Omega_0}\right] + e^{-j\theta}\delta\left[k+\frac{\Omega_1}{\Omega_0}\right]\right)\\
Y_k &= -\frac{jB}{2}\left(e^{j\theta}\delta\left[k-\frac{\Omega_1}{\Omega_0}\right] - e^{-j\theta}\delta\left[k+\frac{\Omega_1}{\Omega_0}\right]\right)\\
X(\omega) &= \pi A\left(e^{j\theta}\delta(\omega-\omega_1) + e^{-j\theta}\delta(\omega+\omega_1)\right)\\
Y(\omega) &= -j\pi B\left(e^{j\theta}\delta(\omega-\omega_1) - e^{-j\theta}\delta(\omega+\omega_1)\right)\\
X_k &= \frac{A}{2}\left(e^{j\theta}\delta\left[k-\frac{\omega_1}{\omega_0}\right] + e^{-j\theta}\delta\left[k+\frac{\omega_1}{\omega_0}\right]\right)\\
Y_k &= -\frac{jB}{2}\left(e^{j\theta}\delta\left[k-\frac{\omega_1}{\omega_0}\right] - e^{-j\theta}\delta\left[k+\frac{\omega_1}{\omega_0}\right]\right)\\
X[k] &= \frac{AN}{2}\left(e^{j\theta}\delta\left[k-\frac{\omega_1}{\omega_0}\right] + e^{-j\theta}\delta\left[k+\frac{\omega_1}{\omega_0}\right]\right)\\
Y[k] &= -\frac{jBN}{2}\left(e^{j\theta}\delta\left[k-\frac{\omega_1}{\omega_0}\right] - e^{-j\theta}\delta\left[k+\frac{\omega_1}{\omega_0}\right]\right)
\end{aligned}
$$

2. Suppose $X(\omega) = 1$ for $|\omega| \le \frac{\pi}{2}$, and $X(\omega) = 0$ elsewhere. What is the DTFT of $y[n] = x[n]\cos\left(\frac{\pi n}{3}\right)$? You may draw the answer, or specify it in an equation, whichever you prefer.

Solution:

$$
Y(\omega) = \begin{cases} 1 & |\omega| \le \frac{\pi}{6} \\ \frac{1}{2} & \frac{\pi}{6} < |\omega| \le \frac{5\pi}{6} \\ 0 & \frac{5\pi}{6} < |\omega| \le \pi \end{cases}
$$