Exam 3 review

Wednesday, November 16, 2022 12:06

$$h_n = \tanh(W_1 s_n) \qquad s_n = [s_{n,1}, \ldots, s_{n,4}]^T$$

$$\hat{t}_n = W_2 h_n$$

$$\mathcal{L} = \frac{1}{2} \sum_n \|\hat{t}_n - t_n\|_2^2$$

$$y = \tanh(x) \Rightarrow \frac{\partial y}{\partial x} = 1 - y^2$$

$$\tanh'(x) = 1 - \tanh^2(x)$$

$$\frac{\partial}{\partial s_{n,k}}\left(\frac{\partial \mathcal{L}}{\partial W_{2,i,j}}\right) \quad \text{FIND} \quad \frac{\partial}{\partial s_{n,k}}\left(\frac{\partial \mathcal{L}}{\partial W_{2,i,j}}\right)$$

SOLUTION

$$\frac{\partial \mathcal{L}}{\partial W_{2,i,j}} = \sum_m \sum_n \frac{\partial \mathcal{L}}{\partial \hat{t}_{n,m}} \frac{\partial \hat{t}_{n,m}}{\partial W_{2,i,j}}$$

0 unless $m = i$

$$= \sum_n \left(\hat{t}_{n,i} - t_{n,i}\right) h_{n,j}$$

$$\hat{t}_n = W_2 h_n \Rightarrow \hat{t}_{n,m} = \sum_{k=1} W_{2,m,k} h_{n,k}$$

$$\Rightarrow \frac{\partial \hat{t}_{n,m}}{\partial W_{2,i,j}} = \begin{cases} 0 & m \neq i \\ h_{n,j} & m = i \end{cases}$$

$$\frac{\partial \mathcal{L}}{\partial W_{2,i,j}} = \sum_n (\hat{t}_{n,i} - t_{n,i}) h_{n,j}$$

Depends on $s_{n,k}$ by way of $\hat{t}_{n,i}$ and $h_{n,j}$

$$\frac{\partial}{\partial s_{n,k}} \frac{\partial \mathcal{L}}{\partial W_{2,i,j}} = \sum_n \left(\frac{\partial \hat{t}_{n,i}}{\partial s_{n,k}} h_{n,j} + (\hat{t}_{n,i} - t_{n,i}) \frac{\partial h_{n,j}}{\partial s_{n,k}} \right)$$

$$\hat{t}_n = W_2 h_n \Rightarrow \hat{t}_{n,i} = \sum_m W_{2,i,m} h_{n,m}$$

$$\frac{\partial \hat{t}_{n,i}}{\partial s_{n,k}} = \sum_m W_{2,i,m} \frac{\partial h_{n,m}}{\partial s_{n,k}}$$

$$\frac{\partial h_{n,m}}{\partial s_{n,k}} =$$

$$h_n = \tanh(W_1 s_n)$$

$$h_{n,m} = \tanh\left(\sum_l W_{1,m,l} s_{n,l} \right)$$

$$\frac{\partial h_{n,m}}{\partial s_{n,l}} = \tanh'\left(\sum_l W_{1,m,l} s_{n,l} \right) W_{1,m,l}$$

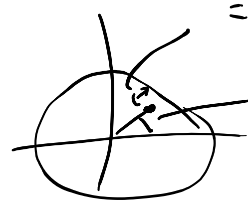
$$\frac{\partial h_{n,m}}{\partial s_{n,l}} = (1 - h_{n,m}^2) W_{1,m,l}$$

$$\frac{\partial}{\partial s_{n,k}}\left(\frac{\partial \mathcal{L}}{\partial W_{2,i,j}}\right) = \sum_n \Big[h_{n,j} \Big(\sum_m W_{2,i,m} \left(1-h_{n,k}^2\right) W_{1,m,k}\Big) + \left(\hat{t}_{n,i} - t_{n,i}\right)\left(1-h_{n,j}^2\right) W_{1,j,k} \Big]$$

(b) $$R_{n,k}(z) = \frac{a_{n,k}}{1 - b_{n,k} z^{-1} - c_{n,k} z^{-2}}$$

↑

FIND $\dfrac{\partial b_{n,k}}{\partial s_{n,j}}$



WHAT IS $b_{n,k}$?

$$(1 - z_k z^{-1})(1 - z_k^* z^{-1}) = 1 - (z_k + z_k^*) z^{-1} + |z_k|^2 z^{-2}$$

↑

$$z_k = e^{-\pi B_k T + j(2\pi F_k t)}$$

$$b_k = z_k + z_k^* = 2\,\mathrm{Re}(z_k) = 2e^{-\pi B_k T} \cos(2\pi F_k T)$$

$$c_k = -|z_k|^2 = -e^{-2\pi B_k T}$$

$$b_{n,k} = 2e^{-\pi B_k T} \cos(2\pi T \hat{t}_{n,k})$$

$$\frac{\partial b_{n,k}}{\partial s_{n,j}} = \frac{\partial b_{n,k}}{\partial \hat{t}_{n,k}} \frac{\partial \hat{t}_{n,k}}{\partial s_{n,j}}$$

$$\frac{\partial b_{n,k}}{\partial \hat{t}_{n,k}} = 2 e^{-\pi B_k T}\left(-2\pi T \sin(2\pi T \hat{t}_{n,k})\right)$$

$$= -4\pi T e^{-\pi B_k T} \sin(2\pi T \hat{t}_{n,k})$$

$$\hat{t}_n = W_2 h_n \Rightarrow \hat{t}_{n,k} = \sum_l W_{2,k,l} h_{n,l}$$

$$h_n = \tanh(W_1 s_n) \Rightarrow h_{n,l} = \tanh\left(\sum_j W_{1,l,j} s_{n,j}\right)$$

$$\frac{\partial \hat{t}_{n,k}}{\partial s_{n,j}} = \sum_l \frac{\partial \hat{t}_{n,k}}{\partial h_{n,l}} \frac{\partial h_{n,l}}{\partial s_{n,j}}$$

$$= \sum_l W_{2,k,l}(1 - h_{n,l}^2) W_{1,l,j}$$

$$\boxed{\frac{\partial b_{n,k}}{\partial s_{n,j}} = -4\pi T e^{-\pi B_k T} \sin(2\pi T \hat{t}_{n,k}) \sum_l W_{2,k,l}\left(1 - h_{n,l}^2\right) W_{1,l,j}}$$

5 $$\mathcal{L} = -\ln \sum_{\pi \in B^{-1}(z)} \prod_{n=1}^{N} y_{n,\pi_n}$$

$$\pi = [\pi_1, \ldots, \pi_N] \qquad z = [z_1, \ldots, z_L]$$

$$l' = [-, z_1, -, z_2, \ldots, -, z_L, -]$$

$$= [l'_1, l'_2, l'_3, \ldots, l'_{2L}, l'_{2L+1}]$$

$l'_i \neq l'_j$ FOR ANY $j \neq i$ — PROBABLY A CORRECT SOLUTION

$$\frac{\partial \mathcal{L}}{\partial y_{n,l'_i}} = -\frac{1}{\sum_{\pi} \prod_{n=1}^{N} y_{n,\pi_n}} \cdot \frac{\sum_{\pi \in \mathcal{B}^{-1}(z)} \prod_{m=1}^{N} y_{m,\pi_m}}{y_{n,l'_i}}$$

OR WE COULD USE:

$$\frac{\partial \mathcal{L}}{\partial y^{\tau}_{k}} = -\frac{\gamma_{\tau}(k)}{y^{\tau}_{k}}$$

$$\gamma_{\tau}(k) = \frac{1}{y^{\tau}} \sum \alpha_{\tau}(l'_{1:s})\,\beta_{\tau}(l'_{s:|l|})$$

$y_k \quad s: l'_s = k \quad ||$

$$\beta_{\tau}(l'_{s:|l|}) = y^{\tau}_{l'_s}\Big(\beta_{\tau+1}(l'_{s:|l|}) + \beta_{\tau+1}(l'_{(s+1):|l|}) + \beta_{\tau+1}(l'_{(s+2):|l|})\,[l'_s \neq - \wedge l'_s \neq l'_{s+2}]\Big)$$

$$\alpha_{\tau}(l'_{1:s}) = y^{\tau}_{l'_s}\Big(\alpha_{\tau-1}(l'_{1:s}) + \alpha_{\tau-1}(l'_{1:(s-1)}) + \alpha_{\tau-1}(l'_{1:(s-2)})\,[l'_s \neq - \wedge l'_s \neq l'_{s-2}]\Big)$$

SOLUTION

$$\Gamma_{\tau}(l'_{\dots}) = y_{\dots}\Big(\alpha_{\tau}(l'_{1:s}) + \alpha_{\tau-1}(l'_{1:(s-1)}) + \alpha_{\tau-1}(l'_{1:(s-2)})\Big)$$

$$\beta_{\tau}(l'_{s:|l'|}) = y_{\tau, l'_s}\left(\beta_{\tau+1}(l'_{s:|l'|}) + \beta_{\tau+1}(l'_{(s+1):|l'|}) + \beta_{\tau+1}(l'_{(s+2):|l'|}\right)$$

$$\gamma_n(l'_i) = \frac{1}{y_{n,l'_i}} \alpha_n(l'_{1:i}) \beta_n(l'_{i:|l'|})$$

$$\frac{\partial \mathcal{L}}{\partial y_{n,l'_i}} = - \frac{\gamma_n(l'_i)}{y_{n,l'_i}}$$