Lecture 2: Loudness, Its Definition, Measurement, and Calculation

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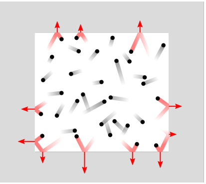
ECE 537: Speech Processing Fundamentals

Outline

Air Pressure

Pressure is the force that a fluid exerts on its container. Units: Pascals $= \frac{\text{Newtons}}{\text{m}^2}$. The **ideal gas law** says that pressure is proportional to density (ρ) and temperature (T):

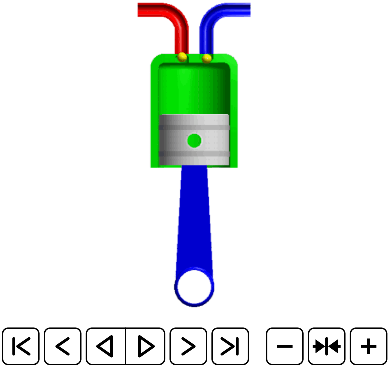
$$p = \rho RT$$



Sound: A Piston of Air

If one chunk of air is moving faster (v) than the chunk ahead of it (i.e., if $\frac{\partial v}{\partial x}$ is negative), that causes an increase in density, which causes an increase in pressure:

$$-\frac{\partial v}{\partial x} = \frac{1}{\rho c^2}\frac{\partial p}{\partial t}$$



GFDL, https://commons.wikimedia.org/wiki/File:Piston.gif

Pressure Gradient

- Suppose that the air pressure (p) at position $x + dx$ is different from the air pressure at x.
- The difference in pressure is a force (f) per unit area (A).
- The force changes the air velocity (v), according to Newton's second law of motion:

$$f = m\frac{\partial v}{\partial t}$$

Divide both sides by volume$=A\partial x$, recognize that $p = \frac{f}{A}$ and $\rho = \frac{m}{A\partial x}$, and you get:

$$-\frac{\partial p}{\partial x} = \rho\frac{\partial v}{\partial t}$$

The Acoustic Wave Equation

The ideal gas law and Newton's second law can be combined to form the 1-dimensional acoustic wave equation:

$$-\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

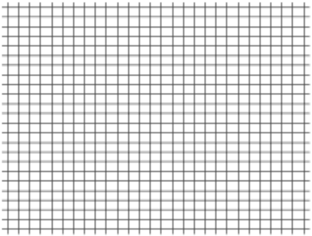
Plane Waves

The solution to the 1d wave equation is any combination of a forward-traveling wave, p_+ and a backward-traveling wave, p_-:

$$p(x,t) = p_+\left(t - \frac{x}{c}\right) + p_-\left(t + \frac{x}{c}\right)$$

Plug into either the ideal gas law or Newton's law, and we find that velocity is just $1/\rho c$ times pressure:

$$v(x,t) = \frac{1}{\rho c}\left(p_+\left(t - \frac{x}{c}\right) - p_-\left(t + \frac{x}{c}\right)\right)$$



Acoustic Intensity

How much energy is there in an acoustic plane wave? The answer is obvious if you think about the units. Use $J = \langle pv \rangle$ to mean "the average, over time, of pressure times velocity." It has these units:

$$\begin{aligned} \text{Pascals} \times \frac{\text{m}}{\text{s}} &= \frac{\text{Newtons}}{\text{m}^2} \times \frac{\text{m}}{\text{s}} \\ &= \frac{\text{Watts}}{\text{m}^2} \end{aligned}$$

So $J = \langle pv \rangle$ has units of Watts per square meter.

Acoustic Intensity of a Pure Tone

Suppose that $p(t)$ is a pure tone, with a root-mean-squared (RMS) amplitude of P Pascals, and a frequency of f Hertz.

$$p(t) = \sqrt{2}P\cos(2\pi ft)$$

$$v(t) = \frac{\sqrt{2}P}{\rho c}\cos(2\pi ft)$$

The intensity of this wave is:

$$\langle pv\rangle = f\int_0^{1/f} p(t)v(t)dt = f\int_0^{1/f}\frac{2P^2}{\rho c}\cos^2(2\pi ft)\,dt$$

$$= \frac{P^2}{\rho c}$$

Acoustic Intensity of a Pure Tone

So if you want to know the intensity (J) of a pure tone, in Watts per square meter, you get it by

$$J = \frac{P^2}{\rho c}$$

where P is the RMS amplitude of the pressure wave, ρ is the density of air, and c is the speed of sound.

Outline

Miles of Standard Cable

On early telephone lines, the power of an electrical signal would drop to about 0.1 of its original value after ten miles of cable. AT&T created a unit of power that they called the "mile of standard cable." The difference between two signal powers, J_2 and J_1, in miles of standard cable (MSC), is

$$\mathrm{MSC} = 10 \log_{10} \left(\frac{J_2}{J_1} \right)$$

decibels

In the 1920s, AT&T proposed a standard unit of "level," where level is defined to be the logarithm of power. If there is some reference power J_r, then the "level" corresponding to J is:

$$\beta = 10 \log_{10} \left(\frac{J}{J_{\text{ref}}} \right)$$

The units of β are "decibels." The idea was that one decibel is a tenth of a Bel, named after Alexander Graham Bell, where the level in Bels is given by:

$$\text{level in Bels} = \log_{10} \left(\frac{J}{J_{\text{ref}}} \right)$$

...but in fact, nobody measures level in Bels, everybody uses decibels.

Sound Pressure Level

The intensity level of a sound can be measured with respect to a standard reference level. The standard reference level is $J_r = 10^{-12}$ Watts per square meter.

The level of a sound, measured w.r.t. 10^{-12} W/m^2, is called its "sound pressure level" (SPL). So

$$\beta = 10 \log_{10} \left(\frac{J}{J_\mathrm{r}} \right)$$

...has units of "dB SPL."

Outline

Loudness adds!

The most revolutionary new idea in Fletcher & Munson's paper was the idea that the loudnesses of different tones add together. If there's a tone at frequency f_1 with loudness level L_1, and a tone at frequency f_2 with loudness level L_2, and if f_2 and f_1 are far enough apart, then

$$G(L) = G(L_1) + G(L_2)$$

where $G(L)$ is the perceived loudness of the whole signal, $G(L_1)$ is the loudness of the first tone, and $G(L_2)$ is the loudness of the second tone.

Loudness adds, except when it doesn't

Fletcher's model of hearing was based on experiments showing that the loudness of a composite tone is equal to the sum of the loudnesses of the individual component tones, unless the frequencies are close together. If they're close together, then the total loudness is less than the sum of the component loudnesses:

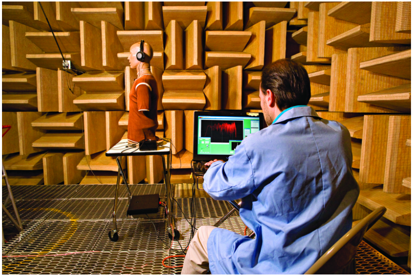
$$G(L) = \sum_k b_k G(L_k)$$

where $b_k \leq 1$. If the frequencies are far apart, then $b_k = 1$. Friday's lecture will consider what happens when they are close together.

Outline

Listening Tests

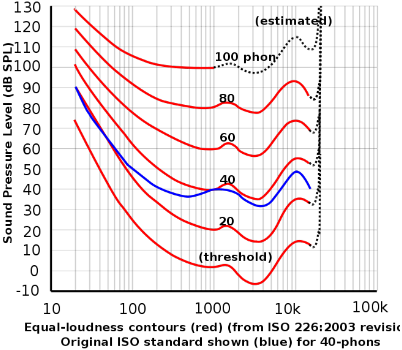
So how do you measure the "loudness" of a sound?
First idea: let's find out how it depends on frequency.
Play two different tones to a person, and ask them to adjust the level of the second tone until it sounds exactly as loud as the first tone.



Loudness Level

The "loudness level" of any pure tone is defined to be the level, in decibels, of a 1000Hz tone that sounds equally loud.

The figure showing loudness levels, as a function of level and frequency, is perhaps the most famous figure from Fletcher & Munson's paper. It's now an ISO standard.



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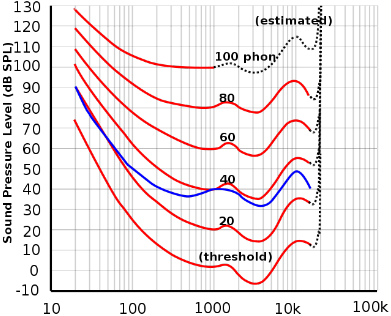
Loudness Level

So if we have a sound with some level β (the y-axis in the figure) at frequency f (the x-axis in the figure), and if the equally loud 1000Hz tone has a level $\beta_1(f,\beta)$ (the numbers written on the curves in the figure), then

$$L = \beta_1(f,\beta)$$



Equal-loudness contours (red) (from ISO 226:2003 revisi
Original ISO standard shown (blue) for 40-phons

Outline

From "Loudness Level" to "Loudness"

Now we know how to normalize for frequency: we can find the loudness level $L(\beta, f)$.

But Fletcher & Munson proposed a harder problem: they proposed to find a transformation $G(L)$ such that

$$G(L) = \sum_k b_k G(L_k)$$

How can we find $G(L)$?

How: Assume that loudness adds!

Suppose that we have two tones with exactly the same loudness level, $L_1 = L_2$. If the model is true — if loudnesses really add — then the loudness of the composite tone should be just exactly twice the loudness of either composite tone!

$$G(L) = G(L_1) + G(L_2) = 2G(L_1)$$

Adding intensities < Adding loudnesses

You might be saying "Yes, that's obvious: 1+1=2!" But in fact, it isn't obvious.

Obviously, the intensity of the composite tone is the sum of the intensities of the two components: $J = J_1 + J_2$.

But if you set a 1000Hz pure-tone to have the same intensity, J, it would **sound quieter!**

If you adjust the intensity of the 1000Hz pure-tone so it sounds equally loud as the composite tone, you would have to adjust it to an intensity louder than J.

How: Use the fact that loudness adds

So the experiment is as follows:

- Play a composite tone, composed of two components with the same loudness level $L_1 = L_2$.
- Ask the listener to adjust the level of a pure 1000Hz tone until it sounds exactly as loud as the composite tone.
- Let the level of the 1000Hz tone be L. Now we know that

$$G(L) = 2G(L_1)$$

Measurement results

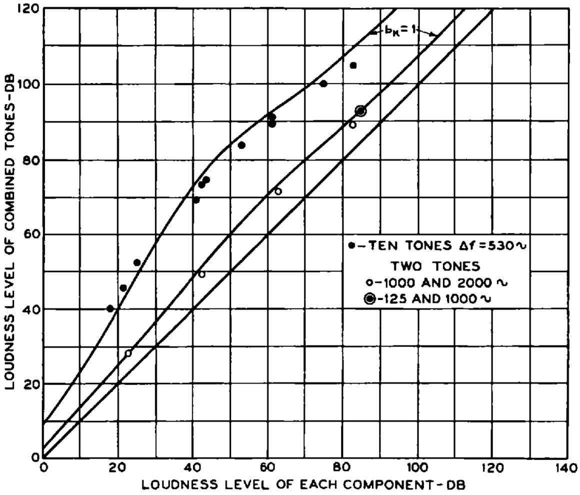
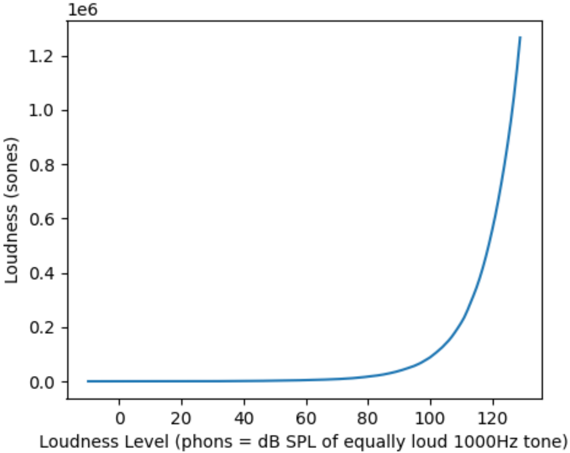


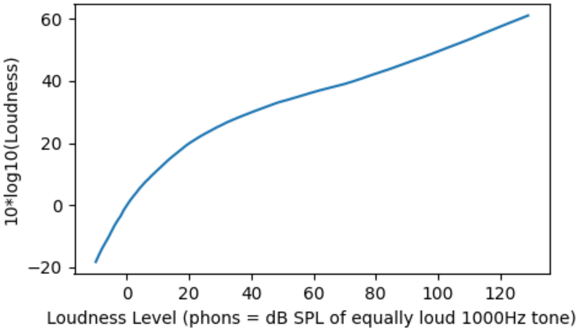
Fig. 6—Complex tones having components widely separated in frequency.

Loudness as a function of Loudness Level



This is the data from Table III in the article (loudness as a function of loudness level).

Log-Loudness as a function of Loudness Level



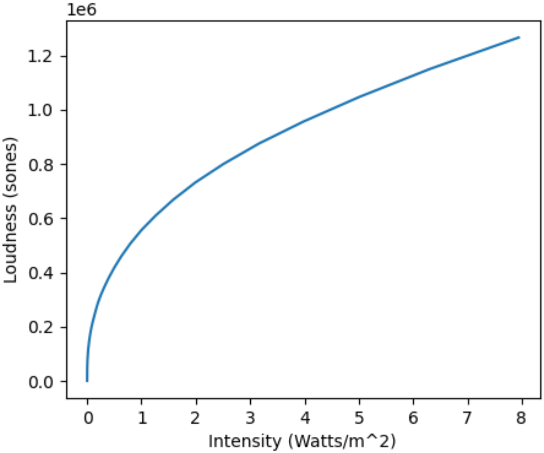
Is Loudness = Intensity?

On the previous slide, notice that, above about 30dB, log-loudness is a linear function of loudness level.

Remember that loudness level is the log of intensity (of an equivalent 1000Hz tone).

Does that mean that loudness = intensity?

Loudness as a function of Intensity (of an equivalently loud 1000Hz tone)



Loudness is a Compressive Function of Intensity

- Loudness looks like an exponential function of level, but...
- Loudness is a compressive function of intensity

A good model is that $G \propto J^{1/3}$. The problem set goes into more detail about that model.

Loudness is a Compressive Function of Intensity

If you adjust the intensity of a 1000Hz pure tone to be J, where $J = J_1 + J_2$ is the same as the total intensity of a composite tone, you will find that the **pure tone is quieter**.
That's because loudness is a compressive nonlinearity:

$$G(J_1 + J_2) < G(J_1) + G(J_2)$$

For example, the cube-root function has this property.

Outline

Conclusions

- Acoustic waves are the alternation of pressure and velocity. RMS pressure, P, is measured in Pascals (Newtons/m^2).
- Intensity (Watts/m^2):
$$J = \frac{P^2}{\rho c}$$
- Level (dB SPL):
$$\beta = 10\log_{10}\left(\frac{J}{J_r}\right)$$

Conclusions

- Loudness Level (phons) is defined to be the level (in dB SPL) of an equally loud 1000Hz tone.
- Loudness (sones) is defined by the equation

$$G(L) = \sum_k b_k G(L_k)$$

. . . where $b_k \leq 1$, and for tones widely spaced in frequency, $b_k = 1$. A not-too-bad approximate model (for tones louder than about 30dB SPL) is $G \propto |J|^{1/3}$.