ECE 537 Practice Exam 1

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

The real exam will be September 28, 2022 in class

- This is a closed-book exam.
- You are allowed to bring one 8.5x11 sheet of handwritten notes (front and back).
- No calculators are allowed. Please do not simplify explicit numerical expressions.
- There are 100 points in the exam. Points for each problem are specified by the problem number.

Name: ______________________________

NetID: ______________________________

Possibly Useful Charts and Formulas

$$G(1000, L) = \sum_{k=1}^{n} b_k G(1000, L_k)$$

$$b_k = \left[\frac{250 + \Delta f}{1000}\right] Q(L_k)$$

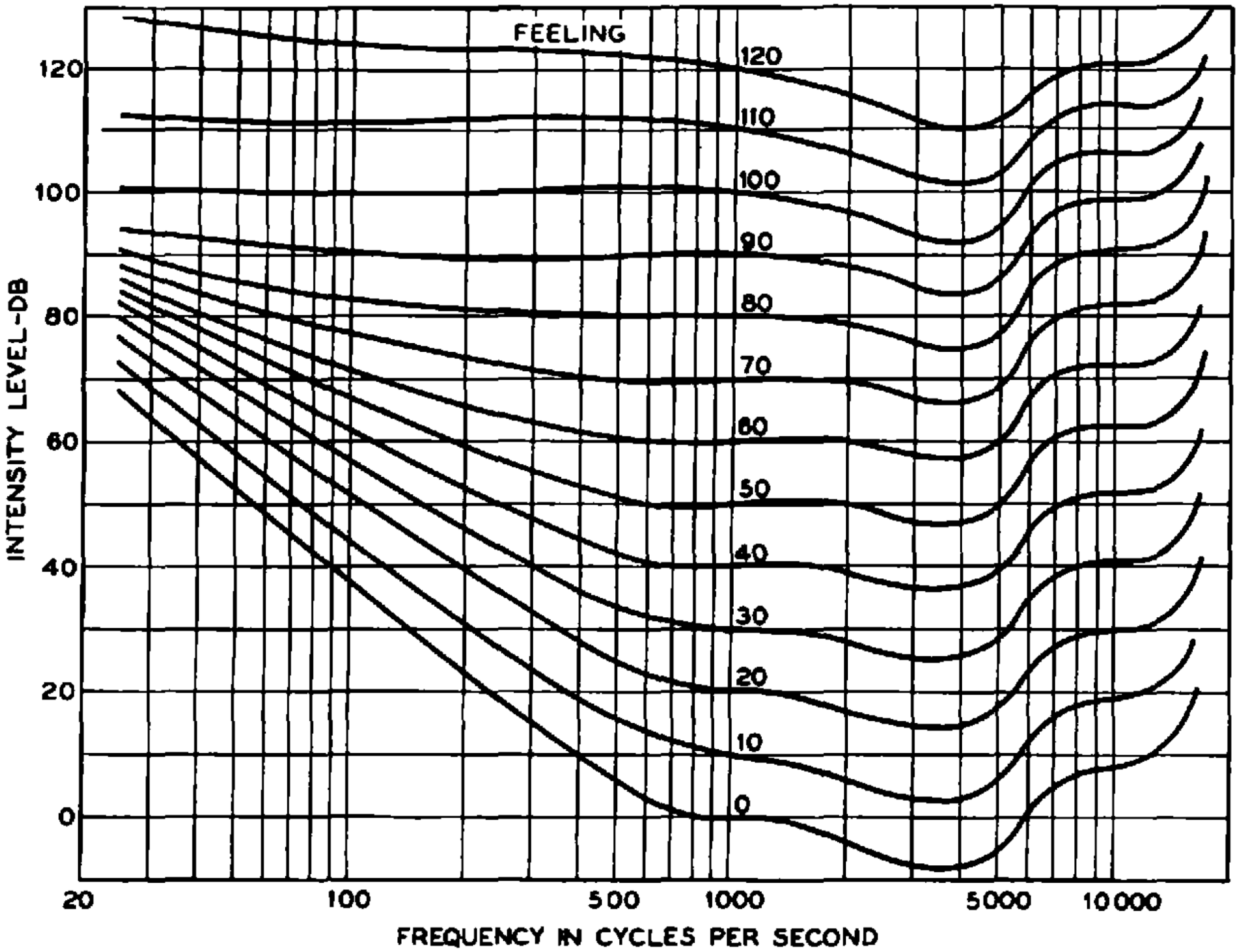
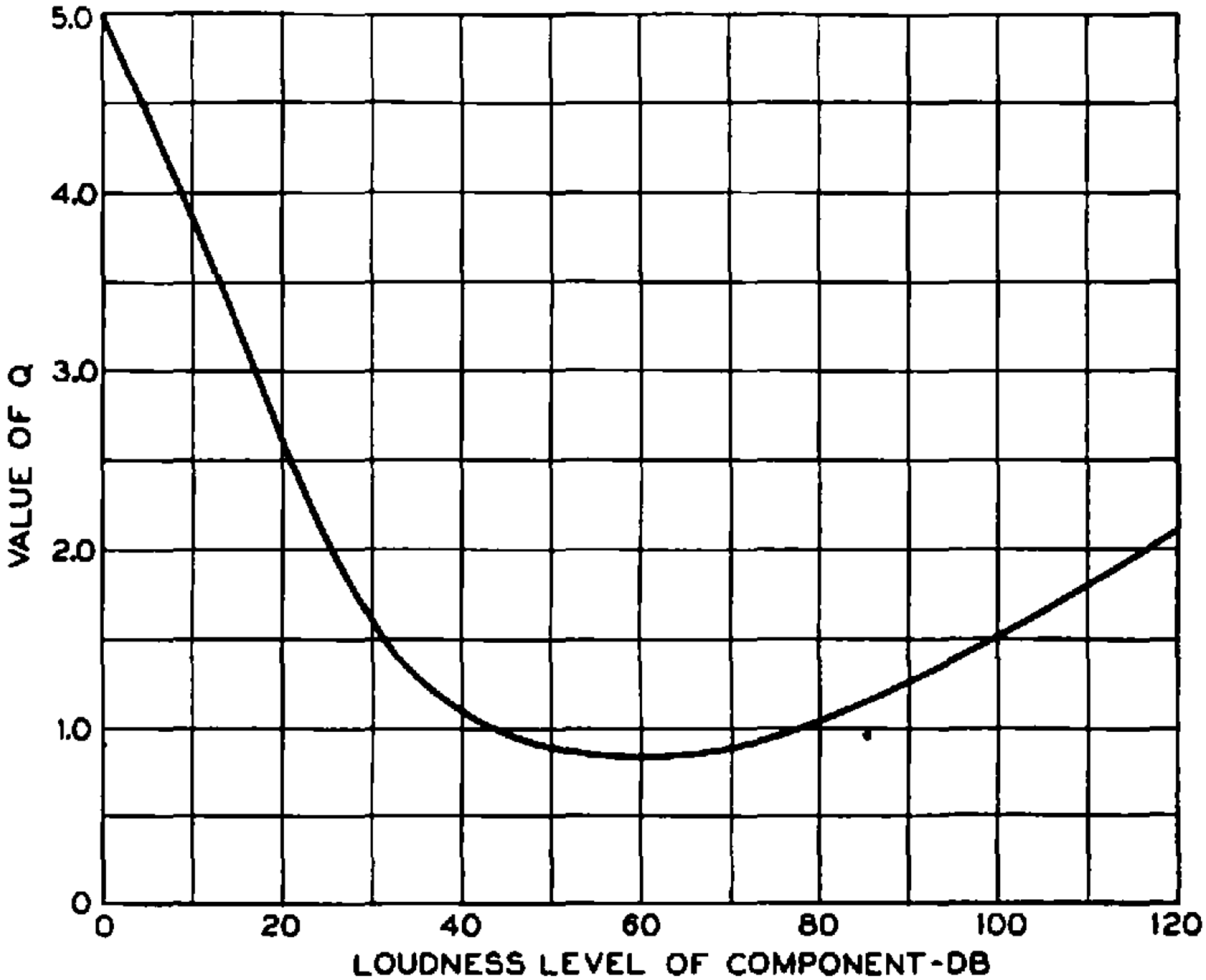


TABLE III

Values of $G(L_k)$.

L	0	1	2	3	4	5	6	7	8	9
−10	0.015	0.025	0.04	0.06	0.09	0.14	0.22	0.32	0.45	0.70
0	1.00	1.40	1.90	2.51	3.40	4.43	5.70	7.08	9.00	11.2
10	13.9	17.2	21.4	26.6	32.6	39.3	47.5	57.5	69.5	82.5
20	97.5	113	131	151	173	197	222	252	287	324
30	360	405	455	505	555	615	675	740	810	890
40	975	1060	1155	1250	1360	1500	1640	1780	1920	2070
50	2200	2350	2510	2680	2880	3080	3310	3560	3820	4070
60	4350	4640	4950	5250	5560	5870	6240	6620	7020	7440
70	7950	8510	9130	9850	10600	11400	12400	13500	14600	15800
80	17100	18400	19800	21400	23100	25000	27200	29600	32200	35000
90	38000	41500	45000	49000	53000	57000	62000	67500	74000	81000
100	88000	97000	106000	116000	126000	138000	150000	164000	180000	197000
110	215000	235000	260000	288000	316000	346000	380000	418000	460000	506000
120	556000	609000	668000	732000	800000	875000	956000	1047000	1150000	1266000

Possibly Useful Charts and Formulas (cont'd)



1. A particular signal contains four component tones at 400, 500, 3000, and 3100Hz. The levels of these four tones are 60, 67, 45, and 40dB SPL, respectively.

   (a) (6 points) Find the loudness levels of all four components.

   > **Solution:** The loudness levels of these four tones are 58, 66, 48, and 43 phons, respectively.

   (b) (6 points) Find the loudness of each of these four components.

   > **Solution:** The loudnesses are 3820, 6240, 1920, and 1250 sones, respectively.

   (c) (6 points) Find the masking coefficients b_k for each of these four components in this combination.

   > **Solution:** The masking coefficients are 1, $0.9\left(\frac{350}{1000}\right)$, 1, and $1.2\left(\frac{350}{1000}\right)$.

   (d) (6 points) What is the total loudness of this signal?

   > **Solution:** The total loudness of this signal is $3820 + 0.9\left(\frac{350}{1000}\right)6240 + 1920 + 1.2\left(\frac{350}{1000}\right)1250$.

   (e) (6 points) Suppose that we deleted the 500Hz and 3100Hz tones, and played only the combination of the two components at 400Hz and 3000Hz. Suppose that you adjusted the level of a 1000Hz reference tone until its loudness was equal to the two-component composite tone. What would be the level of the 1000Hz tone, in dB SPL, that would exactly match this composite tone's loudness?

   > **Solution:** The total loudness of the 400Hz and 3000Hz tones is $3820 + 1920 = 5740$. The level of an equivalently loud 1000Hz tone is 65dB SPL.

2. (20 points) Consider a discrete-time signal, $x[n]$, representing a vowel with a fundamental frequency of $F_0 = 220$Hz, sampled at $F_s = 8000$samples/second. Suppose that this particular vowel only has eight nonzero harmonics. The levels of those eight harmonics are 50, 80, 80, 70, 60, 60, 50, and 80 decibels relative to a reference amplitude of $|x[n]| = 1$ (one least-significant-bit, if the sample values are integers). Suppose you want to reconstruct this vowel using a Dudley vocoder, with an impulse-train excitation. The spectrum is shaped by a set of sub-band amplitudes A_ℓ, where the ℓ^{th} frequency band spans the frequencies $300(\ell-1) \le f < 300\ell$. Find $\{A_1, \ldots, A_{10}\}$.

Solution:

$$A_1 = 10^{5/2}$$
$$A_2 = 10^4$$
$$A_3 = \sqrt{\frac{1}{2}\left(10^8 + 10^7\right)}$$
$$A_4 = 10^3$$
$$A_5 = 10^3$$
$$A_6 = \sqrt{\frac{1}{2}\left(10^5 + 10^8\right)}$$
$$A_7 = 0$$
$$A_8 = 0$$
$$A_9 = 0$$
$$A_{10} = 0$$

3. (20 points) Suppose that a correlogram is computed using ideal bandpass filters with bandwidths $B(f)$ given by the following equation, where $B(f)$ and f are both in Hertz:

$$B(f) = \begin{cases} 100 & 0 \le f \le 1000 \\ 200 & 1000 < f \le 2000 \\ 300 & 2000 < f \le 3000 \\ 400 & 3000 < f \le 4000 \end{cases}$$

Imagine a signal, $x[n] = s[n] + v[n]$, sampled at a sampling rate of $F_s = 10,000$ samples/second. $s[n]$ is a pure tone at 999Hz with a peak amplitude of 10^3, and $v[n]$ is a zero-mean noise signal with the following spectrum

$$E\left[|V(\omega)|^2\right] = \begin{cases} 0 & |\omega| < \frac{2\pi 500}{F_s} \\ 10^5 & \frac{2\pi 500}{F_s} \le |\omega| \end{cases}$$

Write the correlogram $\phi(f,\tau)$ of this signal at the frequencies $f = 700$Hz, $f = 990$Hz, $f = 1020$Hz, and $f = 2400$Hz.

Solution: At $f = 700$Hz, the signal that passes through the auditory filter is an ideal bandpass noise signal with bandwidth of 100Hz, centered at 700Hz. The inverse Fourier transform of such a signal is

$$\phi(700,\tau) = 10^5 \left(\frac{100}{10000}\right) \mathrm{sinc}\left(\frac{2\pi 50}{10000}\tau\right) \cos\left(\frac{2\pi 700}{10000}\tau\right)$$

At $f = 990$Hz, both the noise and the tone pass through the auditory filter, so the correlogram is

$$\phi(990,\tau) = 10^5 \left(\frac{100}{10000}\right) \mathrm{sinc}\left(\frac{2\pi 50}{10000}\tau\right) \cos\left(\frac{2\pi 990}{10000}\tau\right) + \frac{10^6}{2} \cos\left(\frac{2\pi 999}{10000}\tau\right)$$

At $f = 1020$Hz, the noise and the tone both come through, but now the auditory filter bandwidth is 200Hz instead of 100Hz, so

$$\phi(1020,\tau) = 10^5 \left(\frac{200}{10000}\right) \mathrm{sinc}\left(\frac{2\pi 100}{10000}\tau\right) \cos\left(\frac{2\pi 1020}{10000}\tau\right) + \frac{10^6}{2} \cos\left(\frac{2\pi 999}{10000}\tau\right)$$

At $f = 2400$Hz, only the noise comes through, but now the auditory filter bandwidth is 300Hz, so

$$\phi(2400,\tau) = 10^5 \left(\frac{300}{10000}\right) \mathrm{sinc}\left(\frac{2\pi 150}{10000}\tau\right) \cos\left(\frac{2\pi 2400}{10000}\tau\right)$$

4. A particular speaker produces the /ŋ/ consonant with formant frequencies of 400, 1100, 1900, and 2600Hz. Suppose that, during /n/, the mouth cavity of this speaker is a uniform tube of length 5cm. Let $B_i(s)$ be the internal susceptance (the susceptance of nose and pharynx, as viewed from the velum), and let $B_m(s)$ be the susceptance of the mouth as viewed from the velum.

   (a) (6 points) On the same axes, draw a solid curve representing the imaginary part of $B_i(j2\pi f)$ as a function of f, and a dashed curve representing the imaginary part of $B_m(j2\pi f)$ as a function of f, where f is in Hertz, over the range $0 \le f \le 3000$.

   > **Solution:** This should look like the lower plot in Fig. 2 of the Fujimura article. $B_i(j2\pi f)$ is like a negative cotangent, $-B_m(j2\pi f)$ is a negative tangent.

   (b) (6 points) What are the frequencies (in Hertz) of the zero crossings of $B_i(j2\pi f)$, in the range $0 \le f \le 3000$? Include $f = 0$ if $B_i(0) = 0$.

   > **Solution:** The zero crossings of $B_i(j2\pi f)$ are at $f \in \{400, 1100, 1900, 2600\}$Hz.

   (c) (6 points) What is the frequencies (in Hertz) of the zero crossing of $-B_m(j2\pi f)$, in the range $0 \le f \le 3000$? Include $f = 0$ if $B_m(0) = 0$.

   > **Solution:** The zero-crossings of a 5cm tube closed at the opposite end are at $f_k = \frac{(k-1)c}{2\times 0.05}$. Using $c = 354$m/s gives us $f \in \{0, \frac{354}{2\times 0.05}\}$.

(d) (6 points) What is the frequency of the antiformant of /n/ for this speaker?

Solution: The antiformant is at the vertical asymptote of $B_m(j2\pi f)$, which is

$$\frac{c}{4L} = \frac{354}{4 \times 0.05}$$

(e) (6 points) Specify upper and lower bounds for the frequencies of the first five formants of /n/ for this speaker.

Solution: This requires us to simplify the previous two section's answers, giving us a vertical asymptote at 1790Hz and a zero-crossing at 3540Hz, thus

$$\begin{aligned} 0 <& F_1 < 400 \\ 400 <& F_2 < 1100 \\ 1100 <& F_3 < 1790 \\ 1900 <& F_4 < 2600 \\ 2600 <& F_5 < 3540 \end{aligned}$$