ECE 537 Exam 1

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

September 28, 2022

- This is a closed-book exam.
- You are allowed to bring one 8.5x11 sheet of handwritten notes (front and back).
- No calculators are allowed. Please do not simplify explicit numerical expressions.
- There are 100 points in the exam. Points for each problem are specified by the problem number.

Name: ______________________________

NetID: ______________________________

Possibly Useful Charts and Formulas

$$G(1000, L) = \sum_{k=1}^{n} b_k G(1000, L_k)$$
$$b_k = \left[\frac{250 + \Delta f}{1000}\right] Q(L_k)$$

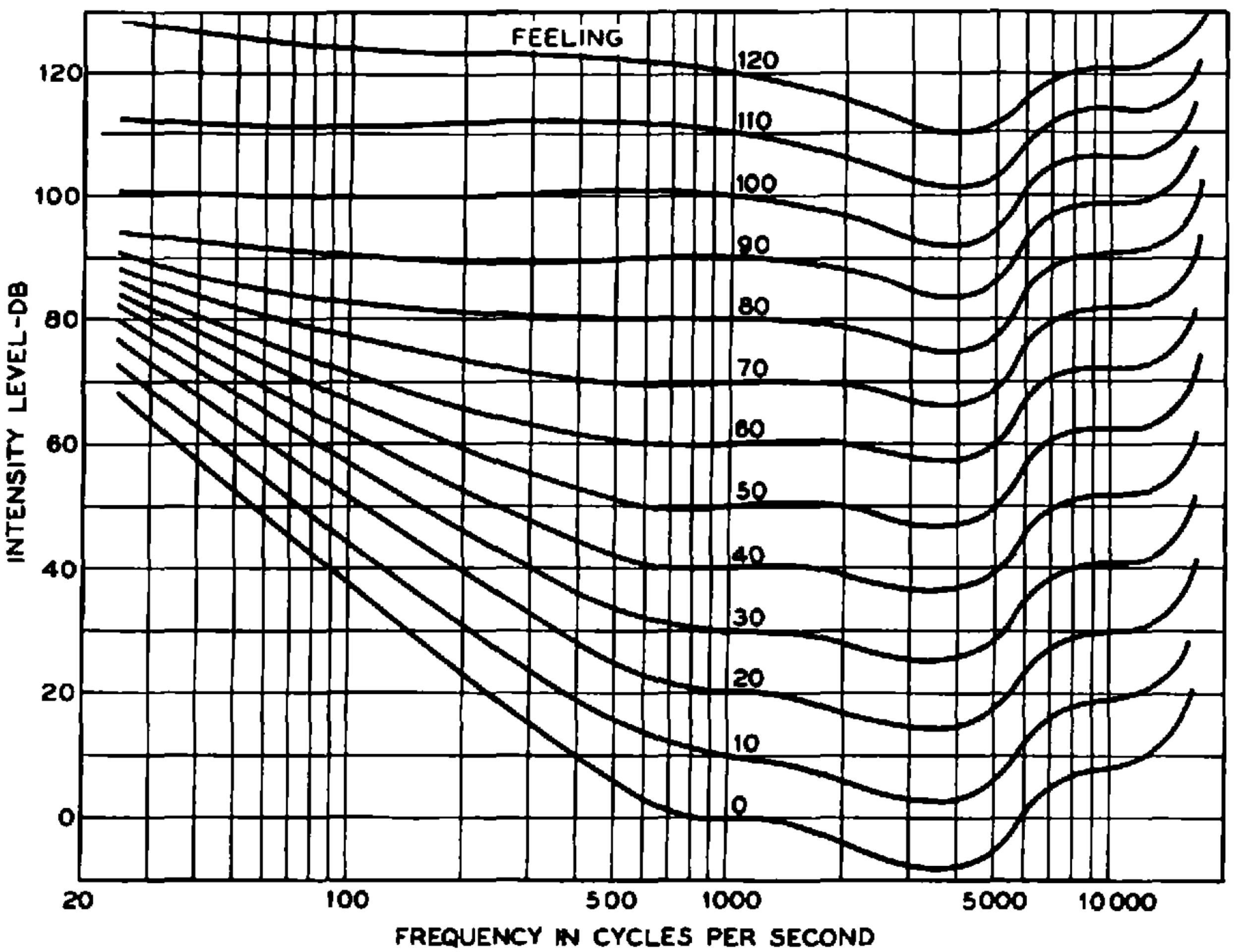
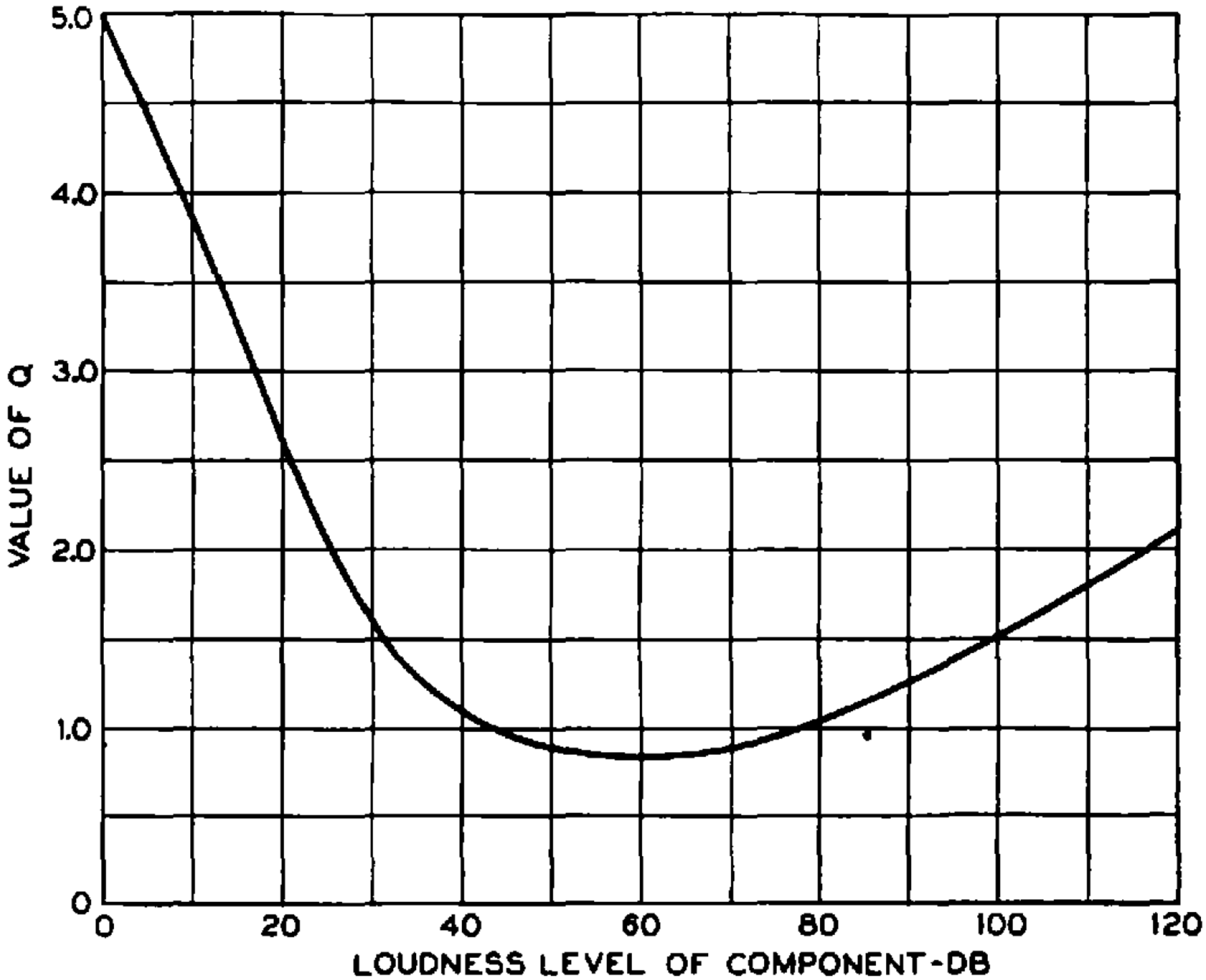


TABLE III

Values of $G(L_k)$.

L	0	1	2	3	4	5	6	7	8	9
−10	0.015	0.025	0.04	0.06	0.09	0.14	0.22	0.32	0.45	0.70
0	1.00	1.40	1.90	2.51	3.40	4.43	5.70	7.08	9.00	11.2
10	13.9	17.2	21.4	26.6	32.6	39.3	47.5	57.5	69.5	82.5
20	97.5	113	131	151	173	197	222	252	287	324
30	360	405	455	505	555	615	675	740	810	890
40	975	1060	1155	1250	1360	1500	1640	1780	1920	2070
50	2200	2350	2510	2680	2880	3080	3310	3560	3820	4070
60	4350	4640	4950	5250	5560	5870	6240	6620	7020	7440
70	7950	8510	9130	9850	10600	11400	12400	13500	14600	15800
80	17100	18400	19800	21400	23100	25000	27200	29600	32200	35000
90	38000	41500	45000	49000	53000	57000	62000	67500	74000	81000
100	88000	97000	106000	116000	126000	138000	150000	164000	180000	197000
110	215000	235000	260000	288000	316000	346000	380000	418000	460000	506000
120	556000	609000	668000	732000	800000	875000	956000	1047000	1150000	1266000

Possibly Useful Charts and Formulas (cont'd)



1. (30 points) A particular signal contains component tones at 150, 300, 1050, and 1200Hz. The levels of these four components are 70, 70, 45, and 50dB SPL, respectively.

   (a) (6 points) Find the loudness levels of all four components.

   > **Solution:** The loudness levels of these four tones are 60, 68, 45, and 50 phons, respectively.

   (b) (6 points) Find the loudness of each of these four components.

   > **Solution:** The loudnesses are 4350, 7020, 1500, and 2200 sones, respectively.

   (c) (6 points) Find the masking coefficients b_k for each of these four components in this combination.

   > **Solution:** The masking coefficients are 1, $0.8\left(\frac{400}{1000}\right)$, 1, and $0.9\left(\frac{400}{1000}\right)$.

(d) (6 points) What is the total loudness of this signal?

Solution: The total loudness of this signal is $4350 + 0.8\left(\frac{400}{1000}\right)7020 + 1500 + 0.9\left(\frac{400}{1000}\right)2200$.

(e) (6 points) Suppose that we deleted the 300Hz and 1200Hz tones, and played only the combination of the two components at 150Hz and 1050Hz. Suppose that you adjusted the level of a 1000Hz reference tone until its loudness was equal to the two-component composite tone. What would be the level of the 1000Hz tone, in dB SPL, that would exactly match this composite tone's loudness?

Solution: The total loudness of the 150Hz and 1050Hz tones is $4350 + 1500 = 5850$. The level of an equivalently loud 1000Hz tone is 65dB SPL.

2. (20 points) Consider a signal $s[n]$, sampled at $F_s = 10,000$samples/second, representing a fricative with the following power spectrum:

$$E\left[|S(\omega)|^2\right] = \begin{cases} 0 & |\omega| \le \frac{2\pi 2000}{F_s} \\ \left|\frac{\omega F_s}{2\pi}\right| - 2000 & 2000 \le \left|\frac{\omega F_s}{2\pi}\right| \le 2500 \\ 500 & \frac{2\pi 2500}{F_s} < |\omega| \end{cases}$$

Suppose you want to model this fricative using a Dudley vocoder, with zero-mean, unit-variance white-noise excitation. The spectral shape is specified using amplitudes A_ℓ, where the ℓ^{th} frequency band spans the frequencies $300(\ell-1) \le \left|\frac{\omega F_s}{2\pi}\right| < 300\ell$Hz.

Find $\{A_1, \ldots, A_{10}\}$.

Solution: $A_1 = A_2 = \cdots = A_6 = 0$.

$$\begin{aligned}
A_7 &= \sqrt{\frac{1}{2\pi 300/10000} \int_{2\pi 1800/10000}^{2\pi 2100/10000} |X(\omega)|^2 d\omega} \\
&= \sqrt{\left(\frac{1}{2}\right) 100} \\
A_8 &= \sqrt{\frac{1}{2\pi 300/10000} \int_{2\pi 2100/10000}^{2\pi 2400/10000} |X(\omega)|^2 d\omega} \\
&= \sqrt{\left(\frac{1}{2}\right) (100 + 400)} \\
A_9 &= \sqrt{\frac{1}{2\pi 300/10000} \int_{2\pi 2400/10000}^{2\pi 2700/10000} |X(\omega)|^2 d\omega} \\
&= \sqrt{\left(\frac{1}{3}\left(\frac{400+500}{2}\right) + \frac{2}{3}500\right)} \\
A_{10} &= \sqrt{\frac{1}{2\pi 300/10000} \int_{2\pi 2700/10000}^{2\pi 3000/10000} |X(\omega)|^2 d\omega} \\
&= \sqrt{500}
\end{aligned}$$

3. (20 points) Suppose that a correlogram is computed using critical bandwidths $B(f)$ given by the following equation, where $B(f)$ and f are both in Hertz:

$$B(f) = \begin{cases} 100 & 0 \le f \le 1000 \\ 200 & 1000 < f \le 2000 \\ 300 & 2000 < f \le 3000 \\ 400 & 3000 < f \le 4000 \end{cases}$$

Imagine a signal $x[n] = s_1[n] + s_2[n]$, sampled at $F_s = 10,000$Hz, where $s_1[n]$ is a pure tone at 1999Hz with a peak amplitude of 10^3, and $s_2[n]$ is a pure tone at 2200Hz with a peak amplitude of 10^2. Write the correlogram $\phi(f,\tau)$ of this signal as a function of the autocorrelation lag, τ (in samples), at the frequencies $f = 1900$Hz, $f = 2100$Hz, and $f = 2300$Hz.

(a) (6 points) $\phi(1900, \tau) =$

> **Solution:** At $f = 1900$Hz,
>
> $$\phi(1900, \tau) = \frac{10^6}{2} \cos\left(\frac{2\pi 1999}{10000}\tau\right)$$

(b) (7 points) $\phi(2100, \tau) =$

> **Solution:** At $f = 2100$Hz,
>
> $$\phi(2100, \tau) = \frac{10^6}{2} \cos\left(\frac{2\pi 1999}{10000}\tau\right) + \frac{10^4}{2} \cos\left(\frac{2\pi 2200}{10000}\tau\right)$$

(c) (7 points) $\phi(2300, \tau) =$

> **Solution:** At $f = 2300$Hz,
>
> $$\phi(2300, \tau) = \frac{10^4}{2} \cos\left(\frac{2\pi 2200}{10000}\tau\right)$$

4. (30 points) A particular speaker produces the /ŋ/ consonant with formant frequencies of 300, 1000, 1800, and 2500Hz. Suppose that, during /m/, the mouth cavity of this speaker is a uniform tube of length 10cm. Let $B_i(s)$ be the internal susceptance (the susceptance of nose and pharynx, as viewed from the velum), and let $B_m(s)$ be the susceptance of the mouth as viewed from the velum.

   (a) (6 points) On the same axes, draw a solid curve representing the imaginary part of $B_i(j2\pi f)$ as a function of f, and a dashed curve representing the imaginary part of $B_m(j2\pi f)$ as a function of f, where f is in Hertz, over the range $0 \leq f \leq 3000$.

   Solution: This should look like the lower plot in Fig. 2 of the Fujimura article. $B_i(j2\pi f)$ is like a negative cotangent, $-B_m(j2\pi f)$ is a negative tangent.

   (b) (6 points) What is/are the frequency/ies (in Hertz) of the zero crossing(s) of $B_i(j2\pi f)$, in the range $0 \leq f \leq 3000$? Include $f = 0$ if $B_i(0) = 0$.

   Solution: The zero crossings of $B_i(j2\pi f)$ are at $f \in \{300, 1000, 1800, 2500\}$Hz.

(c) (6 points) What is/are the frequency/ies (in Hertz) of the zero crossing(s) of $-B_m(j2\pi f)$, in the range $0 \le f \le 3000$? Include $f = 0$ if $B_m(0) = 0$.

Solution: The zero-crossings of a 10cm tube closed at the opposite end are at $f_k = \frac{(k-1)c}{2\times 0.1}$. Using $c = 354$m/s gives us $f \in \{0, \frac{354}{2\times 0.1}\}$.

(d) (6 points) What is the frequency of the antiformant of /m/ for this speaker?

Solution: The antiformant is at the vertical asymptote of $B_m(j2\pi f)$, which is

$$\frac{c}{4L} = \frac{354}{4 \times 0.1}$$

(e) (6 points) Specify upper and lower bounds for the frequencies of the first four formants of /m/ for this speaker.

Solution: This requires us to simplify the previous two section's answers, giving us a vertical asymptote at 895Hz and a zero-crossing at 1790Hz, thus

$$\begin{aligned} 0 <& F_1 < 300 \\ 300 <& F_2 < 885 \\ 1000 <& F_3 < 1770 \\ 1800 <& F_4 < 2500 \end{aligned}$$