ECE 537 Exam 1

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

September 28, 2022

• This is a closed-book exam.
• You are allowed to bring one 8.5x11 sheet of handwritten notes (front and back).
• No calculators are allowed. Please do not simplify explicit numerical expressions.
• There are 100 points in the exam. Points for each problem are specified by the problem number.

Name: ________________________________________________________________

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Possibly Useful Charts and Formulas

\[ G(1000, L) = \sum_{k=1}^{n} b_k G(1000, L_k) \]

\[ b_k = \left[ \frac{250 + \Delta f}{1000} \right] Q(L_k) \]

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1. (30 points) A particular signal contains component tones at 150, 300, 1050, and 1200Hz. The levels of these four components are 70, 70, 45, and 50dB SPL, respectively.

(a) (6 points) Find the loudness levels of all four components.

(b) (6 points) Find the loudness of each of these four components.

(c) (6 points) Find the masking coefficients $b_k$ for each of these four components in this combination.
(d) (6 points) What is the total loudness of this signal?

(e) (6 points) Suppose that we deleted the 300Hz and 1200Hz tones, and played only the combination of the two components at 150Hz and 1050Hz. Suppose that you adjusted the level of a 1000Hz reference tone until its loudness was equal to the two-component composite tone. What would be the level of the 1000Hz tone, in dB SPL, that would exactly match this composite tone’s loudness?
2. (20 points) Consider a signal $s[n]$, sampled at $F_s = 10,000$ samples/second, representing a fricative with the following power spectrum:

$$E \left[ |S(\omega)|^2 \right] = \begin{cases} 0 & |\omega| \leq \frac{2\pi 2000}{F_s} \\ \left| \frac{\omega F_s}{2\pi} \right| - 2000 & 2000 \leq \left| \frac{\omega F_s}{2\pi} \right| \leq 2500 \\ 500 & 2\pi 2500 < |\omega| \end{cases}$$

Suppose you want to model this fricative using a Dudley vocoder, with zero-mean, unit-variance white-noise excitation. The spectral shape is specified using amplitudes $A_\ell$, where the $\ell^{th}$ frequency band spans the frequencies $300(\ell - 1) \leq \left| \frac{\omega F_s}{2\pi} \right| < 300/\text{Hz}$.

Find $\{A_1, \ldots, A_{10}\}$. 

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3. (20 points) Suppose that a correlogram is computed using critical bandwidths \( B(f) \) given by the following equation, where \( B(f) \) and \( f \) are both in Hertz:

\[
B(f) = \begin{cases} 
100 & 0 \leq f \leq 1000 \\
200 & 1000 < f \leq 2000 \\
300 & 2000 < f \leq 3000 \\
400 & 3000 < f \leq 4000 
\end{cases}
\]

Imagine a signal \( x[n] = s_1[n] + s_2[n] \), sampled at \( F_s = 10,000 \text{Hz} \), where \( s_1[n] \) is a pure tone at 1999Hz with a peak amplitude of \( 10^3 \), and \( s_2[n] \) is a pure tone at 2200Hz with a peak amplitude of \( 10^2 \). Write the correlogram \( \phi(f,\tau) \) of this signal as a function of the autocorrelation lag, \( \tau \) (in samples), at the frequencies \( f = 1900 \text{Hz} \), \( f = 2100 \text{Hz} \), and \( f = 2300 \text{Hz} \).

(a) (6 points) \( \phi(1900, \tau) = \)

(b) (7 points) \( \phi(2100, \tau) = \)

(c) (7 points) \( \phi(2300, \tau) = \)
4. (30 points) A particular speaker produces the /ŋ/ consonant with formant frequencies of 300, 1000, 1800, and 2500Hz. Suppose that, during /m/, the mouth cavity of this speaker is a uniform tube of length 10cm. Let \( B_i(s) \) be the internal susceptance (the susceptance of nose and pharynx, as viewed from the velum), and let \( B_m(s) \) be the susceptance of the mouth as viewed from the velum.

(a) (6 points) On the same axes, draw a solid curve representing the imaginary part of \( B_i(j2\pi f) \) as a function of \( f \), and a dashed curve representing the imaginary part of \( B_m(j2\pi f) \) as a function of \( f \), where \( f \) is in Hertz, over the range \( 0 \leq f \leq 3000 \).

(b) (6 points) What is/are the frequency/ies (in Hertz) of the zero crossing(s) of \( B_i(j2\pi f) \), in the range \( 0 \leq f \leq 3000 \)? Include \( f = 0 \) if \( B_i(0) = 0 \).
(c) (6 points) What is/are the frequency/ies (in Hertz) of the zero crossing(s) of $-B_m(j2\pi f)$, in the range $0 \leq f \leq 3000$? Include $f = 0$ if $B_m(0) = 0$.

(d) (6 points) What is the frequency of the antiformant of /m/ for this speaker?

(e) (6 points) Specify upper and lower bounds for the frequencies of the first four formants of /m/ for this speaker.