Random Processes: Quiz

February 16th, 2026

This is a closed notebook quiz. Please write all your derivations out carefully, but note that you will not be graded for this assignment.

Problem 1: Define a probability space and specify the axioms of probability. Given an example of a probability space.

Problem 2: Define a random variable. Explain the concept of a discrete and continuous random variable. Give an example of a pmf and a pdf you saw in your undergraduate courses.

Problem 3: This problems is about Bayes' rule.
A certain disease affects 2% of the population. A medical test is available with the following properties:

– If a person *has* the disease, the test is positive with probability 0.95.

– If a person *does not have* the disease, the test is positive with probability 0.10.

Let $D = \{\text{the person has the disease}\}$, and $T = \{\text{the test is positive}\}$.

1. Using the *law of total probability*, compute the overall probability that a randomly selected person tests positive, $P(T)$.
2. Using *Bayes' rule*, compute the probability that a person actually has the disease given that their test result is positive, $P(D \mid T)$.
3. Briefly explain in words why the answer in part (2) is or is not close to 0.95.

Problem 4: Let $X \sim \text{Uniform}(0, 1)$.

(a) Find the cumulative distribution function (CDF) of X, that is,

$$F_X(x) = \mathbb{P}(X \leq x), \qquad x \in \mathbb{R}.$$

(b) Sketch the graph of $F_X(x)$.

(c) Let X and Y be two independent random variables, each uniformly distributed on $[0, 1]$. Compute the joint probability density function, CDF and find the CDF and pdf of $X + Y$.

Problem 5: Let X, Y be two independent, standard Gaussian random variables (i.e., with zero mean and variance one). Find $E[3X^2 + 2Y - 7]$, $\text{cov}(2X-1, -3Y+4)$. Are $2X$ and $3Y$ correlated? Are they independent?

Problem 6: Prove the continuity of probability result for a sequence of nested non-decreasing events over the same probability space.