Random Processes Spring 2026: Homework I

January 29th, 2026

Problem 1: Countable vs uncountable. Show that the rational numbers are countable, while the reals are uncountable. I am going to discuss the diagonalization proof by Cantor in class, so it is a good starting point to look up the lecture notes.

Problem 2: Sigma Algebras. Describe the axioms of σ-algebras and prove that the intersection of two σ-algebras is another σ-algebra.

Problem 3: Random Variables. Show that if the measurable space is $(\Omega, \mathcal{F})$, with $\mathcal{F} = \{\Omega, \emptyset\}$, then all random variable over this space are constants.

Problem 4: Continuity of Probability Measures. Given a measurable space $(\Omega, \mathcal{F})$ and a sequence of nested events $A_1, A_2, A_3, ...$ in $\mathcal{F}$ such that $A_{i+1} \subset A_i \ \forall i$, prove that

$$\lim_{n\to\infty} P(A_n) = P(\cap_{i=1}^{\infty} A_i).$$

Problem 5: CDFs. Prove that in general, CDFs do not have to be left continuous but have to be right continuous.

Problem 6: More on RVs. Let X be a nonnegative extended real-valued random variable, $X : \Omega \to [0, \infty]$, such that

$$\lim_{n\to\infty} \mathbb{P}(X \geq n) = 0.$$

Show that

$$\mathbb{P}(X < \infty) = 1.$$

Problem 7: Discrete random variables. Let X be a binomial random variable with parameters (n, p), where $p \in (0, 1)$. Find $P\{X \text{ even }\}$, $E[X]$, and $E[X^2 + 3]$.

Problem 8: Continuous random variables. Let X be a Gaussian random variable with parameters (μ, σ^2). Find $P\{X^2 \leq 3\}$. Can you come up with simple bounds on this probability?