ECE 534: SPRING 2022
HOMEWORK 4
ISSUED: MARCH 29TH. DUE APRIL 15TH BY 11AM, GRADESCOPE

Problem 1: Let X_1 and X_2 be the outcomes of fair die throws. Calculate $E[X_1|X_1 > X_2]$.

Problem 2: Consider a class in which grades are based entirely on midterm and final exams. In all, the exams have 100 separate parts, each worth 5 points. A student scoring at least 85%, or 425 points in total, is guaranteed an A score. Throughout this problem, consider a particular student, who, based on performance in other courses and amount of effort put into the class, is estimated to have a 90% chance to complete any particular part correctly. Problem parts not completed correctly receive zero credit.

Assuming that the scores on different parts are independent, what total score for the semester are we likely to see?

Under the same assumptions as in the statement, calculate the approximate probability the student scores enough points for a guaranteed A score.

Problem 3: Let X be a binomial RV with parameters n and p. Using Chernoff bound to upper bound $P\{X \geqslant \alpha n\}$, where $p < \alpha < 1$. Evaluate the bound for $p = 1/2$ and $\alpha = 3/4$. Compare the Chernoff bound with the Markov and Chebyshev bounds.

Problem 4: Problem 4.7 from Prof. Hajek's text.

Problem 5: Suppose (X_n) and (Y_n) are martingales with respect to the same filtration. Are the following two processes necessarily martingales: $Z_n = X_n + Y_n$ and $W_n = max(X_n, Y_n)$?