Tue 9/8

HW 1 released. Due Fri after.

Game plan for rest of semester.

Dynamic analysis → Static analysis → Nonlinear equations → Linear equations

Transient stability ← Power flow ← \( F(x) = 0 \) ← \( Ax = b \)

Small signal stability (if time)

Textbooks & readings:
* Crow
* Milano
* Strang
* Boyd & Vandenberghe

Today: * Review circuits prereqs.
  → Nodal analysis.
  → Inductors & capacitors.
  → Phasors and complex power.
  → Reactive power.
  → Three-phase.
  → Per unit.

* If time: Derivation of Ybus model.
Nodal Analysis

**Example**

\[ R = 1 \Omega \]
\[ V = 5 \Omega \]

\[ i = ? \]

Current at 1:
\[ \frac{V - V_1}{R} + \frac{V_2 - V_1}{R} + \frac{0 - V_1}{R} = 0 \]
\[ \Rightarrow 3V_1 - V_2 = V \] \hspace{1cm} (1)

Current at 2:
\[ \frac{V - V_1}{R} + \frac{V_1 - V_2}{R} + \frac{0 - V_2}{R} = 0 \]
\[ \Rightarrow -V_1 + 3V_2 = V \] \hspace{1cm} (2)

(1) + (2)
\[ 2V_1 + 2V_2 = 2V \]
\[ \Rightarrow V_1 + V_2 = V \] \hspace{1cm} (3)

(1) - (2)
\[ 4V_1 - 4V_2 = 0 \]
\[ \Rightarrow V_1 - V_2 = 0 \] \hspace{1cm} (4)

Sub (4) into (3)
\[ 2V_1 = V \Rightarrow V_1 = V/2 \]
\[ \Rightarrow V_2 = V/2 \]

Output Solution

\[ i = \frac{V - V_1}{R} + \frac{V - V_2}{R} = \frac{5 - 5/2}{1 + 5/2} \]
\[ = 5 \, A \]

**How to program in a computer?**

1) **Label nodes and edges**
2) **For each node, define nodal voltage. Some of these are already known.**
   Once all of these are known, problem is fully solved. Desired solution is trivially recovered. “unknown”
3) **For each node, enforce KCL:**
   Sum of all branch currents into node is zero. Write branch currents directly in terms of nodal voltages.
   “unknown”
4) **For circuit w/ n nodes, yields**
   \[ n \text{ equations over } n \text{ variables}. \rightarrow \text{ standard linear algebra library.} \]
Inductors and Capacitors.

\[ L \frac{di}{dt} + V = L \frac{di}{dt} \quad C \frac{1}{i} \frac{dv}{dt} \]

Example

\[ C = 2 \, \text{F} \]
\[ R = 1 \, \Omega \]
\[ V(t) = \begin{cases} 
5 \, \text{V} & t > 0 \\
0 \, \text{V} & t \leq 0
\end{cases} \]

\[ i(t) \]

Solution via nodal analysis.

\[ \frac{V - V_c}{R} = i_c \]

where \( i_c = C \frac{dv_c}{dt} \)

\[ \frac{dv_c}{dt} = -\frac{1}{RC} V_c + \frac{1}{RC} V \]

\[ = 0 \quad \text{at steady-state.} \]

\[ \Rightarrow V_c = V \]

\[ \frac{dx}{dt} = ax + b \quad \Rightarrow \quad x(t) = x_0 e^{at} + \frac{b}{a} \left(1-e^{at}\right) \]

Closed-form solution.

In general, numerical solution to ODEs.
Phasors.

\[ V(t) = V \cos(\omega t + \phi) \]
\[ = \text{Re} \left( V e^{j(\omega t + \phi)} \right) \]
\[ = \text{Re} \left( V e^{j\phi} \cdot \frac{V}{\sqrt{2}} e^{j\omega t} \right) \]
\[ = \text{Re} \left( \sqrt{2} e^{j\phi} \cdot \frac{V}{\sqrt{2}} e^{j\omega t} \right) \]
\[ = \text{RMS phasor} \]

Example

\[ V(t) = 5 \sin(\omega t + 45^\circ) \]
\[ \bar{V} = \frac{5}{\sqrt{2}} \]
\[ V(t) = 5 \cos(\omega t + 45^\circ - 90^\circ) \]
\[ \bar{V} = \frac{5}{\sqrt{2}} \cos 45^\circ \]

Complex power.

\[ V = 140^\circ \begin{array}{c} \text{?} \end{array} \]
\[ P(t) = 2 \cos^2 \omega t \]
\[ = 1 + \cos 2\omega t \]
\[ \langle P \rangle = \frac{1}{T} \int_0^T 1 + \cos 2\omega t \, dt \]
\[ = 1 \, \text{W} \]
\[ p(t) = V \cos(\omega t + \Theta_v) I \cos(\omega t + \Theta_i) \]
\[ = VI \left[ \cos(\Theta_v - \Theta_i) + \cos(2\omega t + \Theta_v + \Theta_i) \right] \]
\[ \langle p \rangle = P = VI \cos(\Theta_v - \Theta_i) = \text{Re} \bar{V} \bar{I}^* \]

\( \text{AC world, power } \neq \sqrt{V} \sqrt{I} \)
\( \Rightarrow \) no power very high currents.

\[ \bar{V} = j\omega L \bar{I} \]

Utilization factor.
\[ \frac{\text{Re} \bar{V} \bar{I}^* \overline{\gamma}}{\sqrt{V} \sqrt{I}} = \text{power factor.} \]

Complex power.
\[ \bar{S} = \bar{V} \bar{I} \]
\[ \Phi = \text{power factor angle.} \]

Reactive power is a consequence of AC.
Supply \( Q \) to meet voltage & current limits.