ECE 513: Vector Space Signal Processing

Spring 2025

https://courses.grainger.illinois.edu/ece513/sp2025/

Lectures: Tuesdays and Thursdays, 9:30-10:50 AM, 3015 ECEB.

Instructor: Prof. Minh N. Do (minhdo@illinois.edu).
Office hours: Wednesdays 2-3 PM, 113 Coordinated Science Lab (+ appointments by email).

Teaching Assistant: Nick Bampton (bampton2@illinois.edu).
Office hours: TBD.

Course Overview:
Rigorous presentation of key mathematical tools in a vector space framework, and their applications in signal processing, including: finite and infinite dimensional vector spaces, Hilbert spaces, linear operators, inverse problems (e.g. deconvolution, tomography, Fourier imaging), least-squares methods, conditioning and regularization, matrix decompositions, subspace methods, bases and frames for signal representation (e.g. generalized Fourier series, wavelets, splines), Hilbert space of random variables, random processes, signal and spectral estimation.

Course Purpose:
This course is suitable for graduate students in the areas of signal processing, communications, control, and computational science and engineering. It is also suitable for students in other areas who seek graduate level treatment of vector-space methods that are used in various engineering fields. This course complements ECE 551 and ECE 534, and will be particularly useful for students wishing to pursuit a PhD in the above mentioned areas. It provides additional mathematical tools and rigorous foundation for research, and the latest relevant research will be incorporated to keep the contents current.

Prerequisites: ECE 310, ECE 313, and a linear algebra course; or equivalents, or consent of instructor.

Topics:

- **Inverse problems and matrix theory** (*12 hours*): linear inverse problems; orthogonal projections; minimum-norm least squares solutions; Moore-Penrose pseudoinverse; singular value decomposition; matrix decomposition and approximation; conditioning and regularization.
- **General linear vector spaces** (*15 hours*): finite and infinite dimensional vector spaces; Hilbert spaces; projection theorem; inverse problems in infinite dimensional vector spaces; approximation and Fourier series; pseudoinverse operators; iterative methods for optimization and inverse problems; bases and frames for signal representation;
- **Hilbert space of random variables** (*6 hours*): random processes; least-squares estimation; Wiener filtering; Wold decomposition; discrete-time Kalman filter.
- **Applications in signal processing** (*12 hours, during the course*): deconvolution, optimal filter design, temporal and spatial spectrum estimation, tomography, harmonic retrieval, subspace methods, sensor array processing, extrapolation of band-limited sequences, generalized sampling, wavelets, splines, subset selection, sparse approximation, and kernel learning methods.

Texts:

- Textbook by Bresler, Basu, and Couvreur (required; available online with restricted access).
- S. Axler, Linear Algebra Done Right (4th edition), Springer. https://link.springer.com/content/pdf/10.1007/978-3-031-41026-0.pdf.
- Damelin, S., & Miller, Jr, W. (2011). The Mathematics of Signal Processing (Cambridge Texts in Applied Mathematics). Cambridge: Cambridge University Press, http://www-personal.umich.edu/~damelin/proofdammiller.pdf.
- Byrne, C.L. (2014). Signal Processing: A Mathematical Approach, Second Edition. (2nd edition). Chapman and Hall/CRC, https://www.taylorfrancis.com/books/oa-mono/10.1201/b17672/signal-processing-charles-byrne.
- T. Moon and W. Sterling, *Mathematical Methods and Algorithms for Signal Processing*, Prentice Hall, 2000 (recommended).
- F. Deutsch, *Best Approximation in Inner Product Spaces*, Springer Verlag, 2001. (recommended). Excellent book and good reference.
- A. W. Naylor and G. R. Sell, *Linear Operator Theory in Engineering and Science*, Springer Verlag, 1982 (recommended). Excellent book for the Hilbert Space theory in the course.
- P. Stoica and R. L. Moses, *Spectral Analysis of Signals*, Prentice Hall, 2005.
- T. Kailath, A. H. Sayed, and B. Hassibi, *Linear Estimation*, Prentice Hall, 2000.
- A. H. Sayed, *Inference and Learning from Data*, Vols. 1-3, Cambridge University Press, 2022.
- B. Porat, *Digital Processing of Random Signals: Theory and Methods*, Prentice Hall, 1994.
- J. W. Demmel, *Applied Numerical Linear Algebra*, SIAM, 1997.
- L. Trefethen and D. Bau, *Numerical Linear Algebra*, SIAM, 1997.
- C. R. Vogel, *Computational Methods for Inverse Problems*, SIAM, 2002.
- G. Golub and C. Van Loan, *Matrix Computations*, Johns Hopkins University Press, 4th ed., 2013.
- D. Luenberger, *Optimization by Vector Space Methods*, Wiley, 1969.
- R. Vershynin, *High-Dimensional Probability: An Introduction with Applications in Data Science*. Cambridge University Press, 2018.

Grading:

- Weekly homework from week 1 to 9: Posted on Thursdays of the indicated week on the course schedule. Self-check with provided solutions after the due date in one week.
- In-class quizzes (70%): in the last 15 minutes on Tuesday lectures the week after HW due date. The first quiz will be on Tuesday, Feb 3. The two lowest scores will be dropped.
- Final project (30%): presentation and report, could be in a group of 2 students.

Course Schedule:
BBC = Textbook by Bresler, Basu and Couvreur.
Each homework is assigned on Thursday of the indicated week and due on the following Thursday on Gradescope.

Week 1 (1/21)	Introduction to vector space signal processing. Linear inverse problems in $\mathbb{C}^n$ (BBC 1.1–1.4) *Homework #1*
Week 2 (1/27)	Orthogonal projections. Minimum norm least squares solutions (BBC 1.5–1.11). *Homework #2*
Week 3 (2/3)	Singular value decomposition. Matrix approximation (BBC 2.1-7). *Homework #3*
Week 4 (2/10)	Conditioning and regularization. Total least-squares. Subspace fitting (BBC 2.8-11). *Homework #4*
Week 5 (2/17)	Temporal and spatial spectrum estimation (BBC 3). *Homework #5*
Week 6 (2/24)	General linear vector spaces. Linear operators (BBC 4). *Homework #6*
Week 7 (3/3)	Finite and infinite dimensional vector spaces (BBC 5 & 6). *Homework #7* *Project proposals due on Tuesday, 3/3, in class.*
Week 8 (3/10)	Hilbert spaces. Linear inverse problems in infinite dimensional vector spaces (BBC 7). *Homework #8*
(3/15–3/23)	*Spring Break*
Week 9 (3/25)	Hilbert spaces of random variables. Linear least-squares estimation (BBC 8, 9). *Homework #9*
Week 10 (4/1)	Random processes, Wiener filtering (BBC 10).
Week 11 (4/8)	Bases and frames for signal representations (extra notes).
Week 12 (4/15)	Iterative methods for optimization and inverse problems (extra notes).
Week 13 (4/22)	Approximation and learning in Hilbert spaces (papers).
Week 14 (4/29)	*Project presentations* (in class).
Week 15 (5/6)	*Project presentations* (in class). *Project reports due on Friday 5/16.*