ECE 513: Vector Space Signal Processing

Spring 2021

https://courses.grainger.illinois.edu/ece513/sp2021/

Lectures: Mondays and Wednesdays, 9:30-11 a.m.; Synchronous Online

Instructor: Prof. Yoram Bresler (ybresler@illinois.edu).

Course Overview:
Careful presentation of key mathematical tools in a vector space framework, and their applications in signal processing, including: finite and infinite dimensional vector spaces, Hilbert spaces, linear operators, inverse problems (e.g. deconvolution, tomography, Fourier imaging), least-squares methods, conditioning and regularization, matrix decompositions, subspace methods, bases and frames for signal representation (e.g. generalized Fourier series, wavelets, splines), Hilbert space of random variables, random processes, signal and spectral estimation, and compressed sensing.

Course Purpose:
This course is suitable for graduate students in the areas of signal processing, computer vision, communications, control, machine learning, and computational science and engineering. It is also suitable for students in other areas who seek graduate level treatment of vector-space methods that are used in various engineering fields. This course complements ECE 551 and ECE 534, and will be particularly useful for students wishing to pursue a PhD in the above mentioned areas. It provides additional mathematical tools and rigorous foundation for research, and some of the latest relevant research will be incorporated to keep the contents current.

Prerequisites: ECE310, ECE 313 (introduction to probability), and a linear algebra course; or equivalents, or consent of instructor.

Topics:

- **Inverse problems and matrix theory** (*12 hours*): linear inverse problems; orthogonal projections; minimum-norm least squares solutions; Moore-Penrose pseudoinverse; singular value decomposition; matrix decomposition and approximation; conditioning and regularization.
- **General linear vector spaces** (*15 hours*): finite and infinite dimensional vector spaces; Hilbert spaces; projection theorem; inverse problems in infinite dimensional vector spaces; approximation and Fourier series; pseudoinverse operators; iterative methods for optimization and inverse problems; bases and frames for signal representation.
- **Hilbert space of random variables** (*6 hours*): random processes; least-squares estimation; Wiener filtering; Wold decomposition; discrete-time Kalman filter.
- **Applications in signal processing** (*12 hours, during the course*): deconvolution, optimal filter design, temporal and spatial spectrum estimation, tomography, harmonic retrieval, subspace methods, sensor array processing, extrapolation of band-limited sequences, generalized sampling, wavelets, splines, subset selection, sparse approximation, compressive sensing.

Grading: Homework: 20%; Midterm exam I: 25%; Midterm exam II: 25%; Final project (presentation and report): 30%.

Texts:

- Class notes by Bresler, Basu, and Couvreur (required; available online with restricted access)
- Damelin, S., & Miller, Jr, W. (2011). The Mathematics of Signal Processing (Cambridge Texts in Applied Mathematics). Cambridge: Cambridge University Press. doi:10.1017/CBO9781139003896 (PDF available online)
- Byrne, C.L. (2014). Signal Processing: A Mathematical Approach, Second Edition. (2nd edition). Chapman and Hall/CRC, https://doi.org/10.1201/b17672 (PDF available online)
- T. Moon and W. Sterling, *Mathematical Methods and Algorithms for Signal Processing*, Prentice Hall, 2000.
- F. Deutsch, *Best Approximation in Inner Product Spaces*, Springer Verlag, 2001. (Recommended). Excellent book and good reference.
- A. W. Naylor and G. R. Sell, *Linear Operator theory in Engineering and Science*, Springer Verlag, 1982 (recommended). Excellent book for the Hilbert Space theory in the course.
- P. Bremaud, *Mathematical Principles of Signal Processing, Fourier and Wavelet Analaysis* Springer-Verlag, 2002.
- P. Stoica and R. L. Moses, *Introduction to Spectral Analysis*, Prentice Hall, 1997.
- T. Kailath, A. H. Sayed, and B. Hassibi, *Linear Estimation*, Prentice Hall, 2000.
- B. Porat, *Digital Processing of Random Signals: Theory and Methods*, Prentice Hall, 1994.
- J. W. Demmel, *Applied Numerical Linear Algebra*, SIAM, 1997.
- L. Trefethen and D. Bau, *Numerical Linear Algebra*, SIAM, 1997.
- C. R. Vogel, *Computational Methods for Inverse Problems*, SIAM, 2002.
- G. Golub and C. Van Loan, *Matrix Computations*, Johns Hopkins University Press, 3rd ed., 1996.
- R. Vershynin, High-Dimensional Probability: An Introduction with Applications in Data Science. Cambridge University Press, 2018.
- D. Luenberger, *Optimization by Vector Space Methods*, Wiley, 1969.

Course Schedule:
BBC = Class notes by Bresler, Basu and Couvreur.
Each homework is assigned on Thursday of the indicated week and due on the following Thursday.

Week 1 (1/25)	Introduction to vector space signal processing. Linear vector spaces and Linear inverse problems in $\mathbb{C}^n$ (BBC 1.1–1.4) *Homework #1*
Week 2 (2/1)	Orthogonal projections, and least-squares problems (BBC 1.5–1.9) *Homework #2*
Week 3 (2/8)	Singular value decomposition. Matrix approximation (BBC 2.1-7). *Homework #3*

Week 4 (2/15) Conditioning and regularization. Total least-squares. Subspace fitting (BBC 2.8-11).
Homework #4

Week 5 (2/22) Introduction to compressed sensing
Homework #5

Week 6 (3/1) Temporal and spatial spectrum estimation (BBC 3).
Homework #6

Week 7 (3/8) General linear vector spaces. Linear operators (BBC 4).

First midterm exam: TBD.

Homework #7

Week 8 (3/15) Finite and infinite dimensional vector spaces (BBC 5 & 6).

Homework #8

Week 9 (3/22) Hilbert spaces. Linear inverse problems in infinite dimensional vector spaces (BBC 7).
Homework #9

Week 10 (3/29) Iterative methods for optimization and inverse problems (extra notes).
Homework #10

Week 11 (4/6) Bases and frames for signal representations (extra notes).
Homework #11

Week 12 (4/13) Hilbert spaces of random variables. Linear least-squares estimation (BBC 8+9).
Homework #12

Week 13 (4/20) Random processes, Wiener filtering, and Wold decomposition (BBC 9, 10 & extra notes).

Second midterm exam. TBD.
Homework #13

Week 14 (4/27) Approximation and learning in Hilbert spaces (papers).

Week 15 (5/3) Review. Additional applications (papers).

Exam Period (5/7) *Project presentations.*

Exam Period (5/14) *Project reports due.*