ECE 486 **Assignment # 4**

Issued: February 15 Due: February 22, 2024

Reading Assignment:

FPE, Sections 3.3-3.6, 4.1-4.3

Problems:

(unless otherwise noted, you can use a calculator/computer to arrive at numerical answers)

1. Determine whether or not the following polynomials have any RHP roots:

   (i) $s^4 + 10s^3 + 15s^2 + 20s + 1$

   (ii) $s^6 + 2s^5 - 3s^4 + s^3 + 2s + 3$

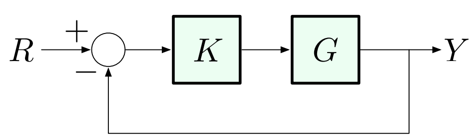
   (iii) $s^8 + 4s^7 + s^6 + 2s^5 + 3s^4 - s^3 + 2s + 3$

   (iv) $s^4 + 10s^3 + 12s^2 + 20s + 1$

   (*Computer use not allowed.*)

2. Consider the following feedback system, where K is a constant gain and

$$G(s) = \frac{1}{s^3 + 3s^2 + s + 1}:$$

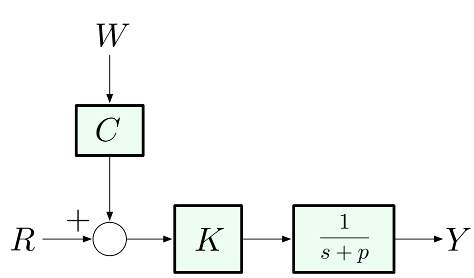


   Using the Routh–Hurwitz criterion, show that the system is stable for $-1 < K < 2$ and unstable for $K \geq 2$. (This illustrates the destabilizing effect of feedback when the gain is too high.)

3. Consider the following open-loop system, consisting of a stable first-order plant

$$G(s) = \frac{1}{s + p}$$

   (thus, $p > 0$) and a scalar-gain controller. There is also a disturbance W that affects the input to the controller, where C is a fixed constant.

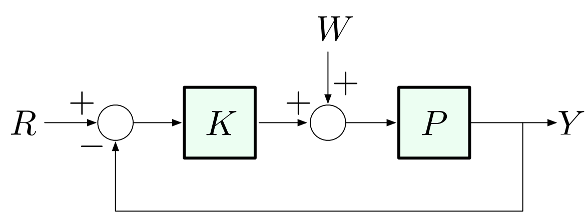


(i) Choose the value of controller gain K to guarantee perfect tracking of a constant reference.

(ii) Show that the resulting system is unable to reject constant disturbances and compute the resulting DC gain from W to Y.

4. In class, we have introduced the notion of system type and described how it relates to reference tracking capabilities (or lack thereof) of closed-loop feedback. In this problem, we will explore the notion of system type with respect to *disturbances.*

   Consider the following unity-feedback configuration that consists of a controller $K(s)$ and a plant $P(s)$, and also includes an additive disturbance W:



   Recall that the forward-loop transfer function $K(s)P(s)$ has system type n with respect to reference input if it has a pole of order n at the origin, or, equivalently, if

$$\lim_{s\to 0} [s^n K(s)P(s)] \neq 0.$$

   Assuming the closed-loop system is stable, this means that n is the lowest degree of a polynomial that cannot be tracked in feedback with zero steady-state error. We will now focus on the control objective of disturbance rejection.

   (i) Show that the system type with respect to reference inputs is equal to n whenever

$$\lim_{s\to 0} \frac{1 - T_{r\to y}(s)}{s^n} = \text{const} \neq 0,$$

   where $T_{r\to y}$ is the transfer function from the reference R to the output Y.

   (ii) Write down the transfer function $T_{w\to y}$ from the disturbance input W to the output Y.

   (iii) We say that the above system has type k with respect to disturbance inputs if

$$\lim_{s\to 0} \frac{T_{w\to y}(s)}{s^k} = \text{const} \neq 0.$$

   Show that this is the case when $T_{w\to y}$ has a *zero* of order k at the origin, i.e., if $T_{w\to y}(s) = s^k \frac{A(s)}{B(s)}$, where A and B are polynomials with real coefficients, such that $A(0) \neq 0$ and $B(0) \neq 0$.

   (iv) Show that the system of type k with respect to disturbances can acehive perfect steady-state rejection of polynomial disturbances of degree $m < k$, but not when $m \geq k$.

(v) Finally, consider the example we have discussed in class, where

$$P(s) = \frac{1}{s^2 + 1}$$

and determine the system type with respect to disturbances for P-control $K(s) = K_{\mathrm{P}}$, PD-control $K(s) = K_{\mathrm{P}} + K_{\mathrm{D}}s$, and PID-control $K(s) = K_{\mathrm{P}} + K_{\mathrm{D}}s + \frac{K_{\mathrm{I}}}{s}$.