ECE 486 *Control Systems* **Spring 2024**

Exam I Information

The first midterm exam will be held ***in class***

1015 ECE Bldg *Thu, March 7, 9:30 – 10:50 a.m.*

The exam is closed-book, no calculators allowed (and you will not need one). You are allowed to bring one two-sided sheet of notes. The exam will cover all of the material covered up to the first half of Lecture 13 given on Feb 29.
In particular, I expect you to understand perfectly the following concepts:

1. General models: State space; transfer function; block diagrams.
2. Linearization.
3. Transient and steady-state response: DC gain; Final Value Theorem.
4. Second-order response and the effect of poles and zeros; time-domain specifications (rise time, overshoot, peak time, settling time) and their relation to pole locations.
5. Stability: definition; necessary condition for stability; Routh–Hurwitz criterion; necessary and sufficient conditions for 2nd- and 3rd-order polynomials.
6. Open-loop and closed-loop feedback control: reference-to-output and reference-to-error transfer functions; tracking error.
7. Simple compensators: PID, lead, lag; effect of controller parameters on time-domain specs and on steady-state response.
8. Root locus methods as developed in class (Rules A—F): Evans' canonical form; phase condition; effect of PD/lead and PI/lag compensation on the root locus.

The bare minimum of the material you need to know will be attached to the exam and reproduced below. *However,* you are responsible for all of the content outlined above.

Useful Facts

Unilateral Laplace transforms:

$$f(t),\ t \geq 0 \quad \xrightarrow{\mathscr{L}} \quad F(s) = \int_0^\infty f(t)e^{-st}\mathrm{d}t,\ s \in \mathbb{C}$$

$$\mathscr{L}\left[f'(t)\right] = sF(s) - f(0)$$

$$\mathscr{L}\left[f''(t)\right] = s^2F(s) - sf(0) - f'(0)$$

Second-order system:

$$\begin{aligned} H(s) &= \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \\ &= \frac{\omega_n^2}{(s+\sigma)^2 + \omega_d^2} \qquad \omega_n, \zeta > 0 \end{aligned}$$

Rise time: $t_r \approx \dfrac{1.8}{\omega_n}$

Peak time: $t_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Overshoot: $M_p = \exp\left(-\dfrac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$

Settling time: $t_s^{5\%} \approx \dfrac{3}{\zeta\omega_n}$

Stability criteria for polynomials:

- a monic polynomial $p(s) = s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n$ is *stable* if all of its roots are in the open LHP
- 2nd-order polynomial

  $$p(s) = s^2 + a_1 s + a_2$$

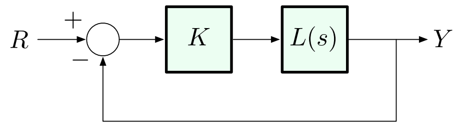
  is stable if and only if $a_1, a_2 > 0$
- 3rd-order polynomial

  $$p(s) = s^3 + a_1 s^2 + a_2 s + a_3$$

  is stable if and only if $a_1, a_2, a_3 > 0$ and $a_1 a_2 > a_3$

Root locus Let L be a proper transfer function of the form

$$L(s) = \frac{b(s)}{a(s)} = \frac{s^m + b_1 s^{m-1} + \ldots + b_{m-1}s + b_m}{s^n + a_1 s^{n-1} + \ldots + a_{n-1}s + a_n}$$



The *root locus* is the set of all $s \in \mathbb{C}$ such that

$$1 + KL(s) = 0 \qquad \Longleftrightarrow \qquad a(s) + Kb(s) = 0$$

Phase condition: a point $s \in \mathbb{C}$ is on the RL if and only if

$$\angle L(s) = \angle \frac{b(s)}{a(s)} = \angle \frac{(s - z_1)\ldots(s - z_m)}{(s - p_1)\ldots(s - p_n)} = 180^\circ \quad \text{mod } 360^\circ$$

Rules for sketching root loci

- Rule A: n branches ($n = \#(\text{open-loop poles})$)
- Rule B: branches start at open-loop poles $p_1, \ldots, p_n$
- Rule C: m of the branches end at open-loop zeros $z_1, \ldots, z_m$ (L is proper: $m \le n$)
- Rule D: a point $s \in \mathbb{R}$ is on the RL if and only if there is an *odd* number of *real* open-loop poles and zeros to the right of it
- Rule E: if $n - m > 0$, the remaining $n - m$ branches approach ∞ along asymptotes departing from the point

  $$\alpha = \frac{\sum_{i=1}^n p_i - \sum_{j=1}^m z_j}{n - m}$$

  at angles

  $$\frac{(2\ell + 1) \cdot 180^\circ}{n - m}, \qquad \ell = 0, 1, \ldots, n - m - 1.$$

- Rule F: $j\omega$-crossings
  - find the critical value(s) of K (if any) that will make the characteristic polynomial $a(s) + Kb(s)$ unstable
  - for each of these critical values, solve

    $$a(j\omega) + Kb(j\omega) = 0$$

    for critical frequencies ω