ECE 486: Control Systems

Lecture 6A: Effect of Extra Poles & Zeros

Key Takeaways

This lecture considers the effect of extra poles and zeros on the step response.

LHP Poles: Increase settling time.

The effects are small if the pole is far in the LHP.

LHP Zeros: Increase overshoot, decrease rise time, and have no effect on settling time.

The effects are small if the zero is far in the LHP.

RHP Zeros: Cause undershoot but no effect on settling time.

The effects are small if the zero is far in the RHP.

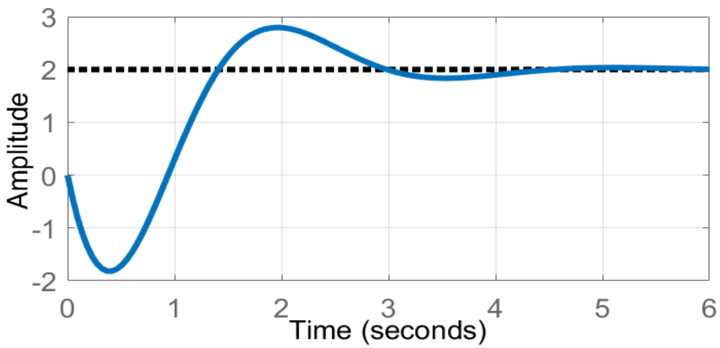
Problem 1

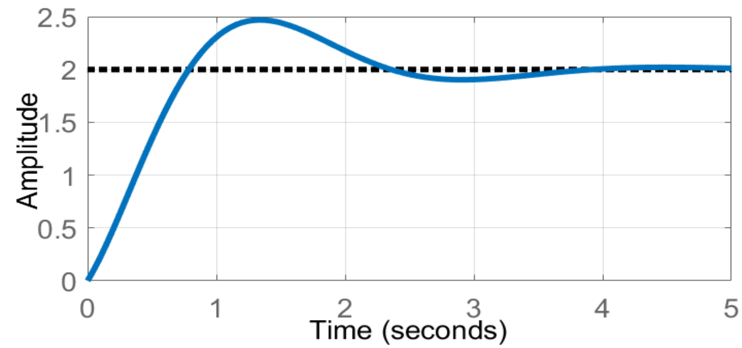
Four systems and four unit step responses are given below. Match each system to its unit step response.

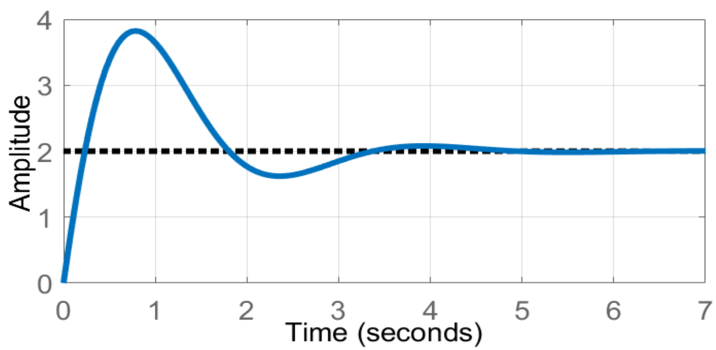
$$G_A = \frac{-2s+10}{s^2+2s+5} \qquad G_C = \frac{2s+10}{s^2+2s+5}$$

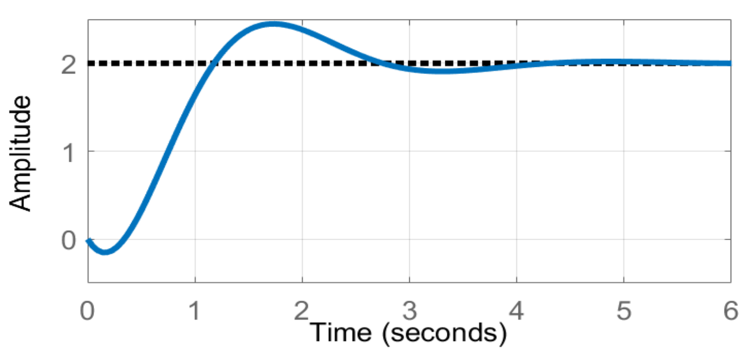
$$G_B = \frac{-10s+10}{s^2+2s+5} \qquad G_D = \frac{10s+10}{s^2+2s+5}$$

What happens if adding a pole at -20?









Solution 1

$$G_A = \frac{-2s+10}{s^2+2s+5},\quad G_B = \frac{\boxed{-10s+10}}{s^2+2s+5},\quad G_C = \frac{\boxed{2s+10}}{s^2+2s+5},\quad G_D = \frac{10s+10}{s^2+2s+5}$$

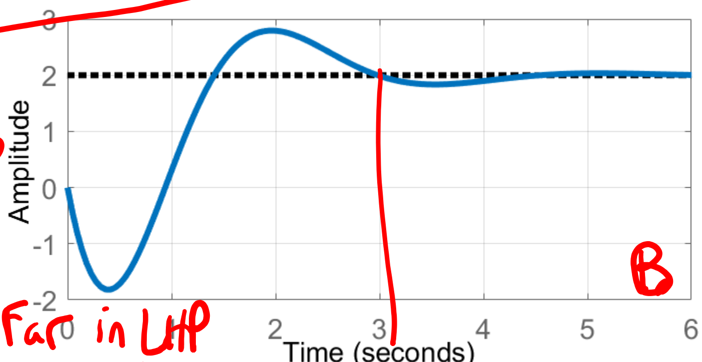
$s = -1 \pm 2j$ $\quad T_s = 3\,sec$, All the DC gain 2

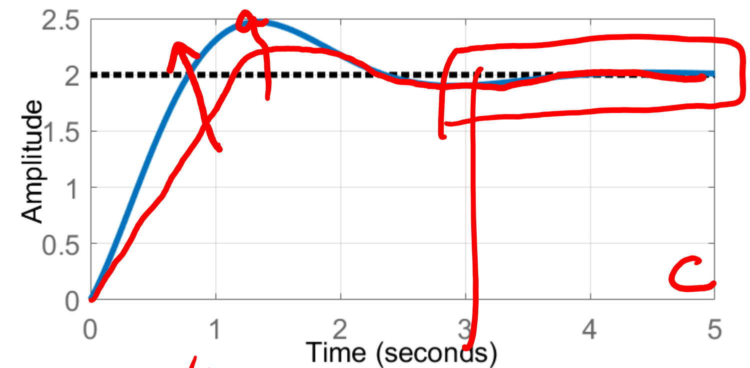
(A) $s = +5$ — RHP Undershoot — Far in RHP (less effect)

(B) $s = +1$ — Nearer to imag axis (more effect)

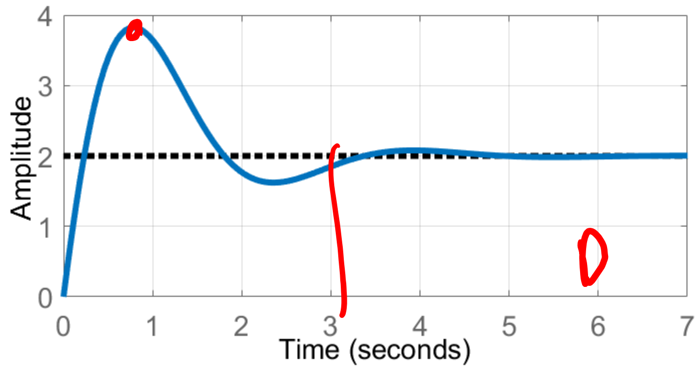
(C) $s = -5$ — LHP — Far in LHP (faster (less effect)

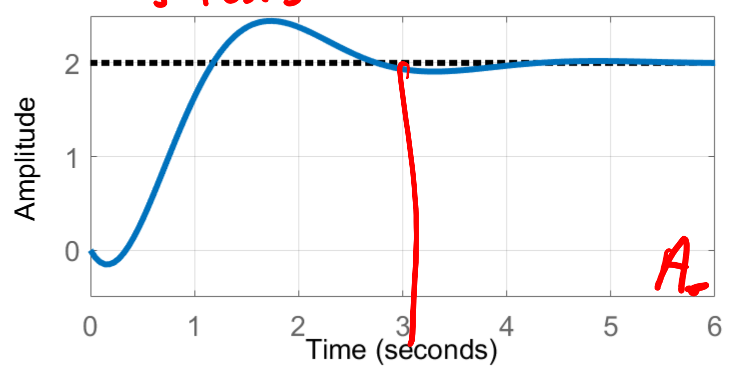
(D) $s = -1$ — Nearer to imag axis (more effect





$G(s) = \frac{10}{s^2+2s+5}$





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Lecture 6B: Stability

Key Takeaways

We study the properties exponential terms e^{st} that appear in the free and forced response.

The lecture covers the following:

1. Response characteristics for real and complex roots
2. Time Constants
3. Internal Stability
4. Bounded-Input, Bounded-Output Stability

Problem 2

For each of the systems below:

- What are the poles? Is the system stable?

$$G_A(s) = \frac{s-2}{s+7}$$

$$G_B(s) = \frac{s+2}{s-7}$$

$$G_C(s) = \frac{-9}{s^2+2s-8}$$

$$G_D(s) = \frac{5}{(s^2+4s+13)(s-5)}$$

Solution 2A

- What are the poles? Is the system stable?

$$G_A(s) = \frac{s-2}{s+7}$$

$s = -7 \leftarrow$ LHP

Stable

$\tau = \frac{1}{|Re(s)|} = \frac{1}{7}$ sec

Solution 2B

- What are the poles? Is the system stable? $G_B(s) = \frac{s+2}{s-7}$

$s = +7 \leftarrow$ RHP

unstable

$\tau = 1/7$ sec

Solution 2C

- What are the poles? Is the system stable?

$$G_C(s) = \frac{-9}{s^2+2s-8}$$

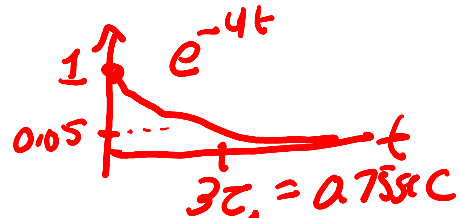
$0 = s^2+2s-8$

RHP

$s_1 = -4, \boxed{2} = s_2$

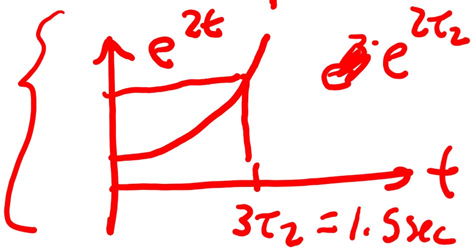
Unstable

$\tau_1 = 1/4$ sec



Slowest (Largest) Time Constant

$$\boxed{\tau_2 = \frac{1}{2} \text{ sec}}$$



$e^{2\tau_2} = e^3 = 20$

Solution 2D

- What are the poles? Is the system stable?

$$G_D(s) = \frac{5}{\underbrace{(s^2+4s+13)}(s-5)}$$

$\underbrace{s_1 = +5}_{\text{RHP}}$

Unstable

$\tau_1 = 1/5$ sec

$\underbrace{s_{2,3} = -2 \pm 3j}_{\text{2 LHP}}$

$\boxed{\tau_{2,3} = \frac{1}{2} \text{ sec}}$ slowest

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Lecture 6C: Routh-Hurwitz Criterion

Problem 3

Without a computer, determine whether or not the following polynomial have any RHP roots:

$$s^4 + 10s^3 + 40s^2 + 20s + 1$$

The Routh table is constructed below

s^4	1	40	1
s^3	10	20	0
s^2	38	1	0
s^1	750/38	0	0
s^0	1	0	0

The polynomial has no RHP roots.