ECE 486: Control Systems

Lecture 5A: Interconnections

Key Takeaways

Transfer functions can be used to derive models for interconnections of LTI systems.

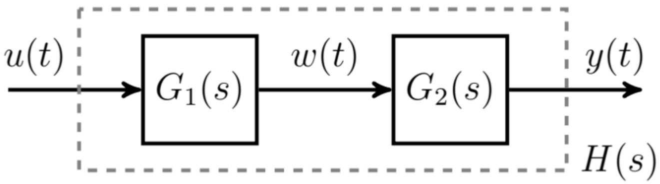
This lecture covers two specific examples:

- The parallel connection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = G_1(s)+G_2(s)$.
- The serial connection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = G_1(s)\, G_2(s)$.
- The negative feedback interconnection of $G_1(s)$ and $G_2(s)$ is given by $H(s) = \frac{G_1(s)}{1+G_1(s)\, G_2(s)}$.

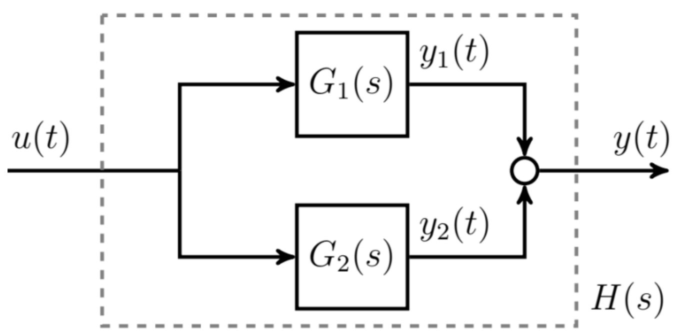
Problem 1

A) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for serial connection $H(s)=G_2(s)\ G_1(s)$?

B) Suppose $G_1(s) = \frac{5}{s+7}$ and $G_2(s) = \frac{3}{s+2}$. What is the ODE for serial connection $H(s)=G_2(s)\ G_1(s)$?



C) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for parallel connection $H(s)=G_1(s) + G_2(s)$?

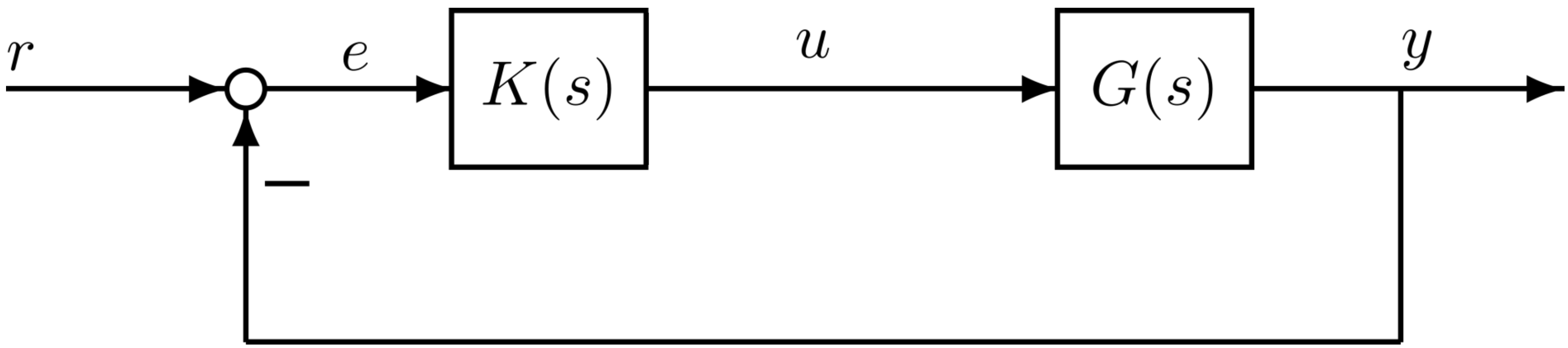


Problem 1

D) Consider the feedback system below with:

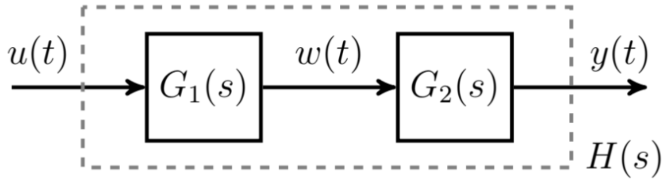
$$\dot{y}(t) + 5y(t) = 5u(t) \text{ and } u(t) = 2e(t) + 4\int_0^t e(\tau)\,d\tau$$

Obtain a model of the closed-loop from *r* to *y* with transfer functions, and compare your answers in Matlab using the function `feedback`.



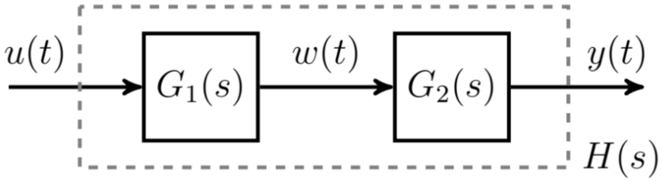
Solution 1A

A) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for serial connection $H(s)=G_2(s)\, G_1(s)$?



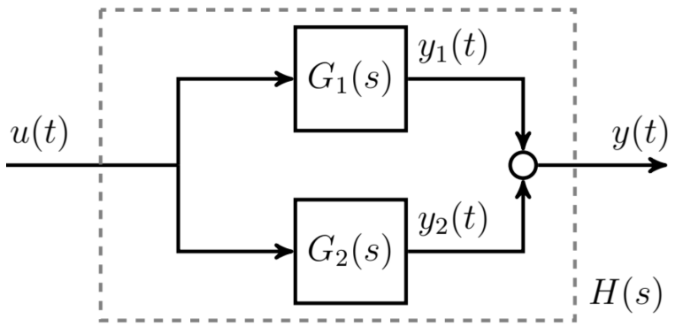
Solution 1B

B) Suppose $G_1(s) = \frac{5}{s+7}$ and $G_2(s) = \frac{3}{s+2}$. What is the ODE for serial connection *H(s)=G$_2$(s) G$_1$(s)*?



Solution 1C

C) Suppose $G_1(s) = \frac{3}{s+2}$ and $G_2(s) = \frac{5}{s+7}$. What is the ODE for parallel connection $H(s)=G_1(s) + G_2(s)$?

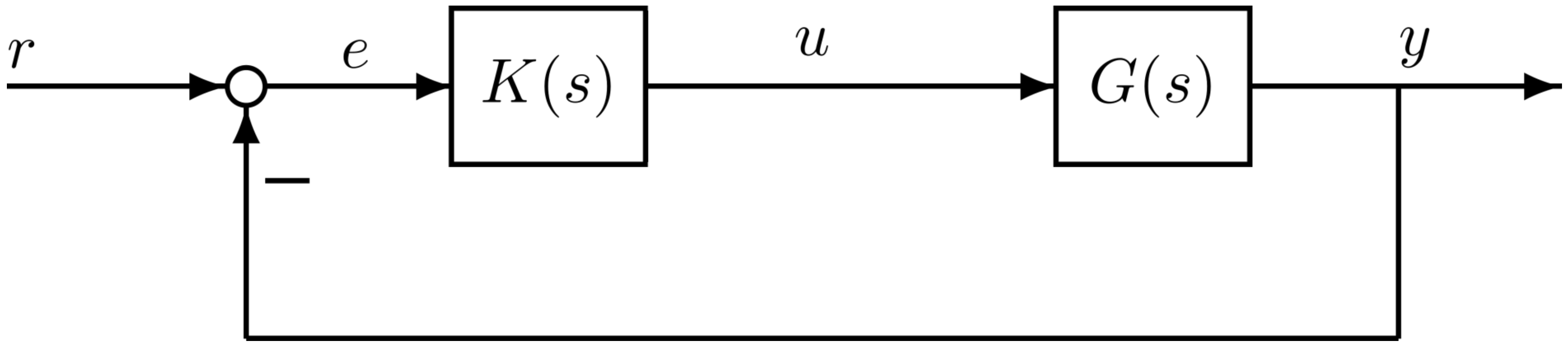


Solution 1D

Consider the feedback system below with:

$$\dot{y}(t) + 5y(t) = 5u(t) \text{ and } u(t) = 2e(t) + 4\int_0^t e(\tau)\,d\tau$$

D) Obtain a model of the closed-loop from *r* to *y* and compare with your answers in Matlab using the function `feedback`.



Solution 1 – Extra Space

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Lecture 5B: Block Diagrams for Linear ODEs

Key Takeaways

- This lecture describes a method to construct block diagrams for linear ODEs with constant coefficients.
- The diagrams are constructed from blocks for:
  - integration,
  - addition/subtraction, and
  - multiplication by a gain
- These diagrams will be used later for:
  - Numerical integration of ODEs using a tool call Simulink
  - Developing state-space models. These provide an alternative to the ODE/TF models that we are using as a starting point.

Problem 2

A) Draw a block diagram for $G_1(s) = \frac{7}{s^2+2s-3}$ using integrator, summation, and gain blocks.

B) Draw a block diagram for $G_1(s) = \frac{5s+6}{s^2+2s-3}$ using integrator, summation, and gain blocks.

Solution 2A

A) Draw a block diagram for $G_1(s) = \frac{7}{s^2+2s-3}$ using integrator, summation, and gain blocks.

Solution 2B

B) Draw a block diagram for $G_1(s) = \frac{5s+6}{s^2+2s-3}$ using integrator, summation, and gain blocks.

Solution 2 – Extra Space

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Lecture 5C: State-Space Models

Key Takeaways

This lecture introduces linear state-space models.

An n^{th}-order linear state-space model expresses the dynamics as a first-order, vector differential equation. It is possible to express as an equivalent n^{th}-order linear ODE.

State-space models have several uses:

- There are different tools for analysis and design of feedback systems based on state-space models.
- They can be used to approximate a nonlinear model by a related linear model.

Problem 3

Find a state-space representation for:

$$y^{[3]}(t) + 2\ddot{y}(t) - 4\dot{y}(t) + 10y(t) = -3\dot{u}(t) + 6u(t)$$

Solution 3

Find a state-space representation for:

$$y^{[3]}(t) + 2\ddot{y}(t) - 4\dot{y}(t) + 10y(t) = -3\dot{u}(t) + 6u(t)$$

Solution 3-Extra Space