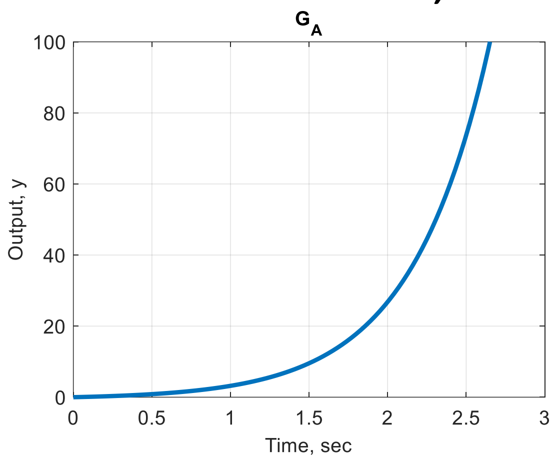
ECE 486: Control Systems

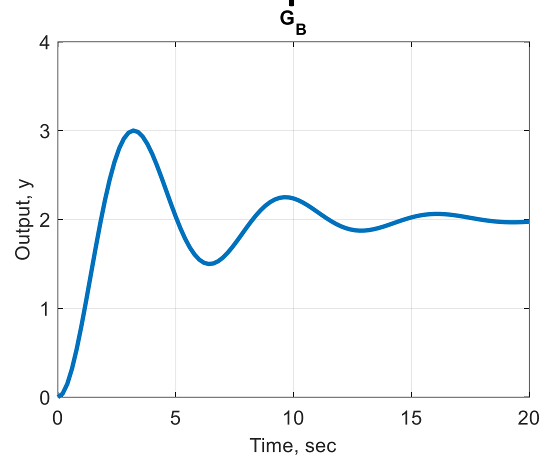
Lecture 4B: Time Domain Performance

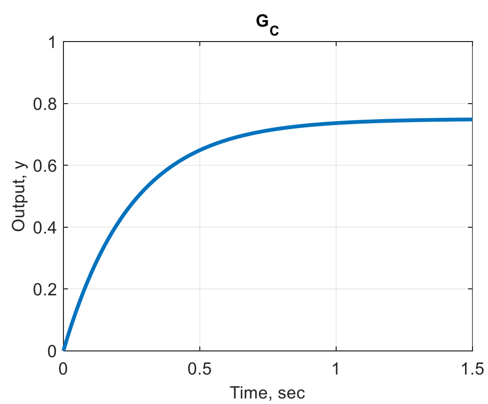
Problem 1

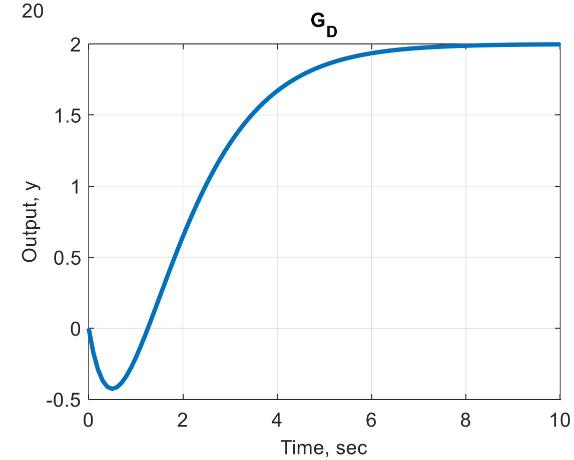
Several unit step responses are shown below. For each:

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



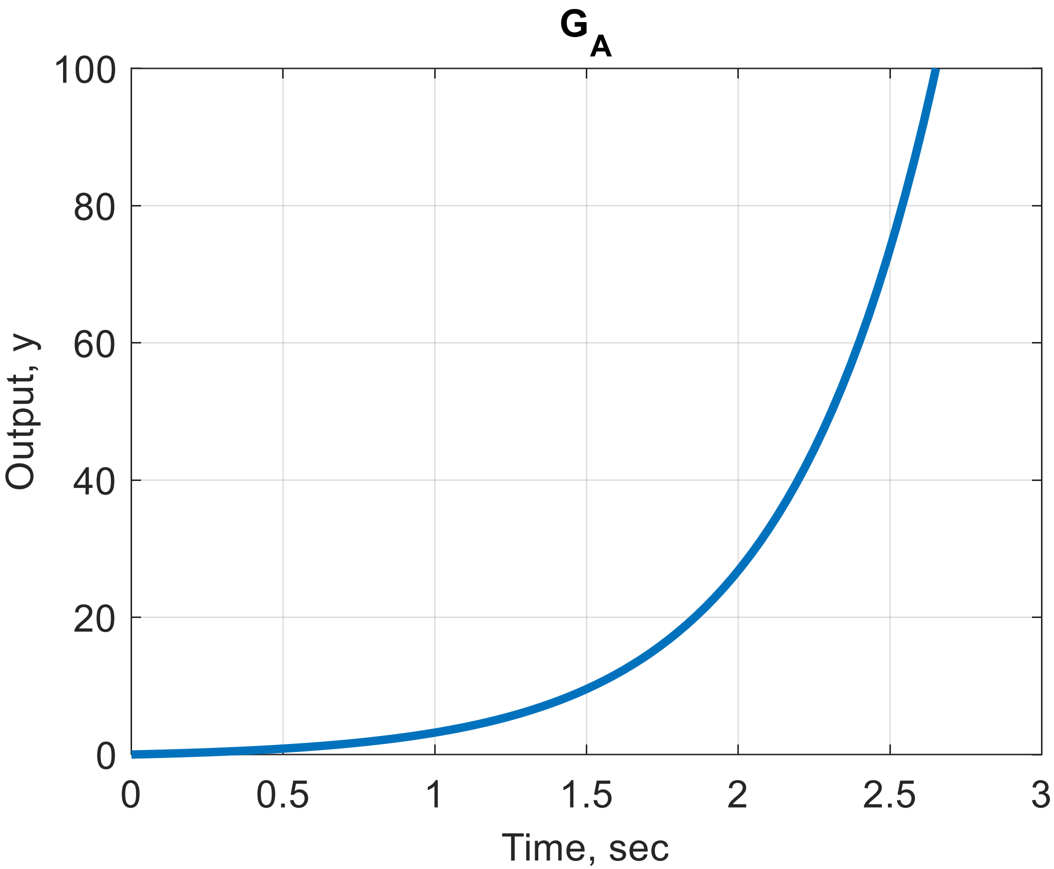






Solution 1A

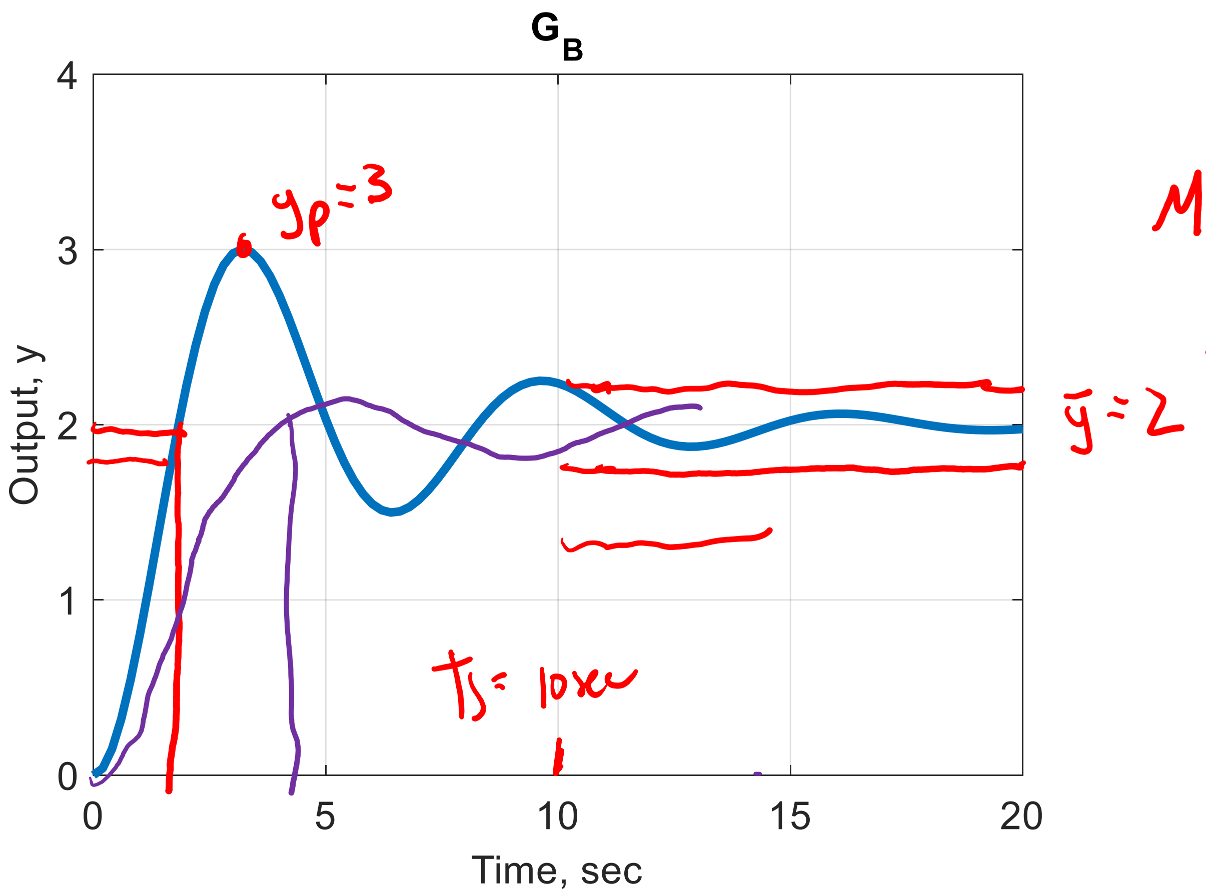
- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



Unstable

Solution 1B

- Is the system stable?
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



Stable

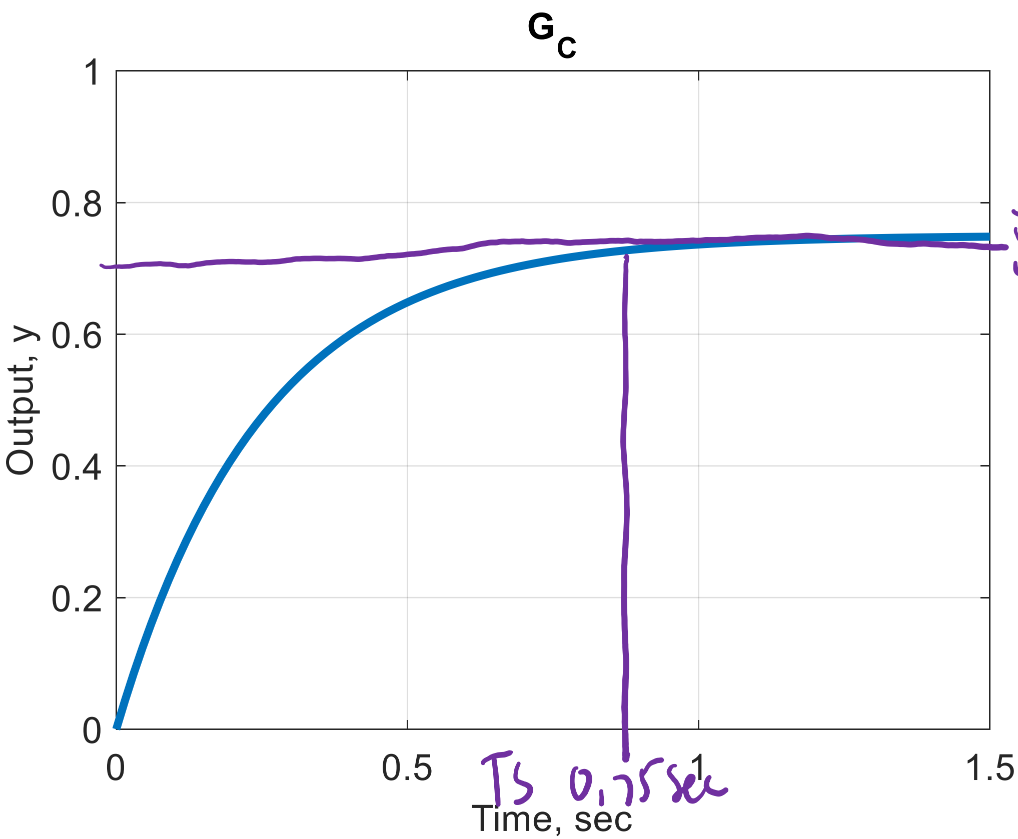
$$M_p = \frac{y_p - \bar{y}}{\bar{y}} = \frac{3-2}{2} = 0.5$$

$100 \times M_p = 50\%$

No undershoot

Solution 1C

- Is the system stable? Yes
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?



Ts 0,75 sec

$\bar{y} = 0.75$

No overshoot

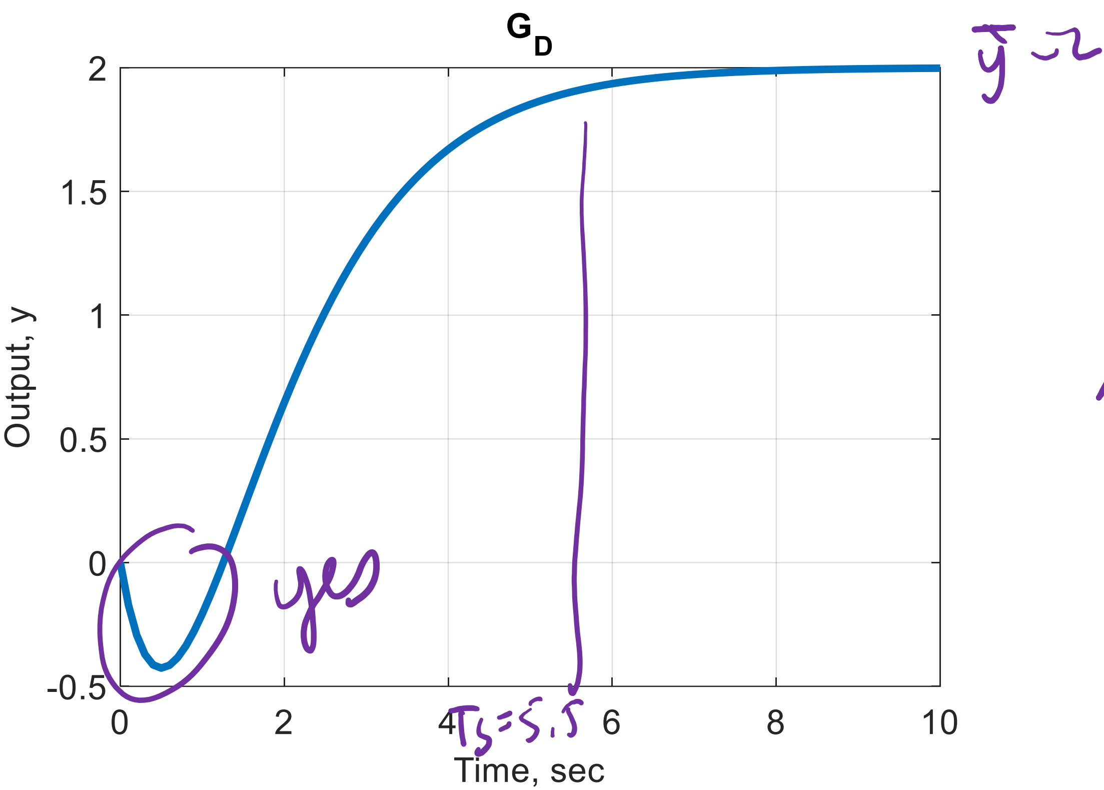
No undershoot

Tr: first time to reach $0.95\,\bar{y}$

Solution 1D

- Is the system stable? Yes
- If the response is stable: What is the final value, settling time, overshoot, rise time? Does the response have undershoot?

no   not Defined



BIBO Stable

All bounded u

$\Rightarrow$ bounded y

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Lecture 4B: First-Order Step Response

Problem 2

A) Roughly sketch the response for the following:

$$\dot{y}(t) + 2y(t) = 4u(t)$$
$$\text{with } y(0) = 0 \text{ and } u(t) = 3 \text{ for all } t \geq 0$$

B) Roughly sketch the response for the following

$$\dot{y}(t) - 3y(t) = 2u(t)$$
$$\text{with } y(0) = 0 \text{ and } u(t) = 1 \text{ for all } t \geq 0$$

Solution 2A

A) Roughly sketch the response for the following:

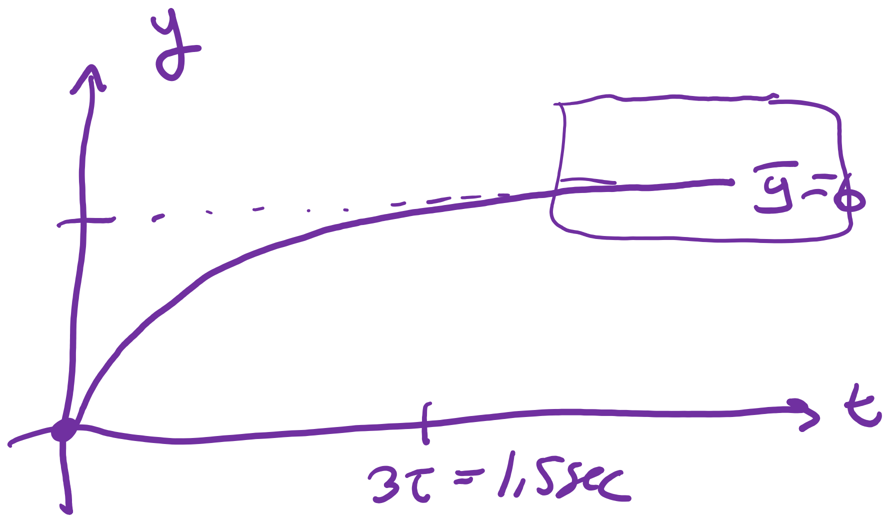
$$\dot{y}(t) + 2y(t) = 4u(t)$$

≈ 0

with $y(0) = 0$ and $u(t) = 3$ for all $t \geq 0$

$G(s) = \frac{4}{s+2}$ $\quad G(0) = \frac{4}{2} = 2$

$s+2=0 \rightarrow s=-2$ $\quad \tau = \frac{1}{2}$ sec



$\frac{dy}{dt} \approx 0$

$2\bar{y} = 4 \cdot 3$

$\bar{y} = \underbrace{\frac{4}{2}}\cdot 3 = 6$

$G(0) = \frac{4}{2}$

Solution 2B

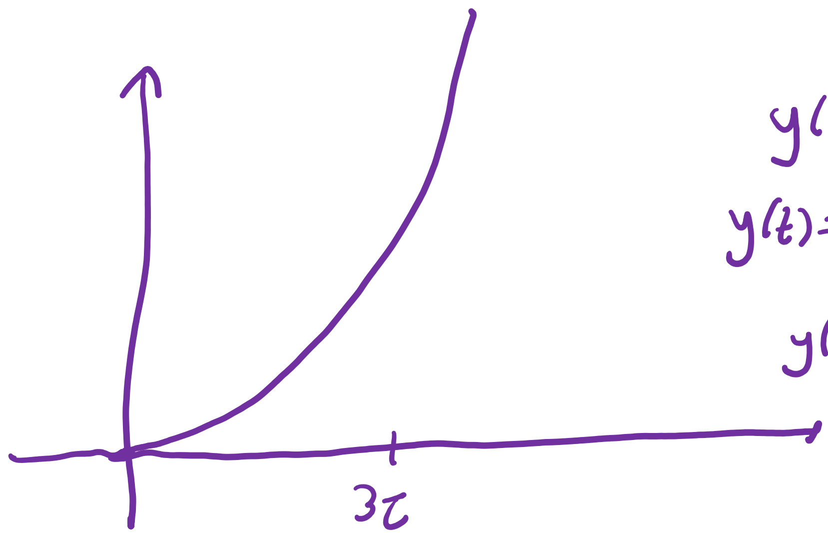
B) Roughly sketch the response for the following

$$\dot{y}(t) - 3y(t) = 2u(t)$$

with $y(0) = 0$ and $u(t) = 1$ for all $t \geq 0$

$G(s) = \frac{2}{s-3}$

$s = +3$ $\quad$ $\tau = \frac{1}{3}$ sec



$y(t) = y_p + ce^{3t}$

$y(t) = -\frac{2}{3} + \frac{2}{3}e^{3t}$

$y(t) = \frac{2}{3}(-1 + e^{3t})$

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Lecture 4C: Second-Order Step Response

Problem 3

Each of the second-order systems below is stable*

For each system:

- What is the natural frequency and damping ratio?
- Is the system under, over, or critically damped?
- Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

$$G_A(s) = \frac{20}{s^2+2s+10} \qquad G_B(s) = \frac{20}{s^2+11s+10}$$

*Recall that $s^2 + a_1 s + a_0 = 0$ has all poles in the LHP if and only if $a_1>0$ and $a_0>0$.

Solution 3A

- What is the natural frequency and damping ratio?
- Is the system under, over, or critically damped?
- Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

$$G_A(s) = \frac{20}{s^2+2s+10}$$

$s^2 + 2\zeta\omega_n + \omega_n^2 = 0$

① $2 = 2\zeta\omega_n \rightarrow \zeta = \frac{2}{2\omega_n} = \frac{1}{\sqrt{10}}$ (unitless) ~ 0.316

$10 = \omega_n^2 \rightarrow \omega_n = \sqrt{10}$ rad/sec ≈ 3.16 rad/sec

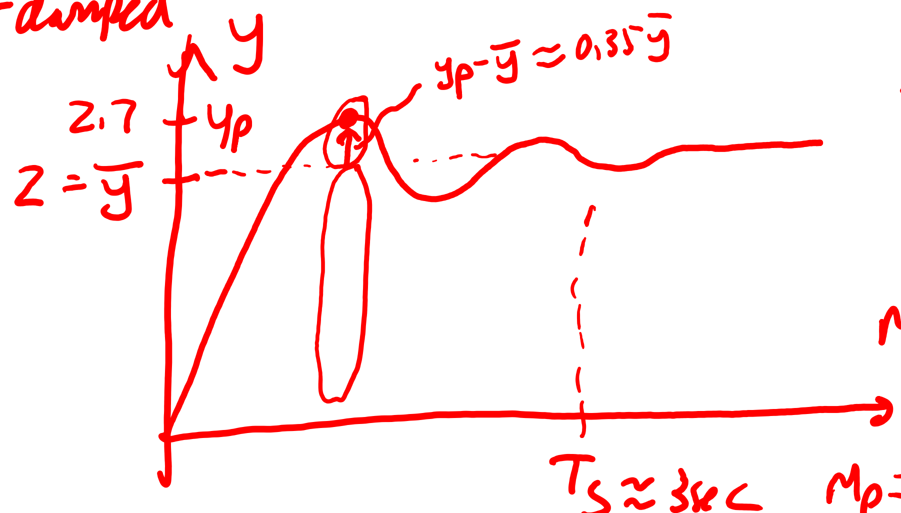
② $\zeta < 1 \rightarrow$ underdamped

$\ddot{y} + 2\dot{y} + 10y = 20u$

$10\bar{y} = 20 \cdot 1$

$\bar{y} = 2$

$\bar{y} = G(0)\bar{u}$



Real $= -\zeta\omega_n$

$s = -1 \pm 3j$

$\tau = 1$ sec

$T_s \approx 3$sec

$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

$= 0.35$

$M_p = \frac{y_p - \bar{y}}{\bar{y}} \rightarrow y_p = 2.7$

Solution 3B

- What is the natural frequency and damping ratio?
- Is the system under, over, or critically damped?
- Roughly sketch the unit step response noting the final time, settling time, and overshoot (if underdamped).

$u \times 1$

$$G_B(s) = \frac{20}{s^2 + 11s + 10}$$

$\omega_n^2 = 10 \rightarrow \omega_n = \sqrt{10}$ rad/sec

$2\zeta\omega_n = 11 \rightarrow \zeta = \frac{11}{2\omega_n} = \frac{11}{2\sqrt{10}} = 1.74 > 1$ Overdamp

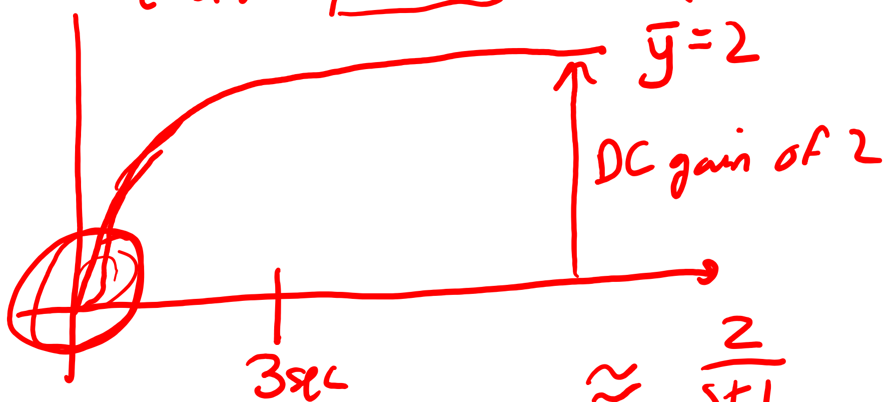
$s_1 = -10, -1$

$\tau = 0.1\,\mathrm{sec}$, $\boxed{\tau = 1\,\mathrm{sec}}$ Slow/Dominant Pole

$y = \bar{y} + \underbrace{c_1 e^{-t}}_{\text{Slow}} + \underbrace{c_2 e^{-10t}}_{\text{Fast}}$

$\tau = 1\,\mathrm{sec}$

$T_s = 3\,\mathrm{sec}$



$\approx \frac{2}{s+1}$

$\ddot{y} + 11\dot{y} + 10y = 20u$

$10\bar{y} = 20 \cdot 1$

$\bar{y} = 2$

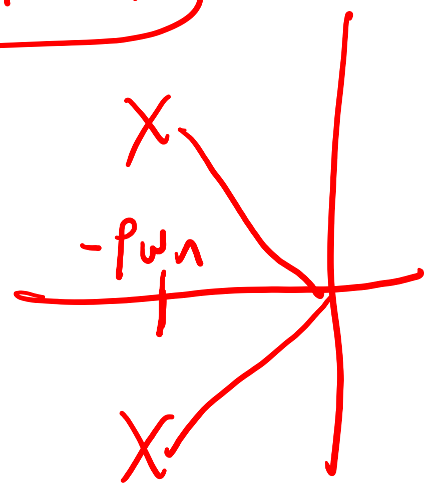
Solution 3-Extra Space

$$\boxed{T_s = \frac{3}{\zeta\omega_n}}$$

Underdamped

$$\tau = \frac{1}{\zeta\omega_n}$$

$$T_s = 3/\zeta\omega_n$$



Overdamped