ECE 486: Control Systems

- Lecture 17C: lead/lag control

Goal: introduce the use of lead and lag dynamic compensators

Reading: FPE, Chapter 5

Approximate PD Using Dynamic Compensation

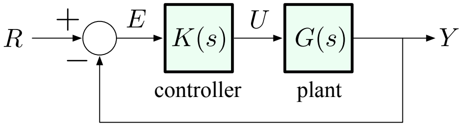
Reminder: we can approximate the D-controller $K_D s$ by

$$K_D \frac{ps}{s+p} \longrightarrow K_D s \text{ as } p \to \infty$$

— here, $-p$ is the *pole* of the controller.

So, we replace the PD controller $K_P + K_D s$ by

$$K(s) = K_P + K_D \frac{ps}{s+p}$$



Closed-loop poles: $1 + \left(K_P + K_D \frac{ps}{s+p} \right) G(s) = 0$

Lead & Lag Compensators

Consider a general controller of the form

$$K\frac{s+z}{s+p} \qquad \text{— } K, z, p > 0 \text{ are design parameters}$$

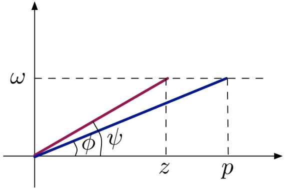
Depending on the relative values of z and p, we call it:

- a lead compensator when $z < p$
- a lag compensator when $z > p$

Why the name "lead/lag?" — think frequency response

$$\angle\frac{j\omega+z}{j\omega+p} = \angle(j\omega+z) - \angle(j\omega+p) = \psi - \phi$$

- if $z < p$, then $\psi - \phi > 0$ (phase lead)
- if $z > p$, then $\psi - \phi < 0$ (phase lag)



Summary on Design Trade-offs

Some deign trade-offs for the lead control:

- p large — good damping, but bad noise suppression (too close to PD)
- p small — noise suppression is better, but worse tracking performance
- intermediate values of p — how to set the control gains?

We will use the Bode plot to do the design.

Lead Compensation: Bode Plot

$$KD(s) = K\frac{s+z}{s+p}, \qquad p \gg z$$

In Bode form:

$$KD(s) = \frac{Kz\left(\frac{s}{z}+1\right)}{p\left(\frac{s}{p}+1\right)}$$

or, absorbing z/p into the overall gain, we have

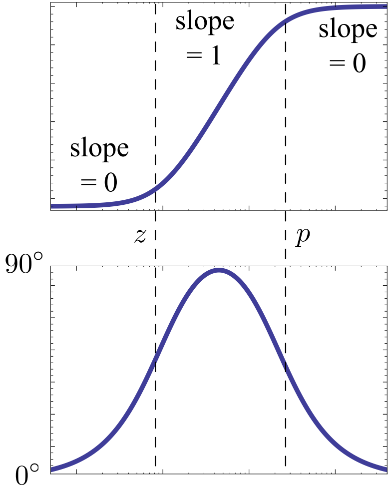
$$KD(s) = \frac{K\left(\frac{s}{z}+1\right)}{\left(\frac{s}{p}+1\right)}$$

Break-points:

- Type 1 zero with break-point at $\omega = z$ (comes first, $z \ll p$)
- Type 1 pole with break-point at $\omega = p$

Lead Compensation: Bode Plot

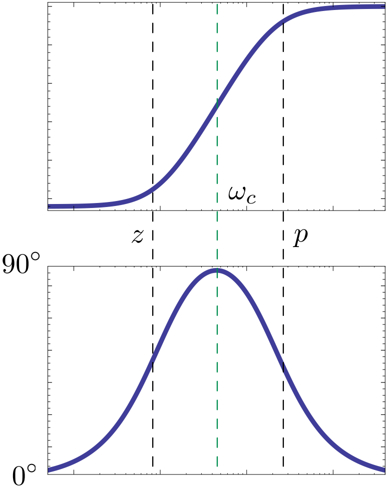
$$KD(s) = \frac{K\left(\frac{s}{z}+1\right)}{\left(\frac{s}{p}+1\right)}$$



- magnitude levels off at high frequencies $\Longrightarrow$ better noise suppression
- adds phase, hence the term "phase lead"

Lead Compensation and Phase Margin

$$KD(s) = \frac{K\left(\frac{s}{z}+1\right)}{\left(\frac{s}{p}+1\right)}$$



For best effect on PM, ω_c should be halfway between z and p (on log scale):

$$\log \omega_c = \frac{\log z + \log p}{2}$$

$$\text{or } \omega_c = \sqrt{z \cdot p}$$

— geometric mean of z and p

Trade-offs: large $p - z$ means

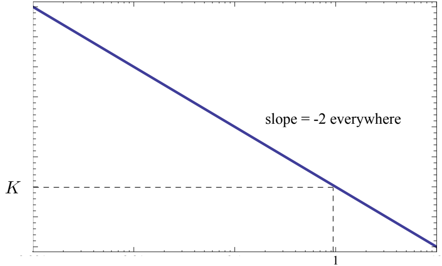
- large PM (closer to 90°)
- but also bigger M at higher frequencies (worse noise suppression)

Back to Our Example: $G(s) = \frac{1}{s^2}$

Objectives (same as before):

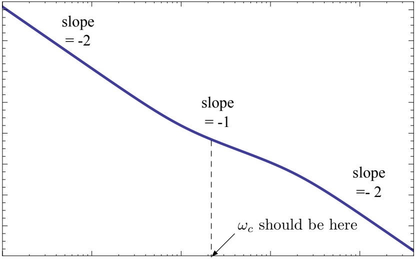
- stability
- good damping
- ω_{BW} close to 0.5

$KG(s) = \frac{K}{s^2}$ (w/o lead):



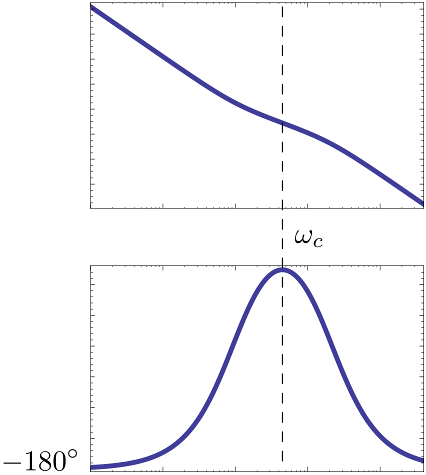
$$\frac{K}{(0.5)^2} = 1 \implies K = \frac{1}{4}$$

after adding lead:



— adding lead will increase ω_c!!

Back to Our Example: $G(s) = \frac{1}{s^2}$



After adding lead with $K = 1/4$, what do we see?

- adding lead increases ω_c
- $\Longrightarrow$ PM $< 90°$
- $\Longrightarrow$ $\omega_{\rm BW}$ may be $> \omega_c$

To be on the safe side, we choose a *new value* of K so that

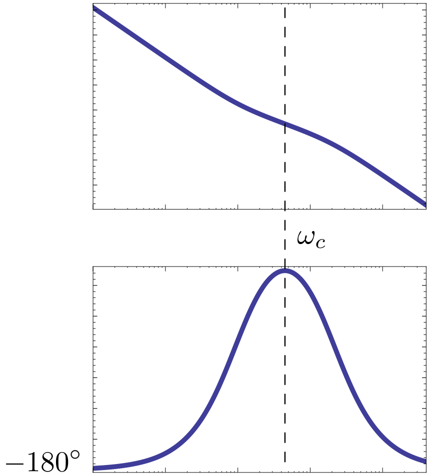
$$\omega_c = \frac{\omega_{\rm BW}}{2}$$

(b/c generally $\omega_c \le \omega_{\rm BW} \le 2\omega_c$)

Thus, we want

$$\omega_c = 0.25 \implies K = \frac{1}{16}$$

Back to Our Example: $G(s) = \frac{1}{s^2}$



Next, we pick z and p so that ω_c is approximately their geometric mean:

e.g., $z = 0.1$, $p = 2$

$$\sqrt{z \cdot p} = \sqrt{0.2} \approx 0.447$$

Resulting lead controller:

$$KD(s) = \frac{1}{16} \frac{\frac{s}{0.1} + 1}{\frac{s}{2} + 1}$$

(may still need to be refined using Matlab)

Lead Controller Design Using Frequency Response

General Procedure

1. Choose K to get desired bandwidth spec w/o lead
2. Choose lead zero and pole to get desired PM
   - in general, we should first check PM with the K from 1, w/o lead, to see how much more PM we need
3. Check design and iterate until specs are met.

This is an intuitive procedure, but it's not very precise, requires trial & error.