ECE 486: Control Systems

Lecture 14C: Introduction to Bode Plots for Higher-Order Systems

Key Takeaways

Consider a system whose transfer function is $G(s) = G_1(s)G_2(s)$.

- The Bode phase plot of $G(s)$ is the sum of the phase plots of $G_1(s)$ and $G_2(s)$.
- The Bode magnitude plot of $G(s)$ (in dB) is the sum of the magnitude plots of $G_1(s)$ and $G_2(s)$.

This can be used to draw Bode plots for higher order systems.

Products of Transfer Functions

- Consider a system whose transfer function is $G(s) = G_1(s)G_2(s)$.
- The response of $G(s)$ at frequency ω is:

$$G(j\omega) = G_1(j\omega)\,G_2(j\omega) = |G_1(j\omega)|e^{j\angle G_1(j\omega)}\,|G_2(j\omega)|e^{j\angle G_2(j\omega)}$$

$$\Rightarrow |G(j\omega)| = |G_1(j\omega)| \cdot |G_2(j\omega)| \qquad \boxed{\angle G(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)}$$

- Next recall that for any real numbers c_1 and c_2:

$$\log_{10}(c_1 c_2) = \log_{10}(c_1) + \log_{10}(c_2)$$

- Thus the magnitude of $G(j\omega)$ in dB is given by:

$$\boxed{|G(j\omega)|_{dB} = |G_1(j\omega)|_{dB} + |G_2(j\omega)|_{dB}}$$

The Bode phase plot of G(s) is the sum of the phase plots of $G_1(s)$ and $G_2(s)$. The Bode magnitude plot of $G(s)$ (in dB) is the sum of the magnitude plots of $G_1(s)$ and $G_2(s)$.

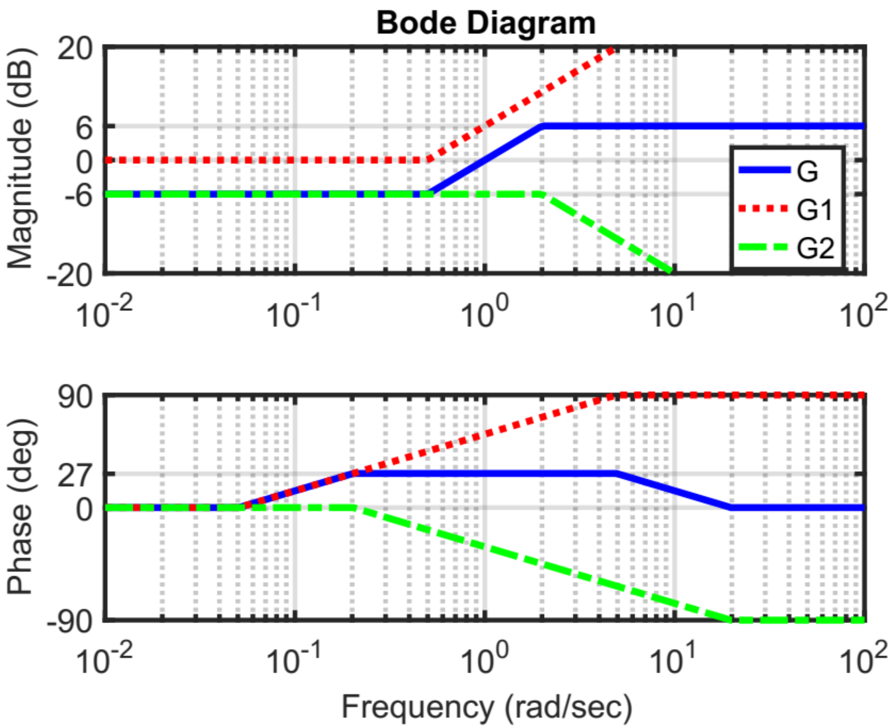
Example: Lead Controller

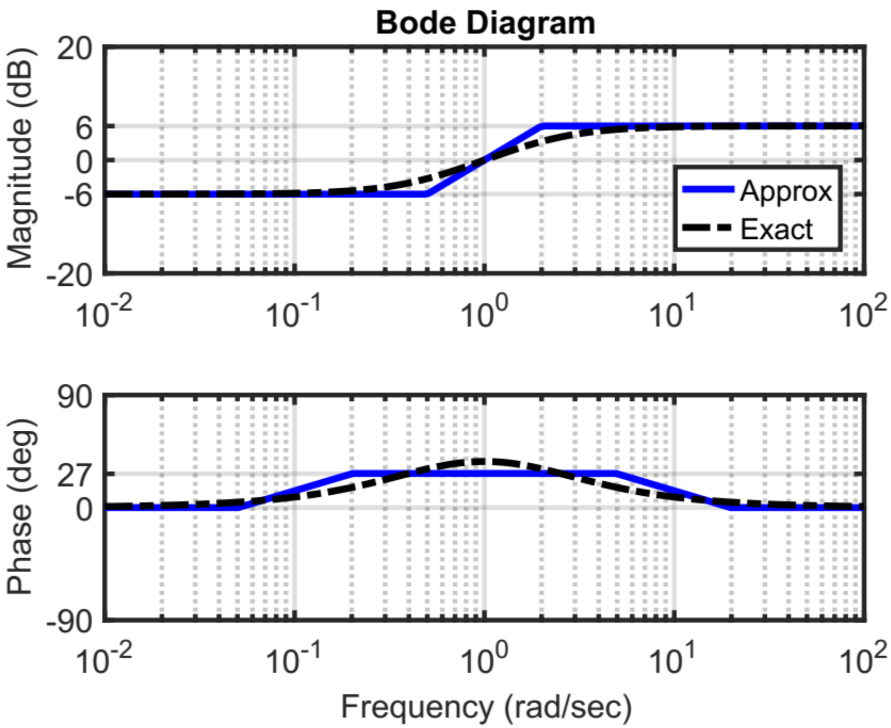
- Consider the first-order system:

$$\dot{u}(t) + 2u(t) = 2\dot{e}(t) + e(t) \qquad G(s) = \frac{2s+1}{s+2}$$

- Express transfer function as a product:

$$G(s) = G_1(s)G_2(s) \text{ where } G_1(s) = 2s+1 \text{ and } G_2(s) = \frac{1}{s+2}.$$





Example: Overdamped Second-Order System

- Consider the first-order system:

$$\ddot{y}(t) + 1.2\dot{y}(t) + 0.2y(t) = 0.5u(t) \qquad G(s) = \frac{0.5}{s^2+1.2s+0.2}$$

- Express transfer function as a product:

$$G(s) = G_1(s)G_2(s) \text{ where } G_1(s) = \frac{1}{s+0.2} \text{ and } G_2(s) = \frac{0.5}{s+1}.$$