ECE 486: Control Systems

Lecture 10B: Integrator Anti-windup

Key Takeaways

This lecture describes impact of actuator saturation and rate limits. These limits:

- Cause slower speed of response and
- Can lead to overshoot and oscillations if the controller does not properly account for the limits.

Anti-windup is one method to reduce the effect of saturation.

- It will prevent overshoot and oscillations.
- However, it does not change the slower speed of response which is a physical limit of the actuators.

Actuator Saturation and Rate Limits

- Actuators often have saturation limits on input values
  - A DC motor might be limited to $u \in [0,3]V$.
  - The steering angle on a car might be limited to $u \in [-30^o, +30^o]$
- Actuators often have rate limits
  - Aircraft have movable surfaces on the wings (e.g. ailerons) to control the motion.
  - These might be limited to $\dot{u} \in [-130^o/sec, +130^o/sec]$
- Effects of saturation and rate limits:
  - The speed of response is slower. The system must be re-designed if desired performance cannot be achieved due to these limits.
  - Large overshoot and oscillations can occur because the control algorithm does not account for these limits.

Anti-windup is one solution to reduce the effect of saturation.

Closed-Loop With Saturation

DC Motor: $G(s) = \frac{b_0}{s+a_0}$ where $a_0 = 0.94 \frac{1}{sec}$ and $b_0 = 766.8 \frac{rad}{sec^2\, V}$

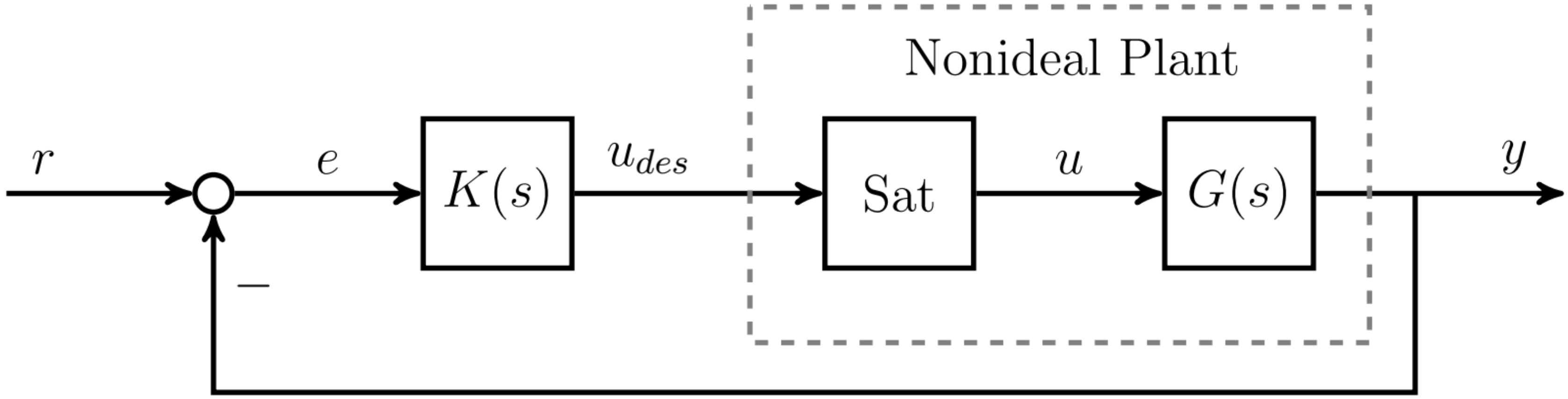
PI Control: Choose gains to place poles with $(\zeta, \omega_n) = (0.7, 10\frac{rad}{sec})$

$$\Rightarrow \quad K_p = \frac{2\zeta\omega_n - a_0}{b_0} = 0.017 \quad \text{and} \quad K_i = \frac{\omega_n^2}{b_0} = 0.13$$

This design assumes the motor input voltage is unlimited.

Consider the effect of saturation $u(t) \in [u_{min}, u_{max}]V$

$$u(t) = \begin{cases} u_{min} & \text{if } u_{des}(t) \leq u_{min} \\ u_{des}(t) & \text{if } u_{min} < u_{des}(t) < u_{max} \\ u_{max} & \text{if } u_{des}(t) \geq u_{max} \end{cases}$$

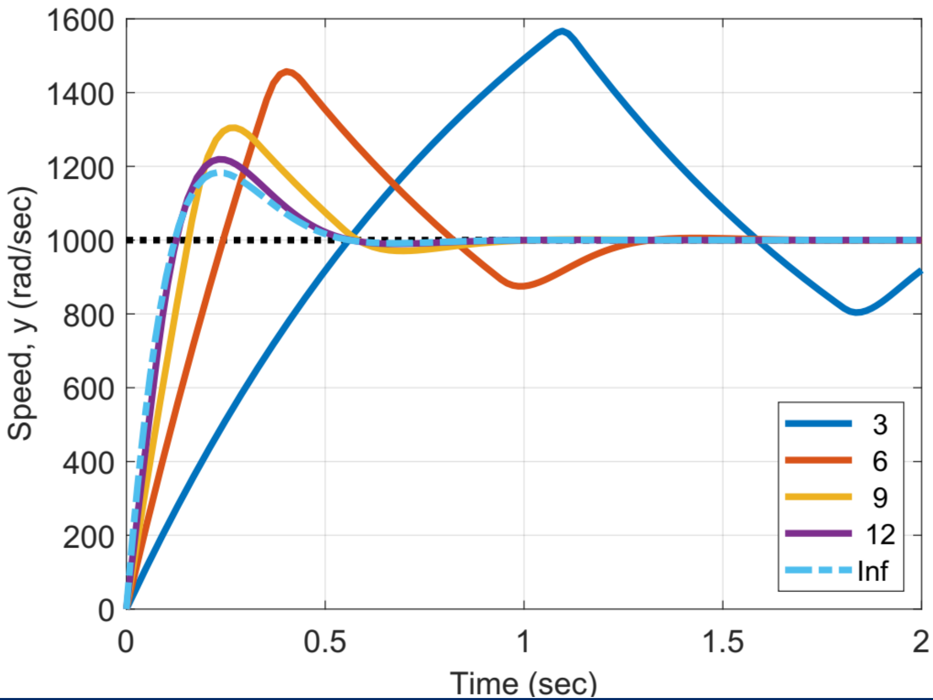


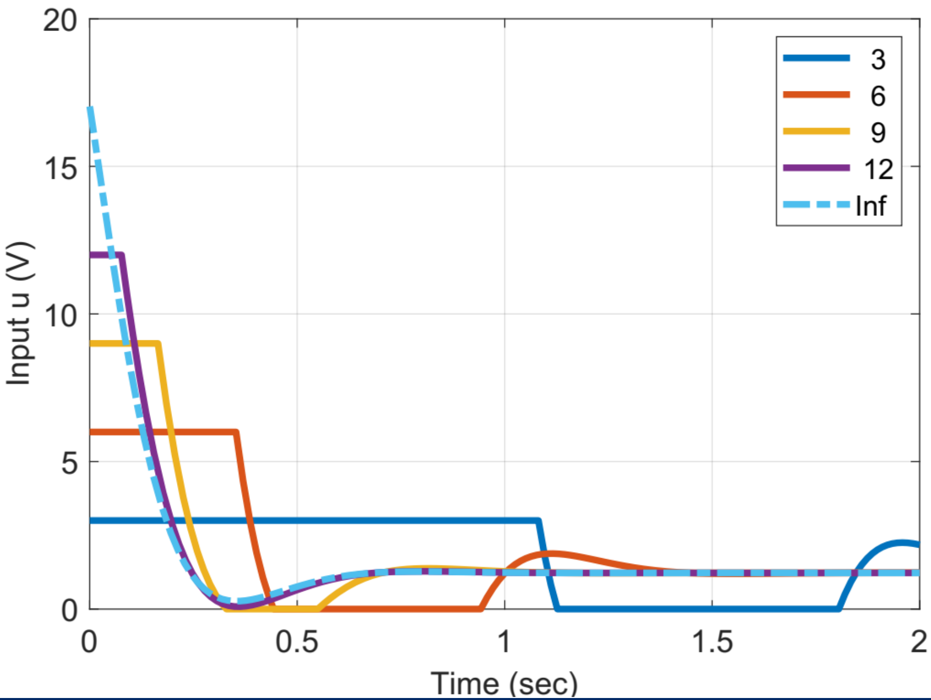
Degraded Performance Due to Saturation

Simulate step responses with

- $r(t) = 1000\frac{rad}{sec}$ for $t \geq 0$
- $u_{min} = 0V$ and several values of u_{max}

Saturation causes both slower response times and larger overshoots / oscillations



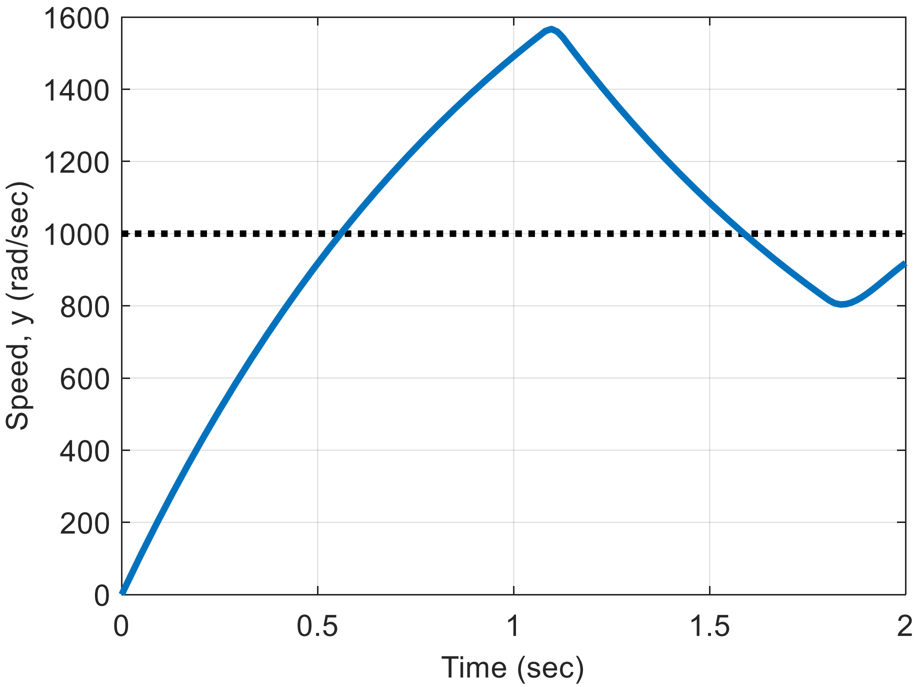


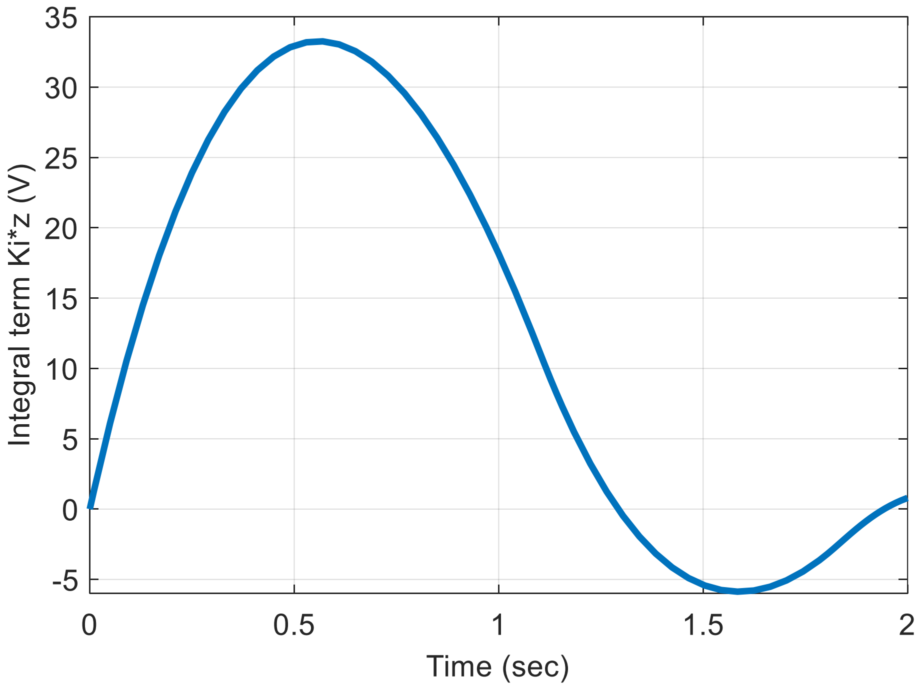
Integrator Windup

PI Controller:

$$u_{des}(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau$$

Simulation below uses $u_{max} = 3V$. The integrator term grows to large value even while input is saturated.





Anti-Windup Protection

Express PI controller as:

$$\dot{z}(t) = e(t)$$
$$u_{des}(t) = K_p e(t) + K_i z(t)$$

Stop the integrator in certain situations when u_{des} is at the saturation limit. If $K_i > 0$:

$$\dot{z}(t) = \begin{cases} 0 & \text{if } u_{des}(t) \geq u_{max} \text{ and } e(t) \geq 0 \\ 0 & \text{if } u_{des}(t) \leq u_{min} \text{ and } e(t) \leq 0 \\ e(t) & \text{otherwise} \end{cases}$$

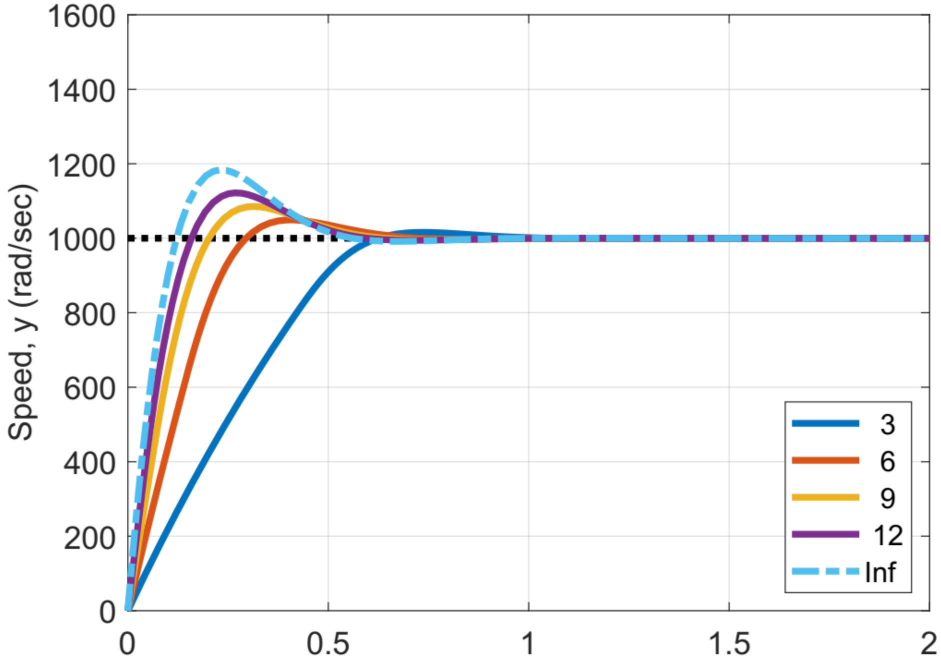
This is called conditional integration or clamping.

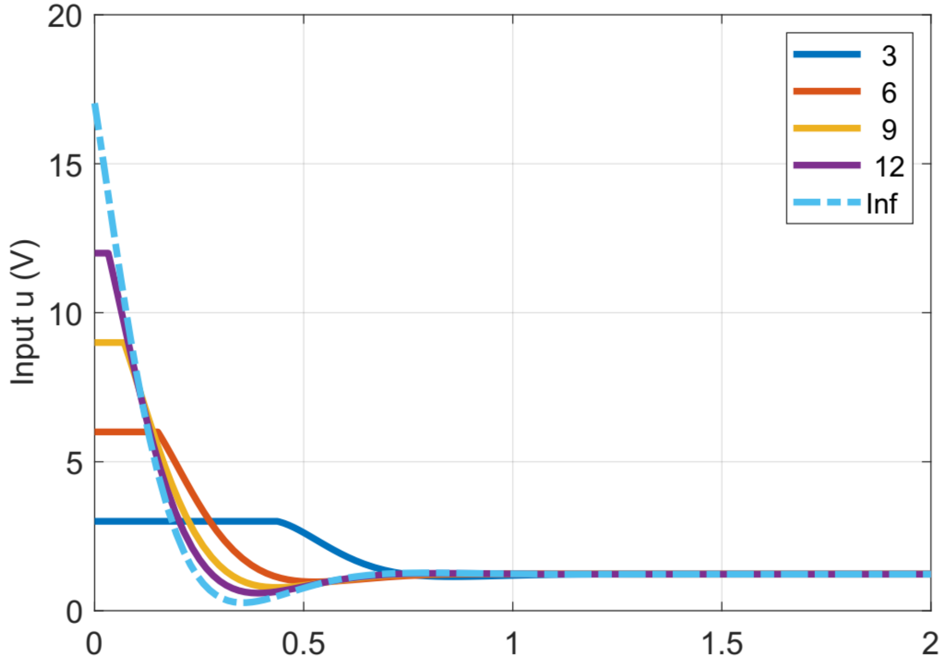
Anti-Windup Protection

Simulate step responses with

- $r(t) = 1000\frac{rad}{sec}$ for $t \geq 0$
- $u_{min} = 0V$ and several values of u_{max}

Anti-windup prevents large overshoots / oscillations. It does not prevent the slower response (which is a physical limit).





Anti-Windup Protection

Simulate step responses with

- $r(t) = 1000\frac{rad}{sec}$ for $t \geq 0$
- $u_{min} = 0V$ and $u_{max} = 3V$
- With and without anti-windup

Anti-windup prevents integrator from growing to large values.