

# ECE 486 (Control Systems) – Homework 9

**Due:** November 23

**Problem 1** (20 points). Suppose we have designed a controller  $K$  for a plant  $G$ . After simulations, we think the response speed for a unit step reference signal is too slow. In this case, shall we decrease or increase the crossover frequency of the open loop transfer function? What issues will there be if we have a very large crossover frequency?

**Problem 2.** DC motors are used in many electro-mechanical systems. A model for a DC motor is:

$$J\dot{y} + by = cV \quad (1)$$

where  $y$  is the angular velocity of the motor shaft (deg/sec), and  $V$  is the input voltage (Volts). The model parameters are:  $J$ =Rotational Inertia (N m sec<sup>2</sup>/deg<sup>2</sup>),  $b$ =rotational damping (Nm sec/deg), and  $c$ =gain from input voltage to applied torque (Nm/Volts). For this problem we'll use the values  $J = 3$ ,  $b = 5$  and  $c = 12$ .

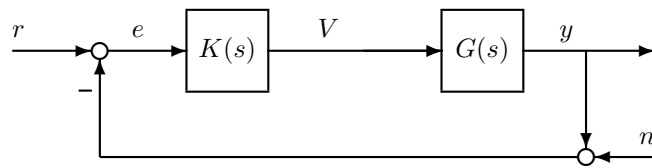


Figure 1: DC Motor Feedback System

Consider the feedback loop in the figure above where  $G(s)$  is the transfer function for the DC motor and  $K(s)$  is the controller transfer function. The specifications are to design a controller so that the closed loop:

- is stable,
- has a loop crossover frequency near 20 rad/sec
- has gain less than 0.01 from reference  $r$  to error  $e$  for frequencies below 0.1 rad/sec.
- has gain less than 0.04 from input  $n$  to output  $y$  for frequencies above 200 rad/sec.

**Tasks.** In this problem, you are asked to perform the following steps:

- (10 points) Translate specifications iii. and iv. into requirements on the loop gain  $|L(j\omega)|$ .
- (15 points) Design a proportional control law  $K(s) = K_p$  to satisfy requirements i. and iii. only. Plot  $G(s)$  and  $L(s) = G(s)K(s)$  on the same Bode plot using the `Matlab` command `bode`. Also, simulate the closed-loop system with  $r(t) = 1$  deg/sec and  $n(t) = 0.1 \sin(200t)$ . Plot the motor speed response  $y(t)$  and the reference command  $r(t)$  on the same plot. Hand in both your Bode plot and your step response plot. For your choice of  $K_p$ , what is the loop cross-over frequency and what is the value of  $|L(j200)|$ ?

**Note:** An m-file and `Simulink` diagram have been posted with this homework. You only need to enter your proportional gain and the m-file will generate both the Bode plot and the step response plot.

- (15 points) Next, design a proportional control law  $K(s) = K_p$  to satisfy requirements i. and iv. only. Generate and hand-in the Bode plot and step response plots as described in the part (b). For your choice of  $K_p$ , what is the loop cross-over frequency and what is the value of  $|L(j0)|$ ?

- (d) (30 points) At this point it should be clear that proportional control will not be able to satisfy the design objectives. Use the loop-shaping procedure described in class to design a controller that satisfies objectives ii., iii., and iv. Verify that the closed-loop is stable with your final control design. Generate and hand-in the Bode plot and step response plots as described in the part (b). Your controller will be specified as a transfer function and you'll need to modify the m-file and **Simulink** diagram.

**Recommendation:** Use a proportional gain to set the loop cross-over frequency (requirement ii.). Then use an integral boost and roll-off to satisfy requirements iii. and iv., respectively. This may require some iteration to get values that meet all design specifications.

- (e) (10 points) What is the ordinary differential equation that models the input-output dynamics of the control law designed in part (d)?