ECE 486 (Control Systems) – Homework 6

Due: October 21

Problem 1. (20 points) Answer the following questions.

(a) Consider the following system $G(s)$ and sinusoidal input:

$$-3\dot{y}(t) - 2y(t) = 7u(t)$$
$$u(t) = 6\cos(t+4)$$

What is the magnitude and phase of $G(1j)$? Is the steady-state output bounded? If yes, what is it?

(b) Consider the following system $G(s)$ and sinusoidal input:

$$\ddot{y}(t) + 0.1\dot{y}(t) + 4y(t) = \dot{u}(t) + 2u(t)$$
$$u(t) = -\cos(2t)$$

What is the magnitude and phase of $G(2j)$? Is the steady-state output bounded? If yes, what is it?

Problem 2. (15 points) Figure 1 shows an input $u(t)$ and the corresponding output $y(t)$ generated by a linear system $G(s)$. The input has the form $u(t) = A_0\cos(\omega_0 t)$.

1. What are the values of A_0 and ω_0 for the input signal?
2. What is the magnitude $|G(j\omega_0)|$?
3. What is the phase $\angle G(j\omega_0)$ in degrees?

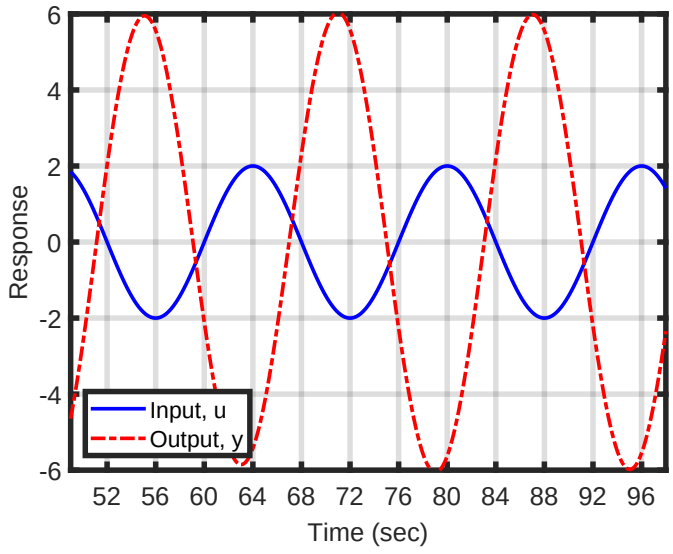

Figure 1: Input $u(t)$ and output response $y(t)$ for system $G(s)$.

Problem 3. (40 points) Sketch the Bode plots by hand for the following systems:

(a) A PI controller with input $e(t)$ and output $u(t)$:

$$\dot{u}(t) = K_p\dot{e}(t) + K_i e(t)$$

with $K_p = 10$ and $K_i = 1$.

(b) A "low frequency boost" controller with input $e(t)$ and output $u(t)$:

$$\dot{u}(t) + u(t) = \dot{e}(t) + 10e(t)$$

This type of controller will be encountered later in the course.

(c) A first-order system with right-half plane zero with input $u(t)$ and output $y(t)$:

$$2\dot{y}(t) + 0.6y(t) = -\dot{u}(t) + 30u(t)$$

(d) A second-order underdamped system with input $u(t)$ and output $y(t)$:

$$\ddot{y}(t) + 0.2\dot{y}(t) + 4.01y(t) = -u(t)$$

After you're done, check your results using `MATLAB`. Turn in both the hand sketches and the `MATLAB` plots.

Problem 4. Now we consider Bode plots for higher-order systems.

(a) (10 points) Consider a feedback loop with the following plant $G(s)$ and PI controller $K(s)$:

$$\dot{y}(t) + 2y(t) = 3u(t)$$
$$u(t) = 10e(t) + 2\int_0^t e(\tau)\,d\tau$$

Sketch the Bode plots of: $G(s)$, $K(s)$, and the product $G(s)K(s)$. What are some of the differences between the Bode plots of $G(s)$ and $G(s)K(s)$?

(b) (15 points) Consider a feedback loop with the following plant $G(s)$ and PD controller $K(s)$:

$$\ddot{y}(t) + 0.4\dot{y}(t) + 4y(t) = 8u(t)$$
$$u(t) = e(t) + 0.5\dot{e}(t)$$

Sketch the Bode plots of: $G(s)$, $K(s)$, and the product $G(s)K(s)$. What are some of the differences between the Bode plots of $G(s)$ and $G(s)K(s)$?

You may check your answers using `Matlab`. However you should turn in sketches of each Bode plot by hand (not computer generated plots).