ECE 486 (Control Systems) – Homework 11

Due: December 8

Problem 1. Consider the single-input, single-output transfer function:

$$Y(s) = \frac{s+1}{s^2+2s+2}U(s)$$

(a) Find a second-order state-space model that represents this transfer function.

(b) For this state-space model, calculate a state-feedback controller $u = -Kx + r$ that places the closed-loop poles at -4 and -25.

(c) Construct a stable observer to estimate x based on the known inputs u and observations y. You may use MATLAB for this part.

(d) With the controller and observer from the previous problems in place, calculate k_r such that $u = -K\hat{x} + k_r r$ yields a closed-loop system Y/R with unity gain. You may use MATLAB.

(e) Plot the step response using MATLAB.

Problem 2. Consider the single-input, single-output transfer function:

$$G_p(s) = \frac{1-s/2}{1+s/2}\frac{1}{s^2}$$

(a) Find a third-order state-space model that represents this transfer function.

(b) For this state-space model, calculate a state-feedback controller $u = -Kx + r$ that places the closed-loop poles at -4, -13, and -25. You may use MATLAB to calculate this controller, but not to find the the state-space model.

(c) Construct a stable observer, and put this together to form a compensator of the form $U = -G_c Y + G_r R$. You may use MATLAB.

(d) Calculate the Nyquist plot of $G_c G_p$. You may use MATLAB to do so. Is the system stable? If so, calculate the gain and phase margins.

Problem 3. Consider the following nonlinear system

$$\dot{x} = -4x - 2u + u^3$$

(a) Find an equilibrium $(\bar{x}, \bar{u})$ with $\bar{u} = 2$.

(b) Linearize the dynamics around $(\bar{x}, \bar{u})$.