ECE 486 (Control Systems) – Homework 10

Due: December 1st

Problem 1 (25 points)**.** Calculate the transfer function for the following state-space model.

$$\dot{x} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 2 \end{bmatrix} x + \begin{bmatrix} 3 \end{bmatrix} u$$

Problem 2. (20 points) For the following transfer function, calculate the controllable canonical form (CCF) state-space model.

$$G(s) = \frac{1}{(s+2)(s^2+2s+5)}$$

Problem 3 (25 points)**.** Prove the following:

i) $(AB)^\top = B^\top A^\top$

ii) $(ABC)^\top = C^\top B^\top A^\top$

iii) $(A^\top)^{-1} = (A^{-1})^\top$

iv) $(I - TAT^{-1})^{-1} = T(I - A)^{-1}T^{-1}$

v) For any integer $k \geq 0$, $(TAT^{-1})^k = TA^kT^{-1}$

Here, $(\cdot)^\top$ denotes the transpose operator and $(\cdot)^{-1}$ denotes the inverse operator. Assume matrices are invertible whenever needed.

Recall that the definition of A^{-1} is the unique matrix such that $AA^{-1} = A^{-1}A = I$, where I is the identity matrix.

You may use previous parts to prove later parts, e.g. you may invoke Part (i) when proving Part (ii).

Problem 4 (30 points)**.** Determine whether or not the following systems are controllable. If they are controllable, put them in controllable canonical form.

i)

$$\dot{x} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 5 & 1 & 3 & 2 \\ 6 & 1 & 3 & 4 \\ 1 & 7 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \end{bmatrix} u$$

ii)

$$\dot{x} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u$$
$$y = \begin{bmatrix} 2 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} u$$