ECE 486 (Control Systems) – Homework 8

Due: November 4

Problem 1. (25 points) Suppose we have the open-loop transfer function $G(s) = \dfrac{K}{s(s+20)(s+85)}$, and we put it through unity feedback, i.e. the closed loop transfer function is $\dfrac{G(s)}{1+G(s)}$.

(a) Set the gain K so that the magnitude is 1 (0 dB) at $\omega = 1$. What value of K achieves this?

(b) We wish to achieve 15% overshoot in the transient response for a step input. What phase margin is required to achieve this?

(c) What frequency on the Bode phase diagram yields this phase margin?

(d) Find the adjusted gain necessary to produce the required phase margin.

(e) Using MATLAB, plot the step response of the compensated system. Did you meet the design specs?

Problem 2. (25 points) For a transfer function $G(s) = \dfrac{K}{s(s+20)(s+85)}$ in unity feedback, design a lag compensator to reduce the steady-state error for a ramp input by a factor of 10 while maintaining a 15% overshoot in the transient response for a step input.

(a) Determine the steady state error of the gain-adjusted system designed in the previous problem.

(b) Find a gain K to satisfy the steady-state specification and plot the Bode plot in MATLAB for this gain.

(c) Determine the phase margin to achieve the desired overshoot. Find the frequency where the phase margin is 10 degrees greater than this phase margin.

(d) Design a lag compensator to achieve a gain of 1 (0 dB) at this frequency. In particular, set a high-frequency asymptote so that the compensated system will have a gain of 1 at this frequency, set the low-frequency asymptote to be at 1 (0 dB), and then connect the two with a -1/decade line (-20 dB/decade line). Find the lag compensator that achieves this.

(e) Why do we increase the phase margin above the desired margin when designing a lag compensator? Did you meet the design specifications? Include relevant plots.

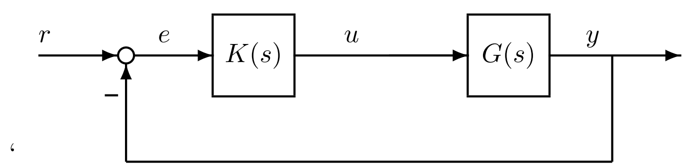

Figure 1: Feedback Loop

Problem 3. (10 points) Consider the loop in Figure 1 with $L(s) := G(s)K(s)$. Define two loop transfer functions:

(a) $L(s) = \dfrac{-8}{s+4}$ (b) $L(s) = \dfrac{2s+9}{s-4}$

Use the `nyquist` command in Matlab to generate the Nyquist plot for each loop transfer function. Then apply the Nyquist stability theorem to predict the number of closed-loop poles of the feedback system in the right-half plane.

Problem 4. (25 points) Consider the following:

$$G(s) = \frac{1}{(s+2)(s^2+2s+5)}$$

Suppose this is in unity feedback with a constant gain controller K. In other words, we have a negative feedback loop where the forward gain is $KG(s)$ and the loop gain is also $KG(s)$.

(a) Use the Routh-Hurwitz stability criterion to determine what values of K stabilize the closed-loop system.

(b) Sketch the Bode plots of $G(s)$ by hand.

(c) Verify your Bode plots using MATLAB.

(d) Using the Bode plot, sketch the Nyquist plot by hand. You should primarily use your sketch of the Bode plot, but you may use MATLAB to calculate exact numerical values as needed.

(e) Using the Nyquist plot, determine what values of K stabilize the closed-loop system. Does this match your answer from the Routh-Hurwitz criterion?

Note: In this problem, I have explicitly provided a step-by-step guide on how to use Nyquist plots to determine stability of a system. You are expected to be able to understand these principles and do this on your own, e.g. a problem on an exam may be "Use the Nyquist plots to determine the values of K which stabilize the closed-loop system." with no further guidance provided.

Problem 5. (15 points) Consider:

$$G(s) = \frac{1}{(s-1)(s+2)(s+4)} \qquad K = 10$$

(a) Sketch the Bode plot of $KG(s)$ by hand, and use the Bode plot to sketch the Nyquist plot.

(b) Use MATLAB to draw an exact Nyquist plot.

(c) Use this Nyquist plot to calculate the gain margin and phase margin.