ECE 486 (Control Systems) – Homework 7

Due: October 28

Problem 1. (40 points) For each of the transfer functions given below, draw the Bode plots (both magnitude and phase) by hand, using the techniques discussed in Lecture 15. Explain all steps in your drawing procedures.

(a) $L(s) = \dfrac{s+8}{s(s+4)}$

(b) $L(s) = \dfrac{8s}{s^2+0.2s+4}$

(c) $L(s) = \dfrac{s^2+0.2s+1}{s(s+0.2)(s+6)}$

(d) $L(s) = \dfrac{s+10}{s(s^2+1.4s+1)}$

After you're done, check your results using `MATLAB`. (Note that the bode command in `MATLAB` plots magnitude in decibels.) Turn in both the hand sketches and the `MATLAB` plots.

Problem 2. Consider the feedback diagram in Figure 1 below. Suppose $L(s) := G(s)K(s)$. Consider two pairs of plants and controllers:

i) $K(s) = \dfrac{10(s+3)}{s}$ and $G(s) = \dfrac{-0.5(s^2-2500)}{(s-3)(s^2+50s+1000)}$

ii) $K(s) = \dfrac{0.4s+1}{s}$ and $G(s) = \dfrac{1}{s+1}$

iii) $K(s) = 2$ and $G(s) = \dfrac{1}{(s+1)^3}$

Perform the following calculations for each plant/controller pair:

(a) (15 points) Verify that the feedback system is stable.

(b) (15 points) Use the Bode plot of $L(s)$ to compute the gain margin(s) of the feedback system.

(c) (15 points) Use the Bode plot of $L(s)$ to figure out the phase margins of the feedback system.

You can use the `allmargin` command to check your answers.

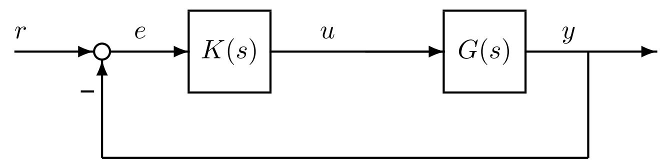

Figure 1: Feedback Loop

Problem 3. (15 points) Show that for the transfer function $KG(s) = \dfrac{\omega_n^2}{s^2+2\zeta\omega_n s}$, the phase margin is independent of ω_n and is given as $\tan^{-1}\left(\dfrac{2\zeta}{\sqrt{\sqrt{4\zeta^4+1}-2\zeta^2}}\right)$.