ECE 486: Control Systems

Lecture 6A: Effect of Extra Poles & Zeros
This lecture considers the effect of extra poles and zeros on the step response.

**LHP Poles**: Increase settling time.
The effects are small if the pole is far in the LHP.

**LHP Zeros**: Increase overshoot, decrease rise time, and have no effect on settling time.
The effects are small if the zero is far in the LHP.

**RHP Zeros**: Cause undershoot but no effect on settling time.
The effects are small if the zero is far in the RHP.
First-Order Step Response

Step Response:
\[ \dot{y}(t) + 1.5y(t) = 1.5u(t) \]
with \( y(0) = 0 \) and \( u(t) = 1 \) for \( t \geq 0 \)

1. Stable: \( s + 1.5 = 0 \) \( \Rightarrow \) \( s_1 = -1.5 < 0 \)

2. Time constant: \( \tau = \frac{1}{|s_1|} = \frac{1}{1.5} = \frac{2}{3} \text{ sec} \)

3. Settling time: \( 3\tau = 2 \text{ sec} \)

4. Final Value: \( \bar{y} = G(0)\bar{u} = 1 \)
First-Order Step Response

Step Response:

\[ \dot{y}(t) + 1.5y(t) = 1.5u(t) \]
with \( y(0) = 0 \) and \( u(t) = 1 \) for all \( t \geq 0 \)

Response:

(i) stable,
(ii) \( 3\tau = 2 \text{ sec} \),
(iii) \( \bar{y} = 1 \)

Matlab:

\[
\begin{align*}
\text{>> } & G = \text{tf}(1.5, [1 1.5]); \\
\text{>> } & [\text{yunit}, t] = \text{step}(G); \\
\text{>> } & \text{plot}(t, yunit);
\end{align*}
\]
Consider the second-order system:

\[
\ddot{y}(t) + (1.5 + p) \dot{y}(t) + (1.5p) y(t) = (1.5p) u(t)
\]

\[
G_p(s) = \frac{1.5p}{s^2 + (1.5+p)s + 1.5p} = \frac{p}{s+p} \cdot G(s) \quad \text{where} \quad G(s) = \frac{1.5}{s+1.5}
\]

\[
\begin{array}{c}
0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \\
0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1 \quad 1.2
\end{array}
\]

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Additional poles in the LHP increase the settling time.

The effect is small if the extra pole is far in the LHP (\( \approx 5 \times \) faster than slowest pole)
Second-Order Step Response

Step Response:
\[ \ddot{y}(t) + 2\dot{y}(t) + 4y(t) = 4u(t) \]
with \( y(0) = 0, \dot{y}(0) = 0 \) and \( u(t) = 1 \) for \( t \geq 0 \)

\[ G(s) = \frac{4}{s^2 + 2s + 4} \]

1. **Underdamped and Stable:**

\[ \omega_n = 2\frac{rad}{sec} \text{ and } \zeta = 0.5 \quad \Rightarrow \quad s_{1,2} = -1 \pm 1.73j \]

2. **Settling time:**

\[ T_s = 3\tau = \frac{3}{|Re(s_{1,2})|} = 3 sec \]

3. **Final Value:**

\[ \bar{y} = G(0)\bar{u} = 1 \]

4. **Peak Overshoot:**

\[ M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} \approx 0.16 \]

\[ \Rightarrow y(T_p) = (1 + M_p)\bar{y} \approx 1.16 \]
Second-Order Step Response

Step Response:

\[ \ddot{y}(t) + 2\dot{y}(t) + 4y(t) = 4u(t) \]

with \( y(0) = 0, \dot{y}(0) = 0 \) and \( u(t) = 1 \) for \( t \geq 0 \)

Response:

(i) stable, underdamped
(ii) \( 3\tau = 3\text{sec} \),
(iii) \( \bar{y} = 1 \)
(iv) \( y(T_p) \approx 1.16 \)

Matlab:

```matlab
>> G=tf(4,[1 2 4]);
>> [yunit,t]=step(G);
>> plot(t,yunit);
```
Effect of a Zero

Consider the second-order system:

\[
\ddot{y}(t) + 2\dot{y}(t) + 4y(t) = (4b) \dot{u}(t) + 4u(t)
\]

\[
G_b(s) = \frac{4b \, s + 4}{s^2 + 2s + 4} = (bs + 1) \cdot G(s) \quad \text{where} \quad G(s) = \frac{4}{s^2 + 2s + 4}
\]
Effect of a Zero

Consider the second-order system:

\[ \ddot{y}(t) + 2 \dot{y}(t) + 4y(t) = (4b) \dot{u}(t) + 4u(t) \]

\[ G_b(s) = \frac{4b s + 4}{s^2 + 2s + 4} = (bs + 1) \cdot G(s) \text{ where } G(s) = \frac{4}{s^2 + 2s + 4} \]

The zero is at:

\[ z = -\frac{1}{b} \]

\[ \Rightarrow y_b = y - \frac{1}{z} \dot{y} \]
Consider the second-order system:

\[
\ddot{y}(t) + 2\dot{y}(t) + 4y(t) = (4b) \dot{u}(t) + 4u(t)
\]

\[
G_b(s) = \frac{4bs + 4}{s^2 + 2s + 4} = (bs + 1) \cdot G(s) \quad \text{where} \quad G(s) = \frac{4}{s^2 + 2s + 4}
\]

\[
b = 0.5 \\
\Rightarrow z = -\frac{1}{b} = -2
\]
Effect of a LHP Zero

Consider the second-order system:

\[ \ddot{y}(t) + 2\dot{y}(t) + 4y(t) = (4b)\dot{u}(t) + 4u(t) \]

\[ G_b(s) = \frac{4bs + 4}{s^2 + 2s + 4} = (bs + 1) \cdot G(s) \text{ where } G(s) = \frac{4}{s^2 + 2s + 4} \]

A zero in the LHP:
- Increases overshoot
- Decreases rise time
- No effect on settling time

The effects are small if the zero is far in the LHP.
Effect of a RHP Zero

Consider the second-order system:

\[ \ddot{y}(t) + 2 \dot{y}(t) + 4y(t) = (4b) \dot{u}(t) + 4u(t) \]

\[ G_b(s) = \frac{4bs+4}{s^2+2s+4} = (bs + 1) \cdot G(s) \text{ where } G(s) = \frac{4}{s^2+2s+4} \]

A zero in the RHP:
- Causes undershoot
- No effect on settling time

The effects are small if the zero is far in the RHP.