

2025 april 30

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$$P(A=.) = \sigma(x) = \frac{1}{1+e^{-x}} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$$

$$P(B=.) = \sigma(y) = \frac{1}{1+e^{-y}} = \frac{1}{1+\frac{1}{3}} = \frac{3}{4}$$

$$P_A = \begin{bmatrix} 1-\sigma(x) \\ \sigma(x) \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix} \quad P_B = \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$$

$$\frac{\partial P_A}{\partial x} = \begin{bmatrix} -\sigma(x)(1-\sigma(x)) \\ \sigma(x)(1-\sigma(x)) \end{bmatrix} = \begin{bmatrix} -\frac{2}{9} \\ \frac{2}{9} \end{bmatrix}$$

$$\frac{\partial P_B}{\partial y} = \begin{bmatrix} -\frac{3}{16} \\ \frac{3}{16} \end{bmatrix}$$

$$\frac{\partial E(r_A)}{\partial x} = \frac{\partial P_A}{\partial x} R_A P_B$$

$$= \begin{bmatrix} -\frac{2}{9} & \frac{2}{9} \end{bmatrix} \begin{bmatrix} 8 & 4 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$$

$$= \frac{2}{36} [-1, 1] \begin{bmatrix} 8 & 4 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

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$$= \frac{1}{18} [-1, 1] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{18} (-3)$$

$$\begin{aligned} \frac{\partial E(r_B)}{\partial y} &= p_A^T R_B \frac{\partial p_B}{\partial y} \\ &= \left[\frac{1}{3}, \frac{2}{3} \right] \begin{bmatrix} 7 & 9 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} -\frac{3}{16} \\ \frac{3}{16} \end{bmatrix} \\ &= \frac{1}{16} [1, 2] \begin{bmatrix} 7 & 9 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} \\ &= \frac{1}{16} [1, 2] \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \frac{1}{16} \cdot 8 = \end{aligned}$$