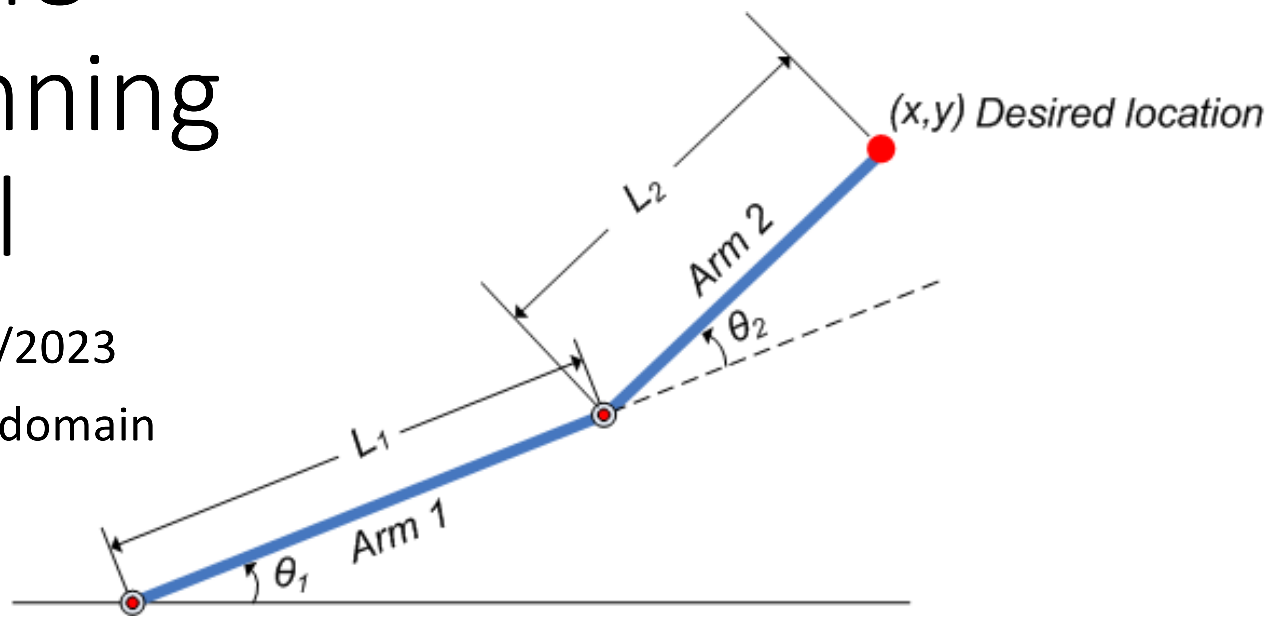
CS440/ECE448 Lecture 35: Planning and Control

Mark Hasegawa-Johnson, 4/2023



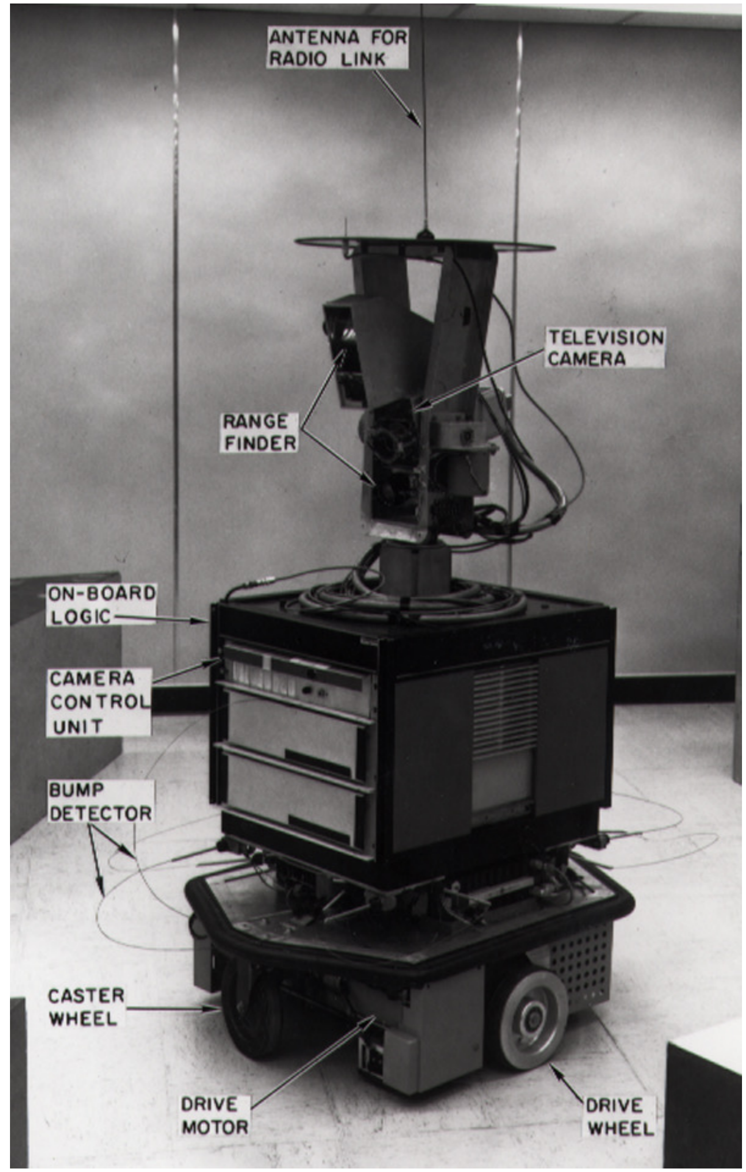
Outline

- The robot path planning problem
- Workspace vs. Configuration space
- Path planning
  - Visibility graph
  - Rapid Random Trees (RRT)
- Trajectory control
  - Proportion-Integral-Derivative (PID) controller
  - Model predictive control
- Model-based and model-free RL

What is a “Robot”?

Example: Shaky the robot, 1972
https://en.wikipedia.org/wiki/Shakey_the_robot

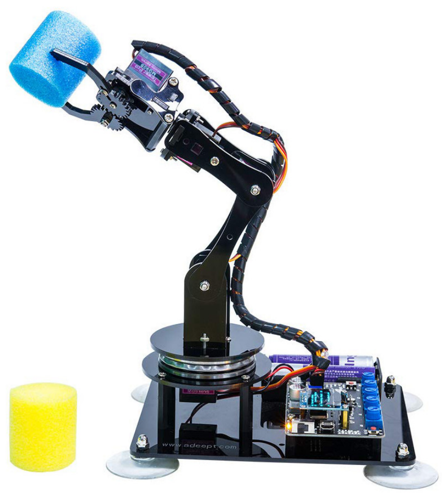
- Planning
  - Antenna for radio link
  - On-board logic
  - Camera control unit
- Perceiving
  - Range finder
  - Television camera
  - Bump detector
- Acting
  - Caster wheel
  - Drive motor
  - Drive wheel



Example: Robot Arm

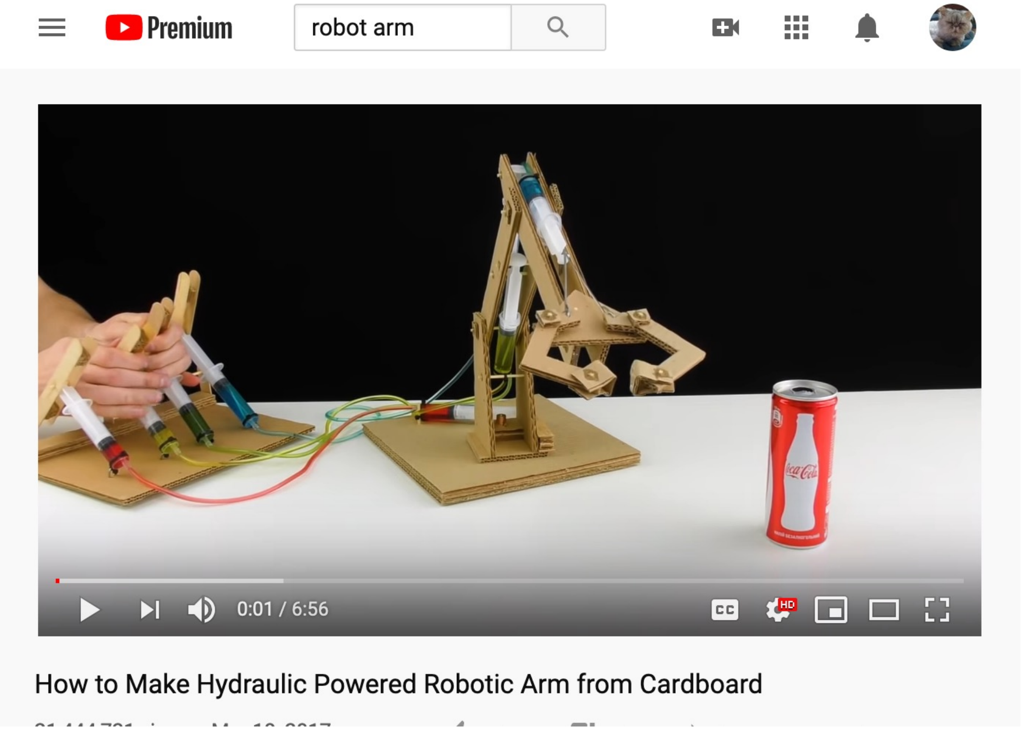
Adeept robot arm for Arduino (from Amazon)

- How does the robot arm decide when it has successfully grasped a cup?
- How does it find the shortest path for its hand?



Configuration Space Example: Robot Arm

https://www.youtube.com/watch?v=P2r9U4wkjcc



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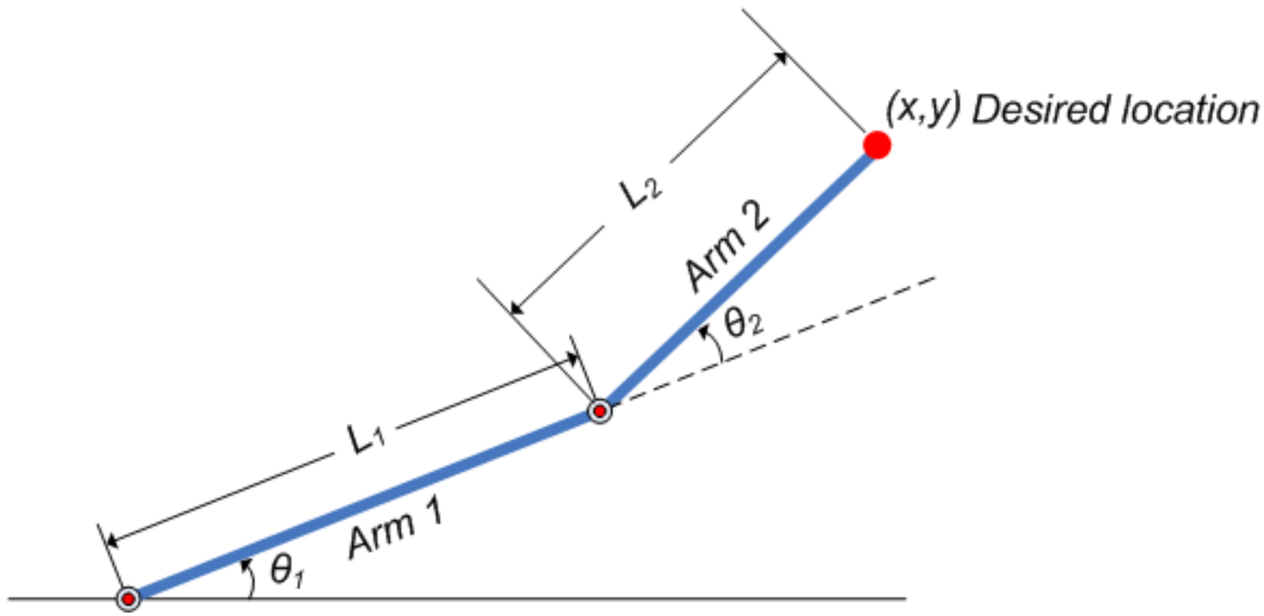
The Robot Arm Reaching Problem

https://www.mathworks.com/help/fuzzy/modeling-inverse-kinematics-in-a-robotic-arm.html

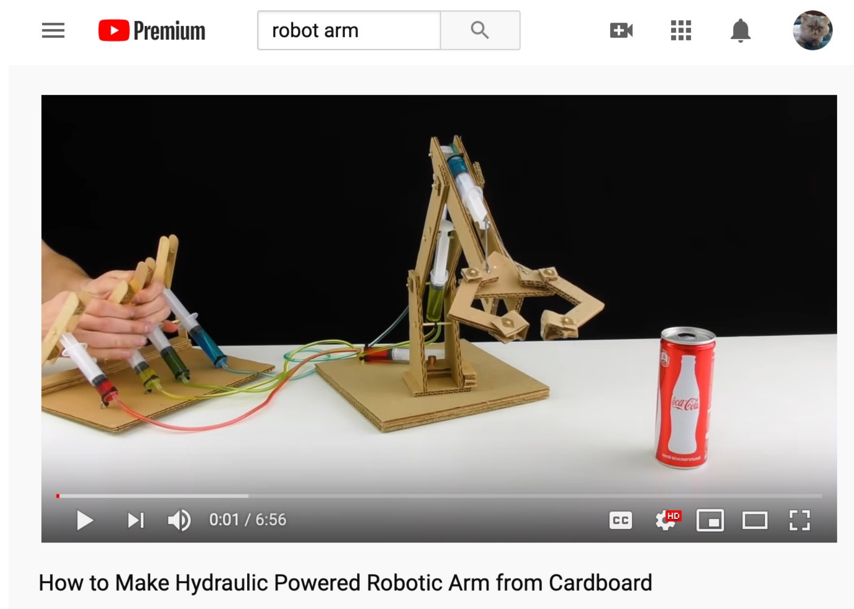
- Our goal is to reach a particular location (x,y)
- But we can't control (x,y) directly! What we actually control is (θ_1, θ_2).



Workspace vs. Configuration space

- A robot's **workspace**, $\mathcal{W}$, is the physical landscape in which it operates, $\mathcal{W} \subset \mathbb{R}^3$.
- **Configuration space**, C, is the set of joint angles that govern the robot's shape. For example, if we have four angles to control, then $C \subset \mathbb{R}^4$:

$$q = \begin{bmatrix} \text{shoulder azimuth} \\ \text{shoulder elevation} \\ \text{elbow elevation} \\ \text{gripper opening} \end{bmatrix} \in C \subset \mathbb{R}^4$$

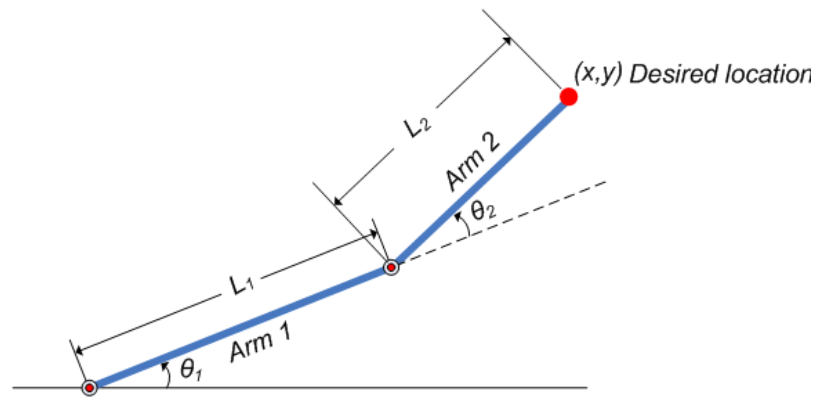


Forward kinematics

The **forward kinematics** function, $\varphi_b(q)$, maps (point on robot, configuration space)→(workspace). This is just geometry. Example:

- b = a particular point on the arm which is b meters from the shoulder, $0 \le b \le L_1 + L_2$
- $q = [\theta_1, \theta_2]$

$$\varphi_b(q) = \begin{cases} \begin{bmatrix} b\cos\theta_1 \\ b\sin\theta_1 \end{bmatrix} & b \le L_1 \\ \begin{bmatrix} L_1\cos\theta_1 + (b-L_1)\cos(\theta_1+\theta_2) \\ L_1\sin\theta_1 + (b-L_1)\sin(\theta_1+\theta_2) \end{bmatrix} & b \ge L_1 \end{cases}$$



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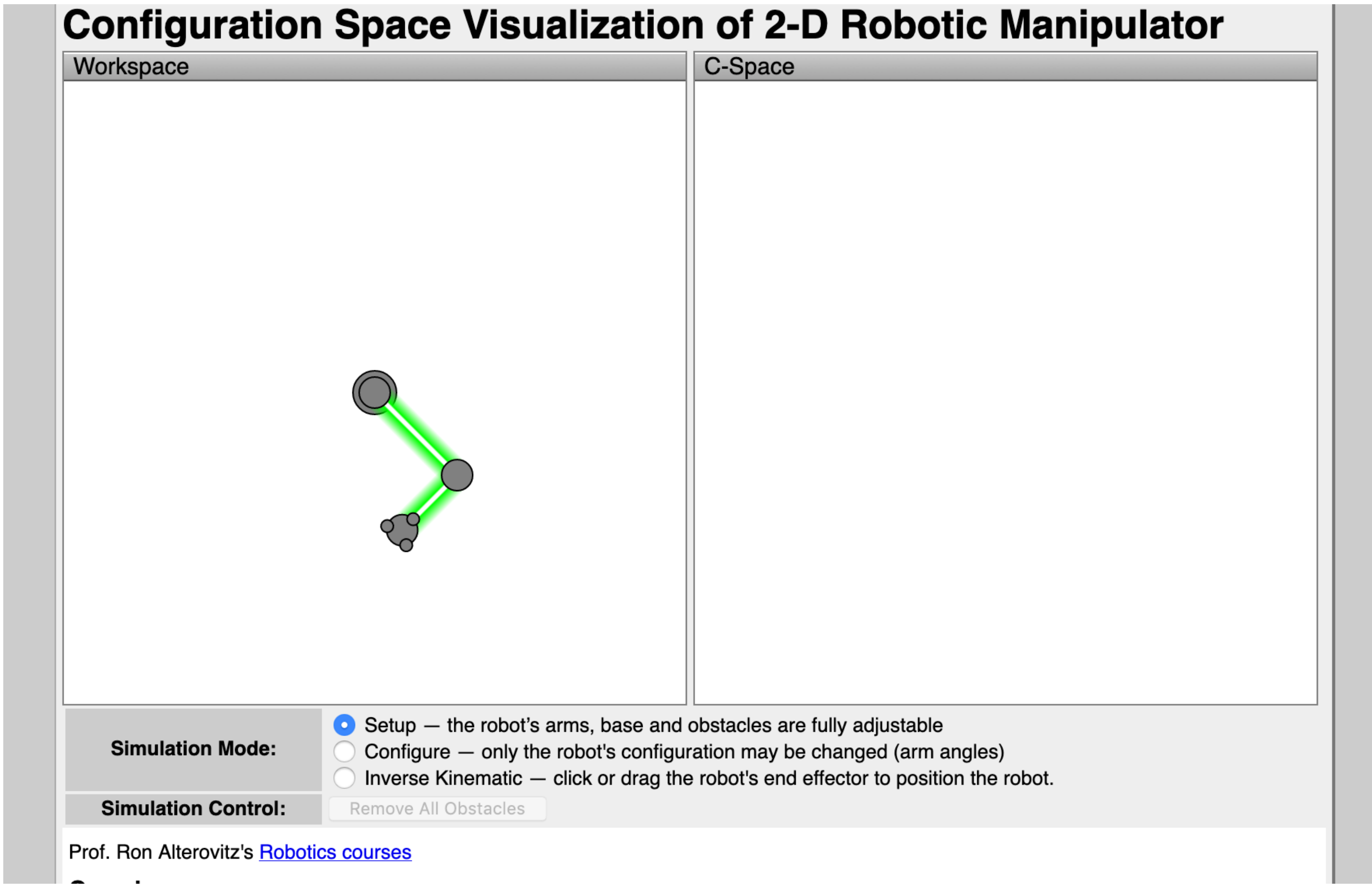
The Robot Arm Reaching Problem

Jeff Ichnowski, University of North Carolina, https://www.cs.unc.edu/~jeffi/c-space/robot.xhtml



Quiz

Try the quiz!

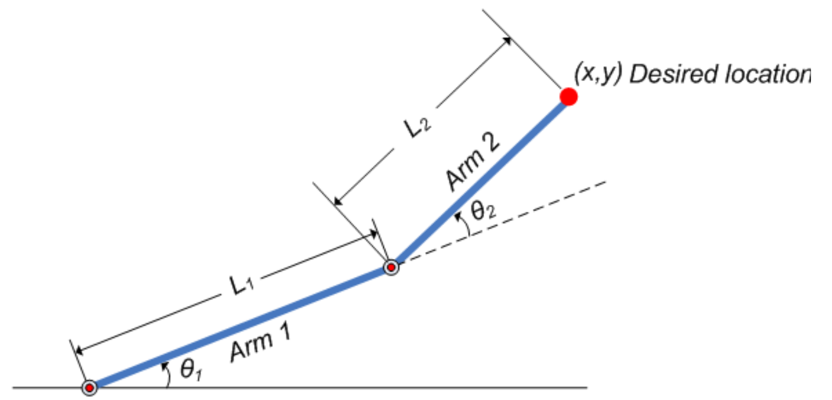
https://us.prairielearn.com/pl/course_instance/129874/assessment/2343758

Obstacles and Inverse kinematics

- Obstacles are things in the workspace, $\mathcal{W}$, that we don't want to run into.
- We want to plan a path through configuration space, C, such that we don't run into any obstacle.
- In order to do that, we need **inverse kinematics**: a function that converts obstacles in the workspace, $\mathcal{W}_{\text{obs}}$, into equivalent obstacles in configuration space, C_{obs}.

$$C_{\text{obs}} = \{q: \exists b: \varphi_b(q) \in \mathcal{W}_{\text{obs}}\}$$

- For example: we usually do this by just exhaustively testing every point in configuration space, to see if it runs into an obstacle.



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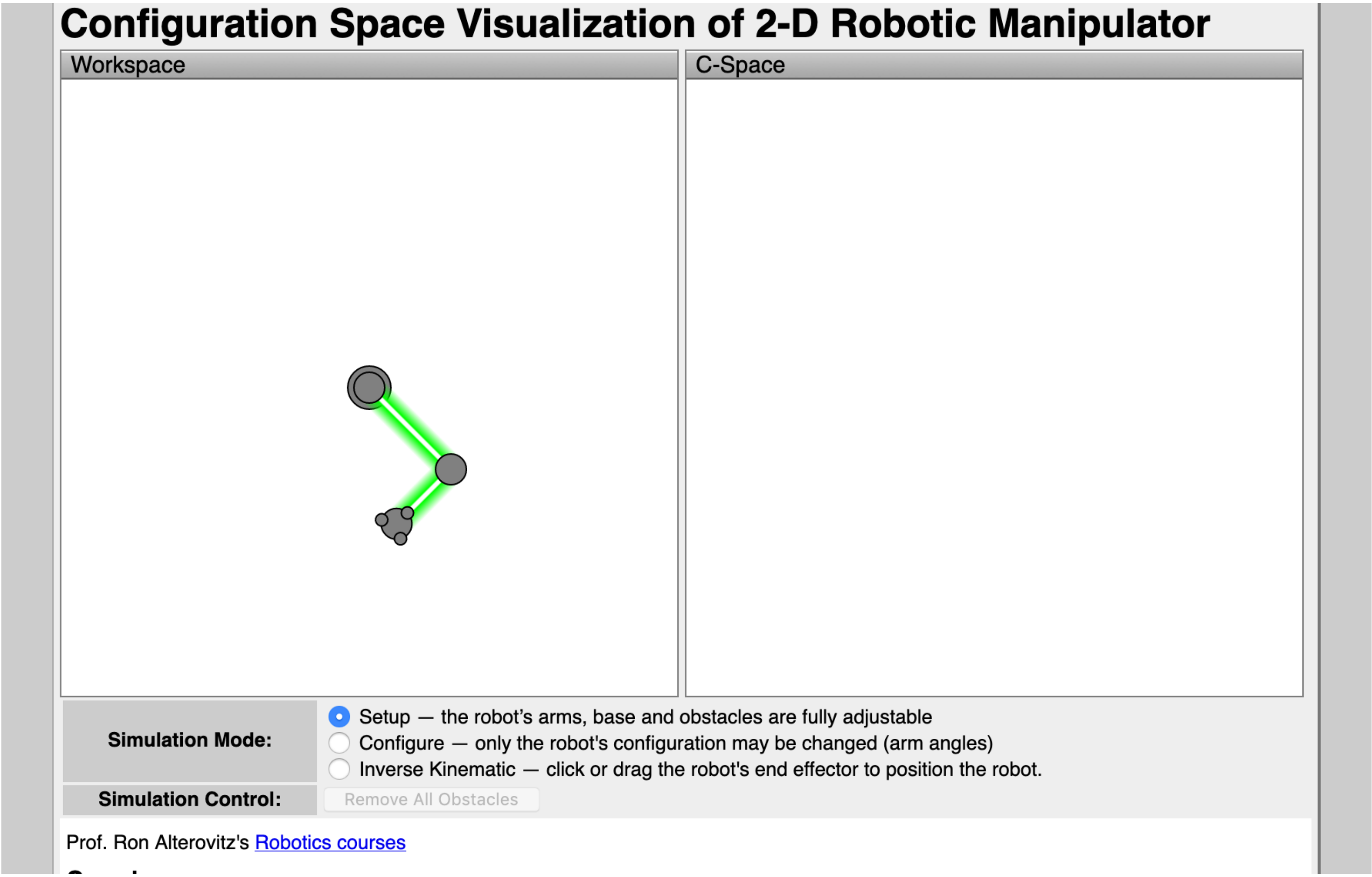

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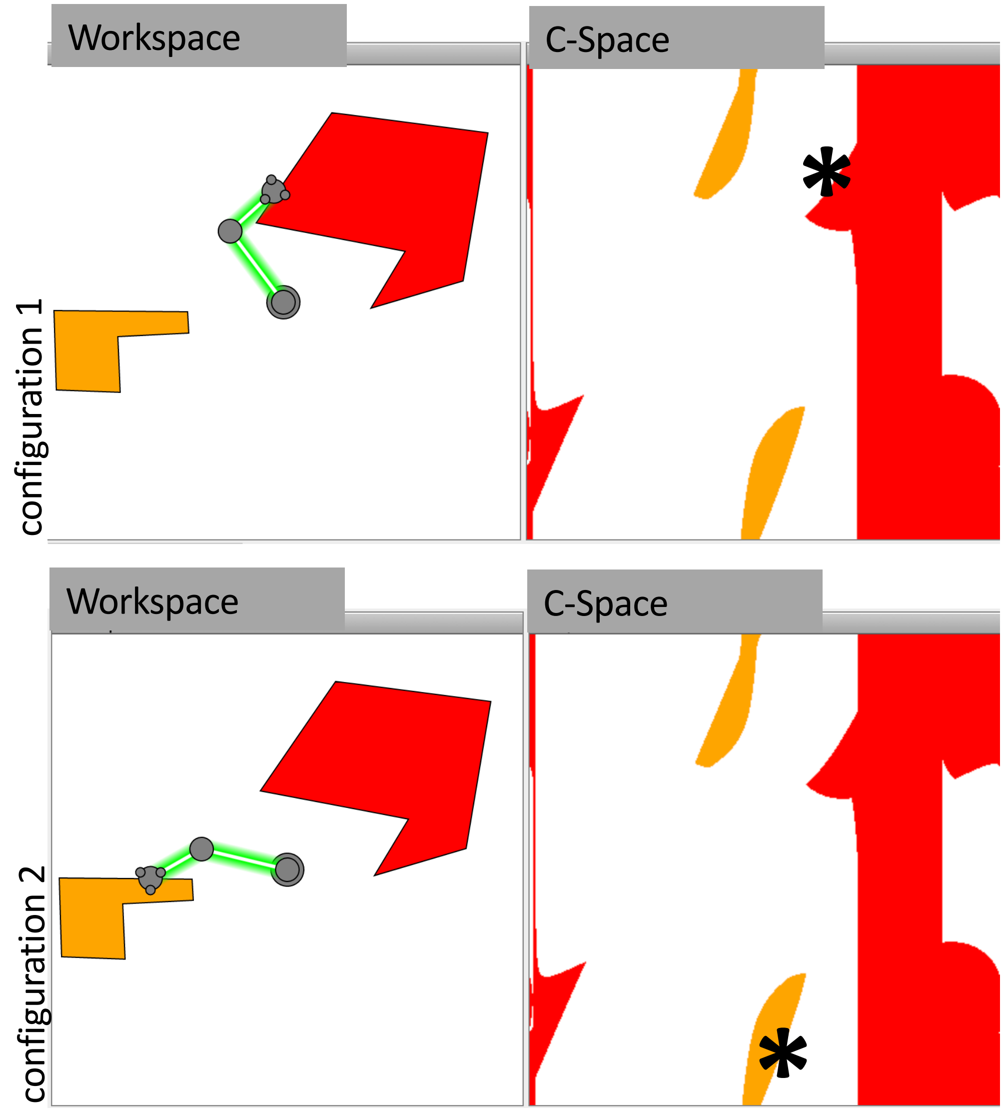
The planning problem

What is the best way to get from configuration 1 to configuration 2?



What is “best”?

We need some way to define the word “best.”

Assumption: The shortest path in C-space is the best way to get from config 1 to config 2.

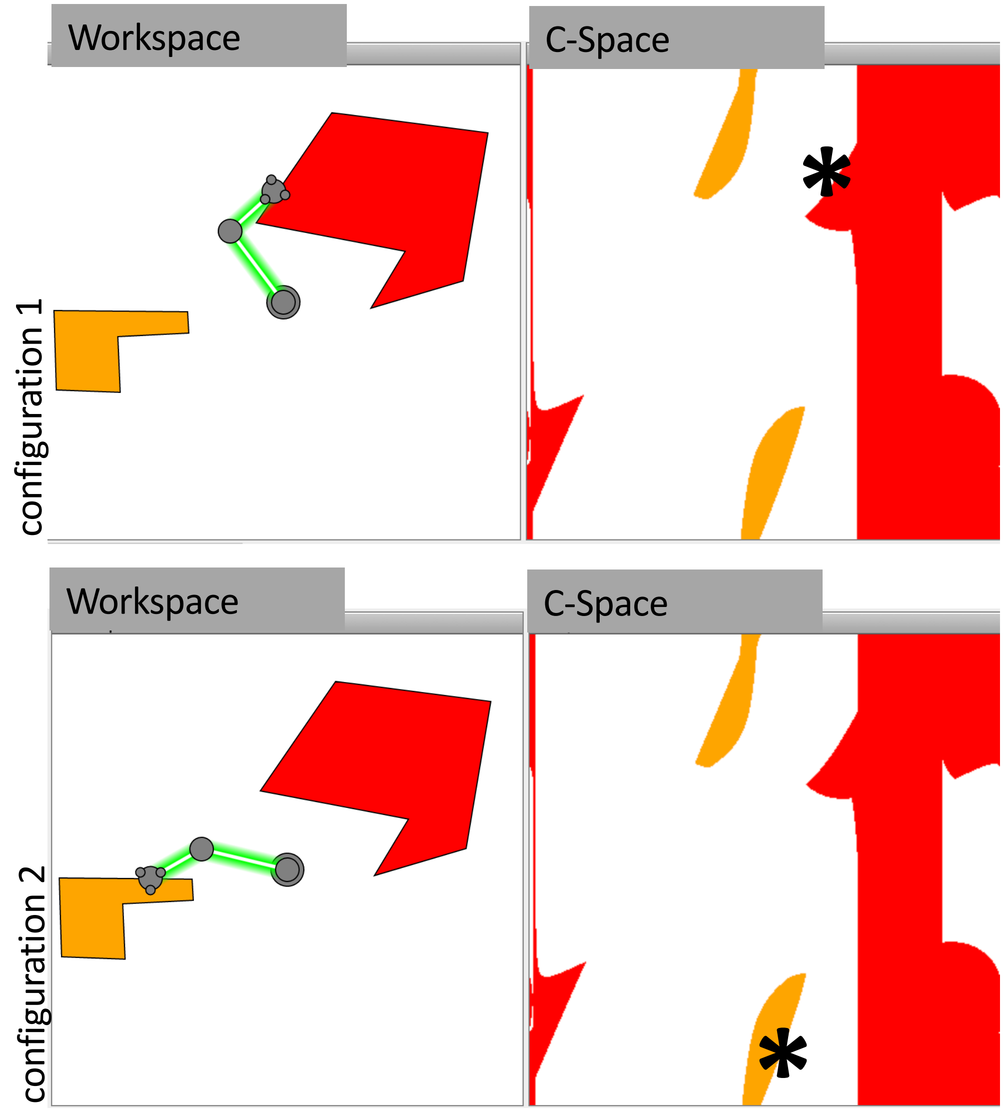
Implied assumption:

Longer path in C-space =

More manipulation of robot motors =

Greater energy expenditure =

Bad.



Finding the shortest path

Here are some algorithms you know that are guaranteed to find the shortest path:

- Dijkstra's algorithm (BFS)
- A* search

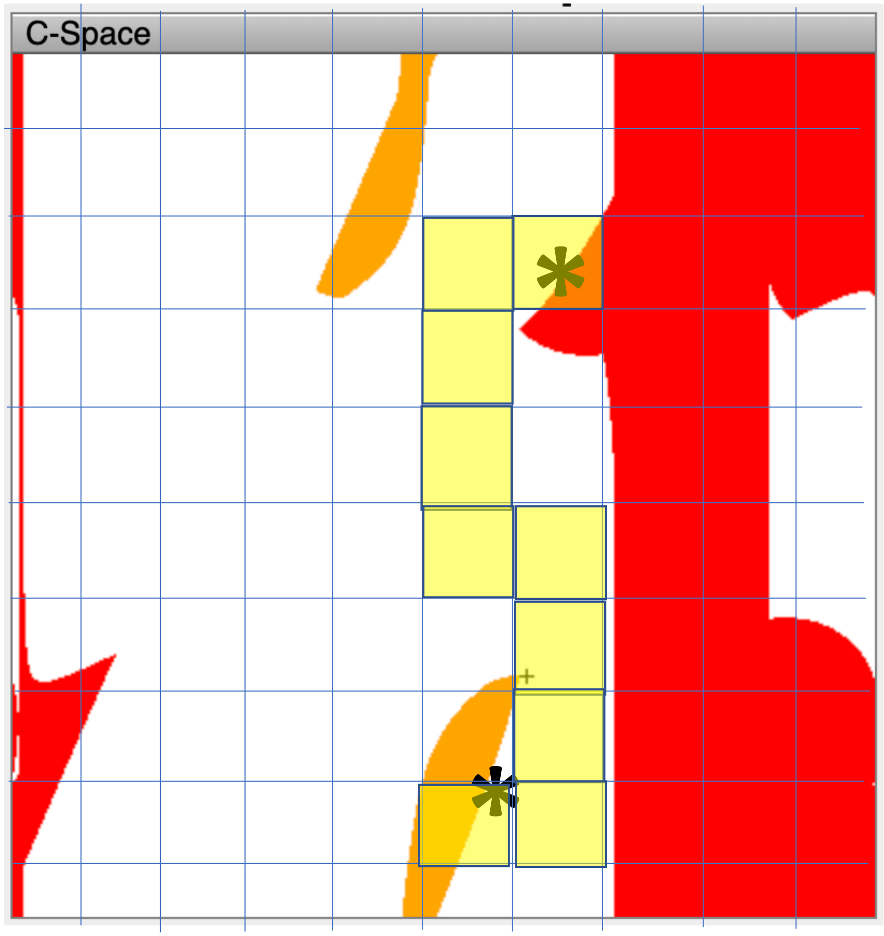
In fact, A* search was invented as a solution to the robot path planning problem. However, A* search is not quite well-suited to this problem, because…

A* requires discretizing the search space

A* assumes a discrete search space.

To apply it to the robot path-planning problem, we first need to discretize C-space.

We can discretize it using a rectangular grid, but doing so reduces the precision of our answer.



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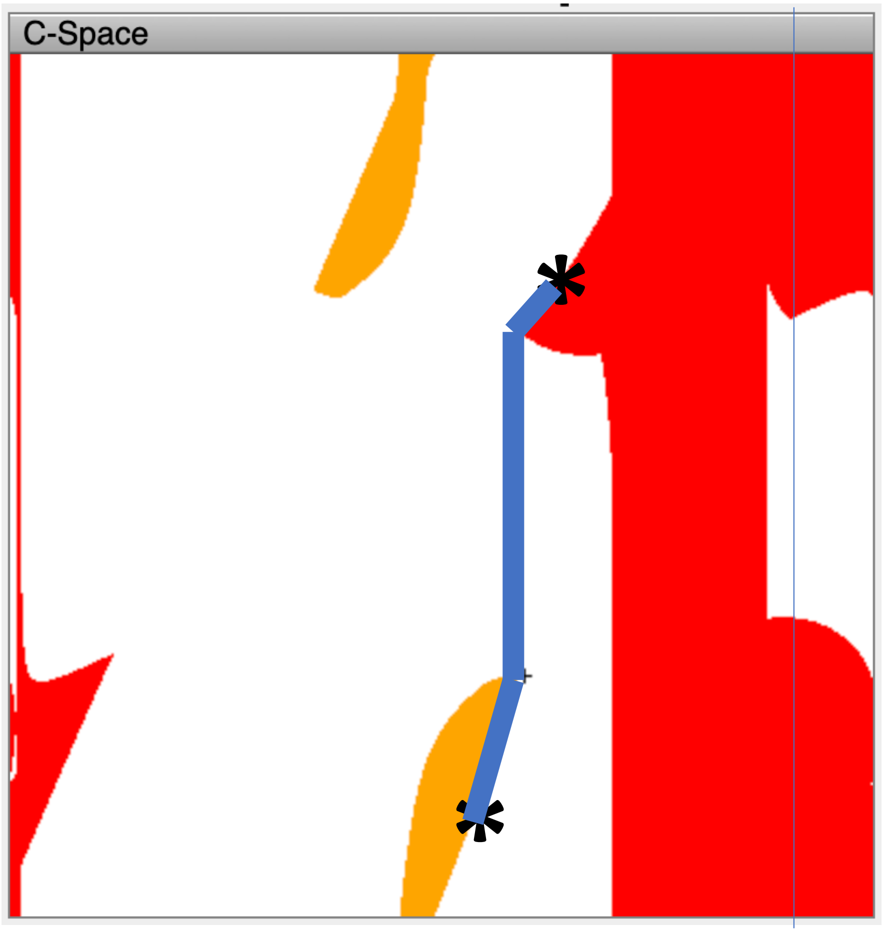
Visibility Graph

Suppose all the obstacles are polygons in C-space. Then the shortest path is guaranteed to be:

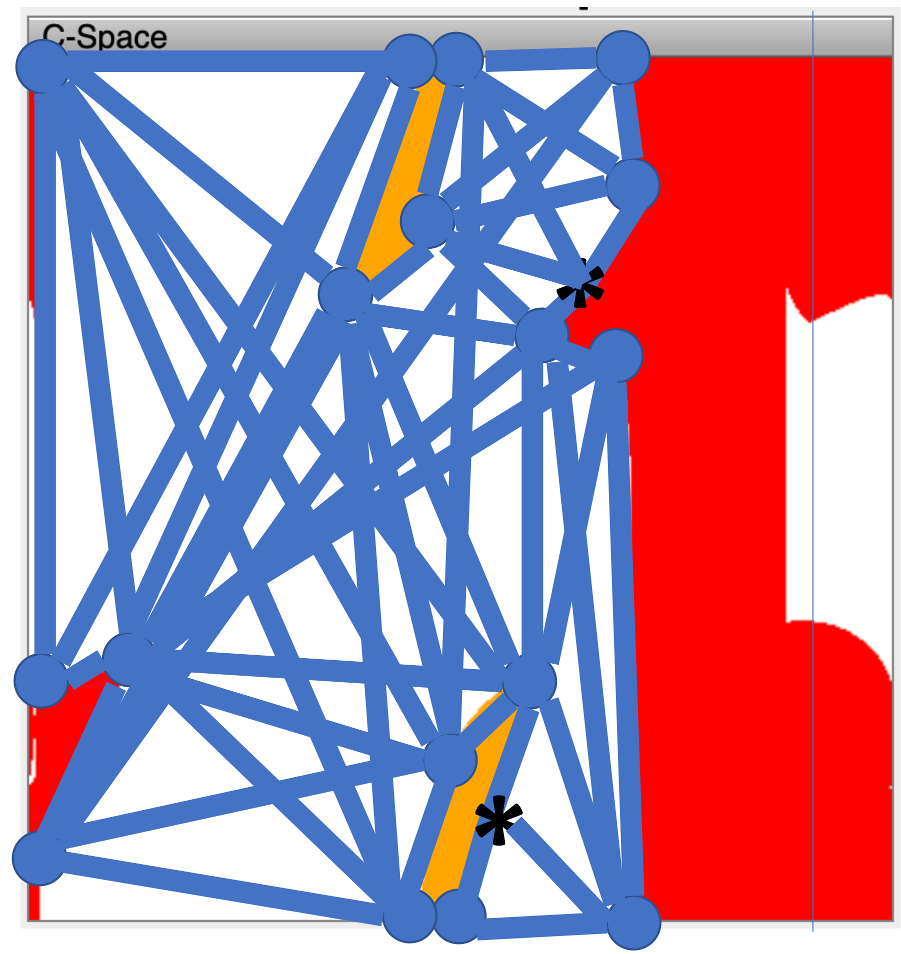
- From starting point to the corner of an obstacle, then…
- …from that corner to another corner, then….
- …from the corner of an obstacle to the goal.



Visibility Graph

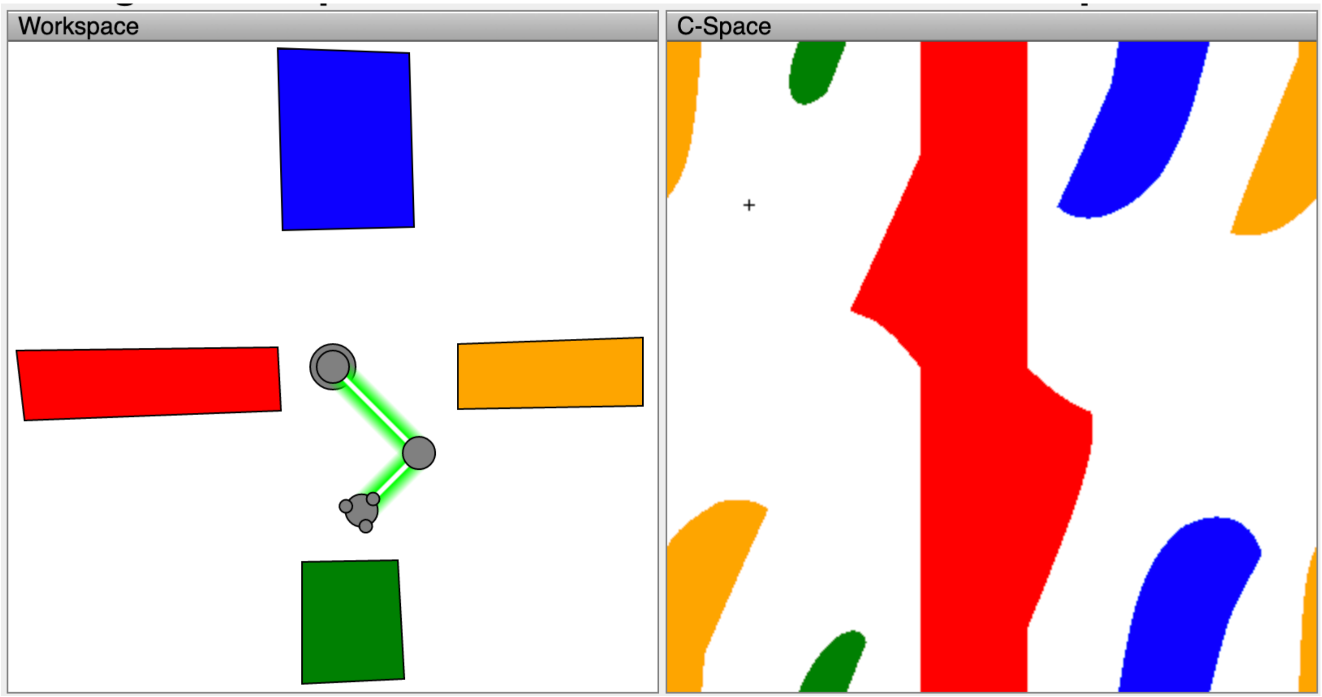
The algorithm, then, is:

1. Find all the corners.
2. Find the distances between every pair of corners.
3. Search that graph, using A*, to find the best path.



Limitations

The limitation of a visibility graph: it only works if the obstacles are polygons in C-space. If obstacles are arcs, they don't have corners.



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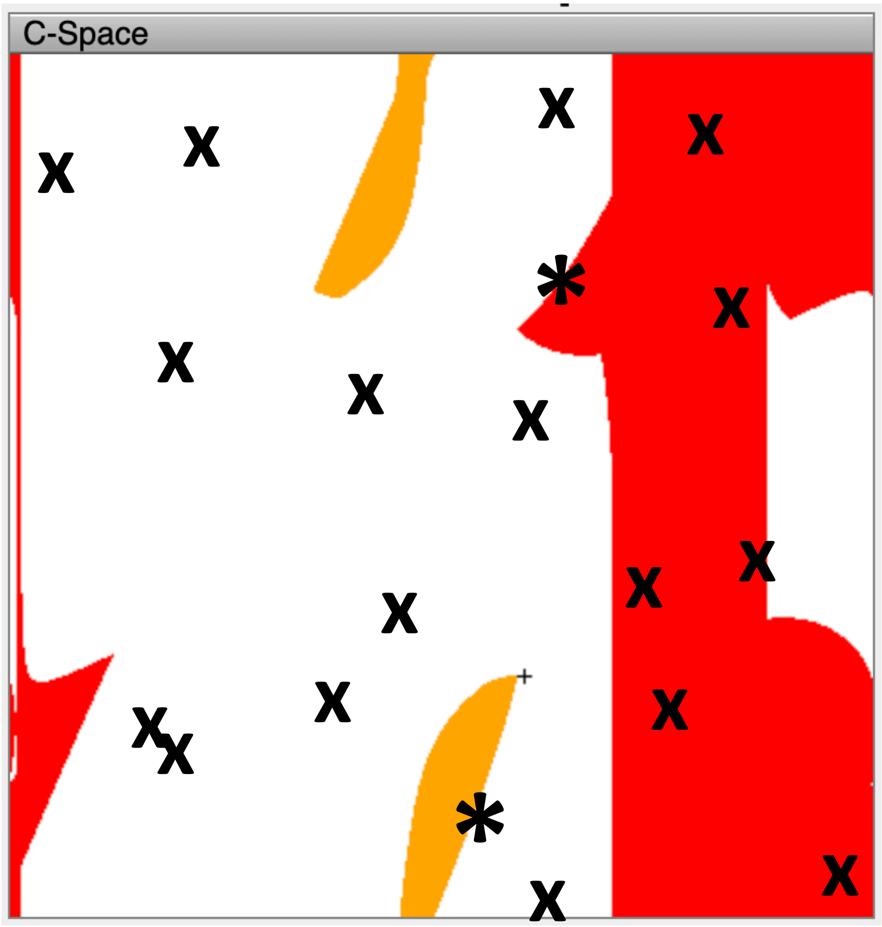
C-Space Best-path algorithms

- A* on a rectangular grid
  - Search nodes: squares on the grid
- A* on a visibility graph
  - Search nodes: obstacle corners
- A* on a graph of rapid random trees (RRT)
  - Search nodes: randomly sampled points

RRT

1. Generate a bunch of randomly sampled points to serve as search nodes
2. Eliminate the points that are inside obstacles
3. Perform A* over the remaining points to find the best path
4. Generate more samples in the vicinity of best points
5. Repeat steps 2 through 4



RRT

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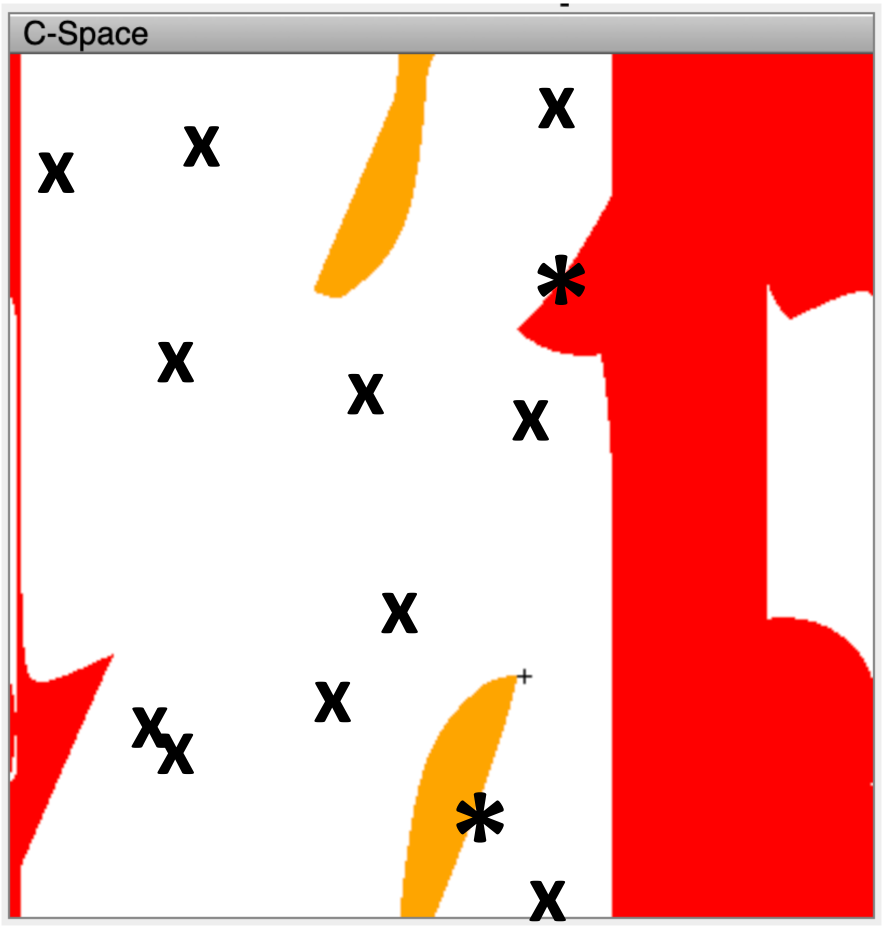

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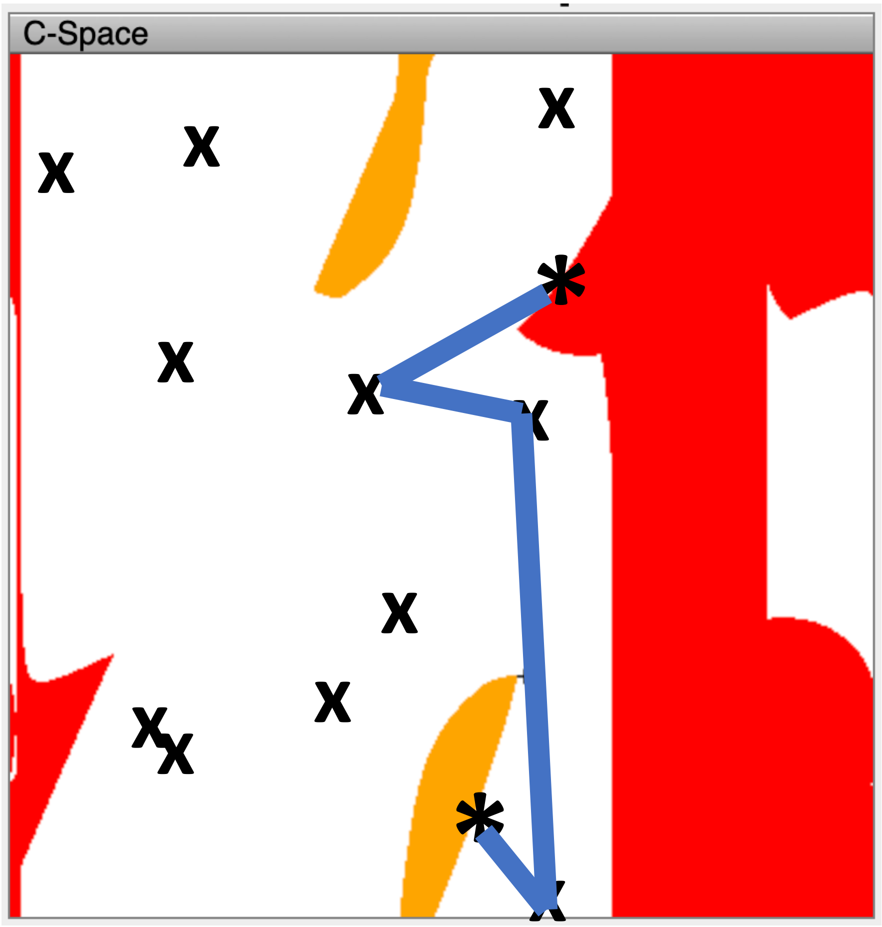

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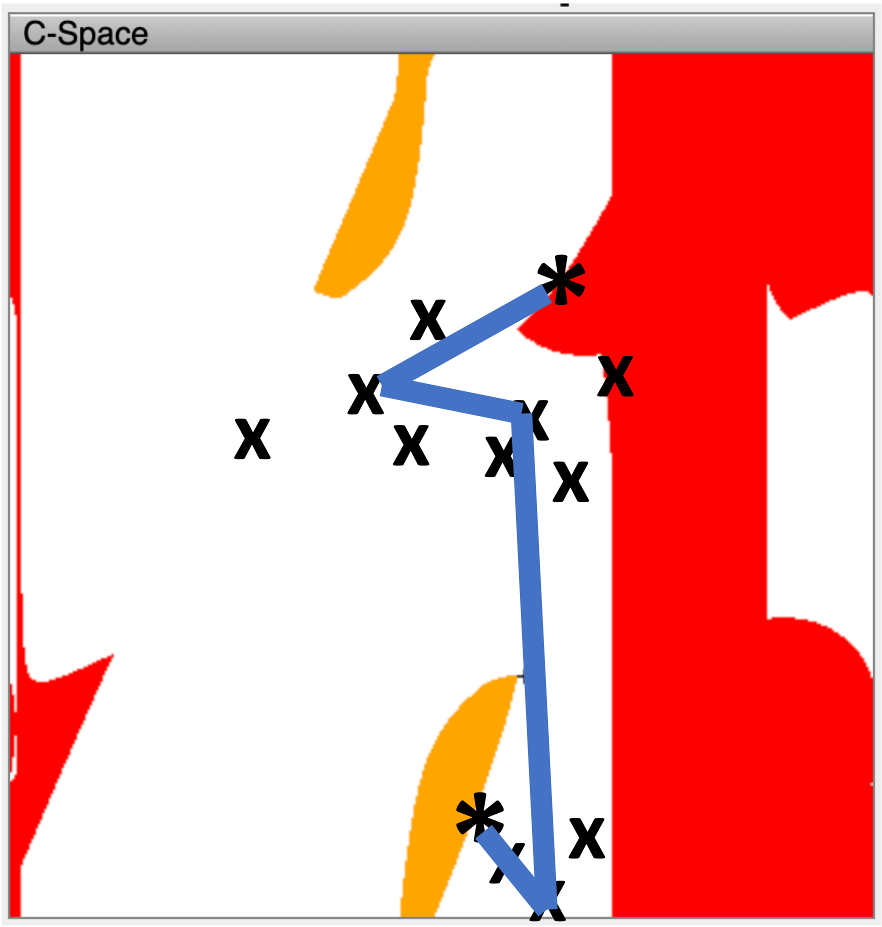

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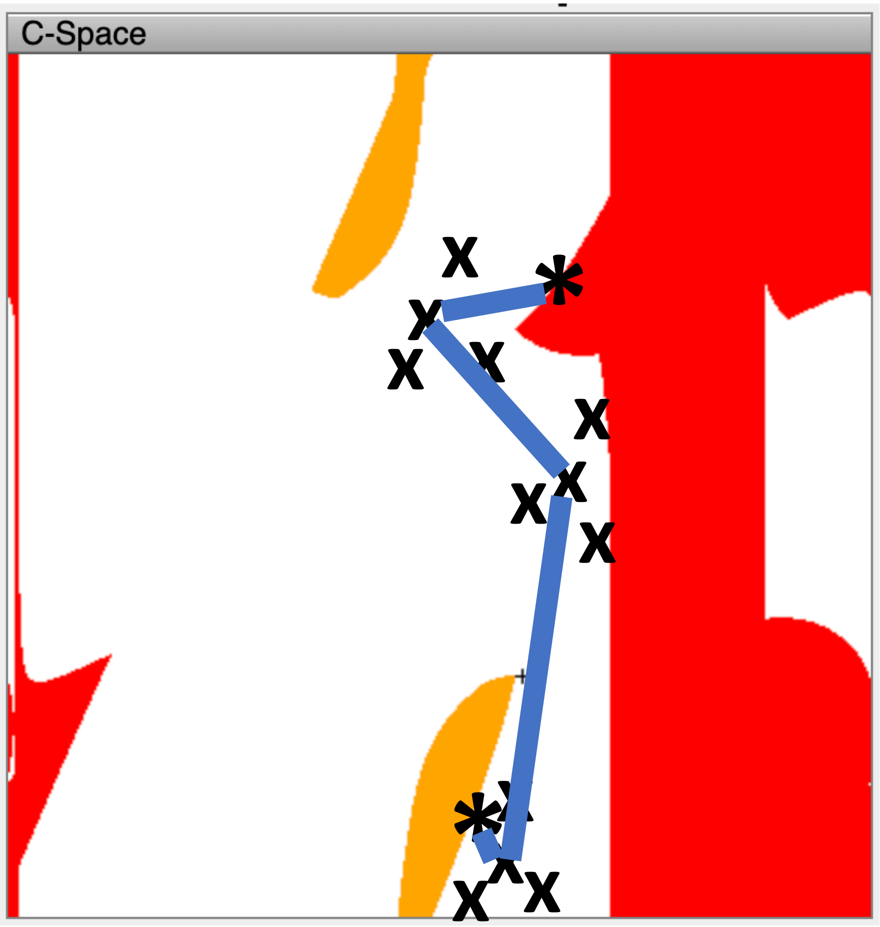

Key benefits of RRT

- Even with very limited computation (e.g., you can only afford one iteration), you still get a path that solves the problem
- In the limit of infinite computation (infinite # iterations), you get the best possible continuous-space path

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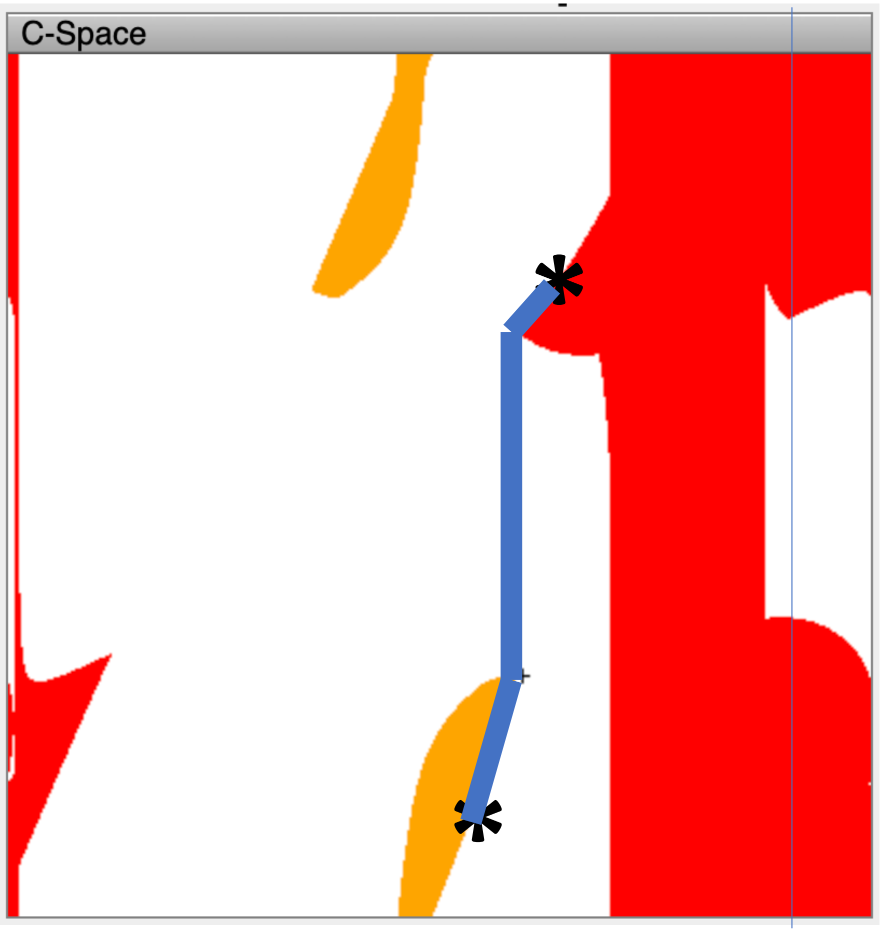
Trajectory control: maximum torque

Now that you have an optimum path, how fast should the robot travel along that path?

Consideration #1: maximum torque.

Find $q(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix}$ so that

$$\left|\frac{d^2\theta_1}{dt^2}\right| \le max_1, \left|\frac{d^2\theta_2}{dt^2}\right| \le max_2$$



Trajectory control: maximum safe velocity

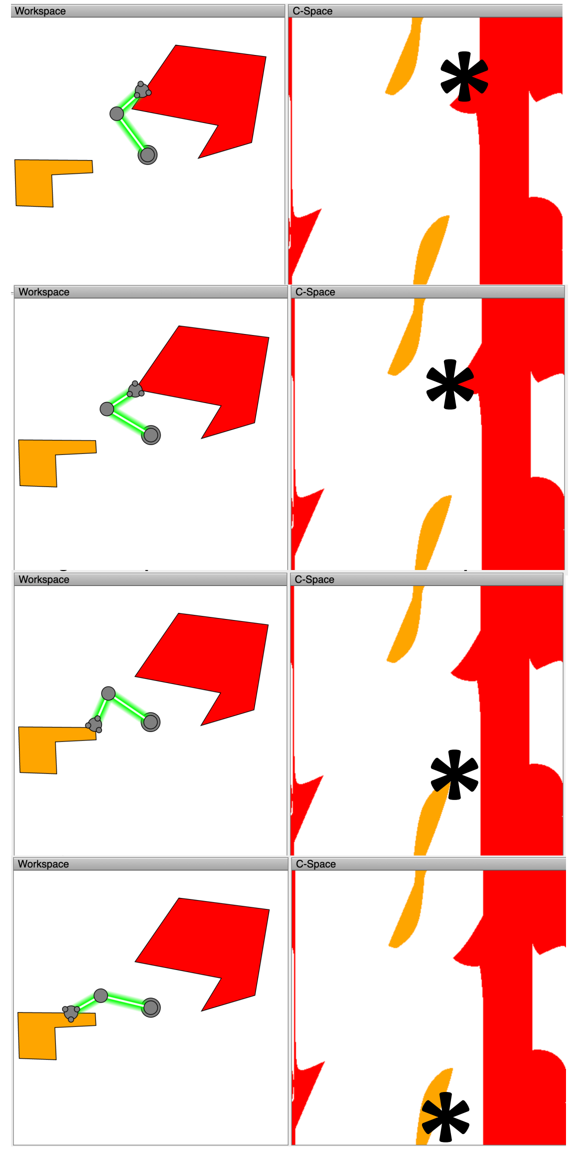
Consideration #2: maximum safe velocity.

Find $q(t) = \begin{bmatrix}\theta_1(t)\\ \theta_2(t)\end{bmatrix}$ so that

$$\sqrt{\left(\frac{dw_1}{dt}\right)^2 + \left(\frac{dw_2}{dt}\right)^2} \le v_{max}$$

...where $w(t)$ is the solution to the inverse kinematics:

$$\begin{bmatrix}w_1(t)\\ w_2(t)\end{bmatrix} \in \{w: \exists b: \varphi_b(q(t)) = w(t)\}$$



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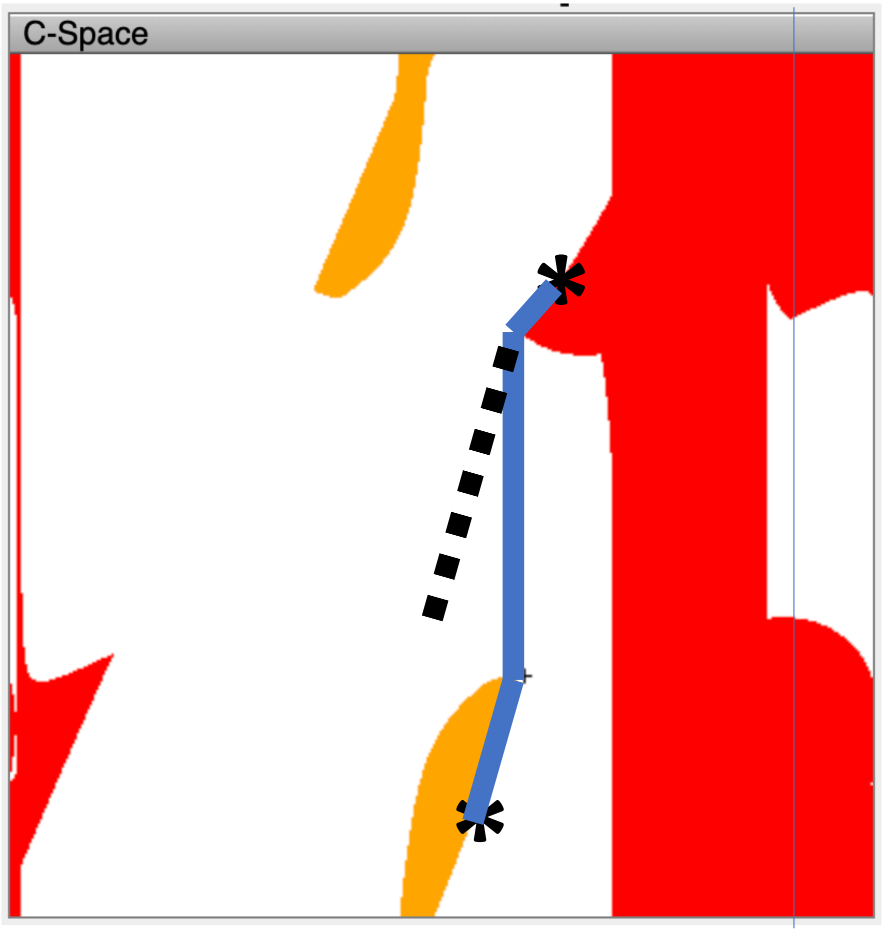
Trajectory control: error management!!!

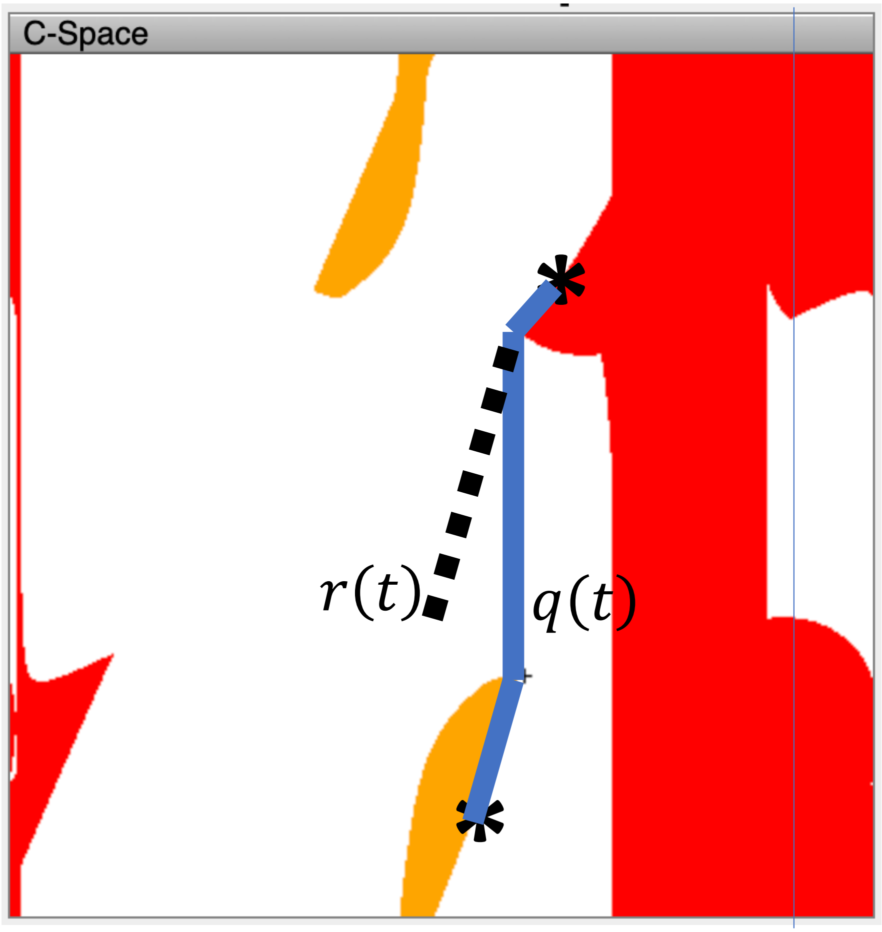
Consideration #3: what do you do if you start on a path but discover that your motor is miscalibrated and you're going the wrong direction?



P-controller

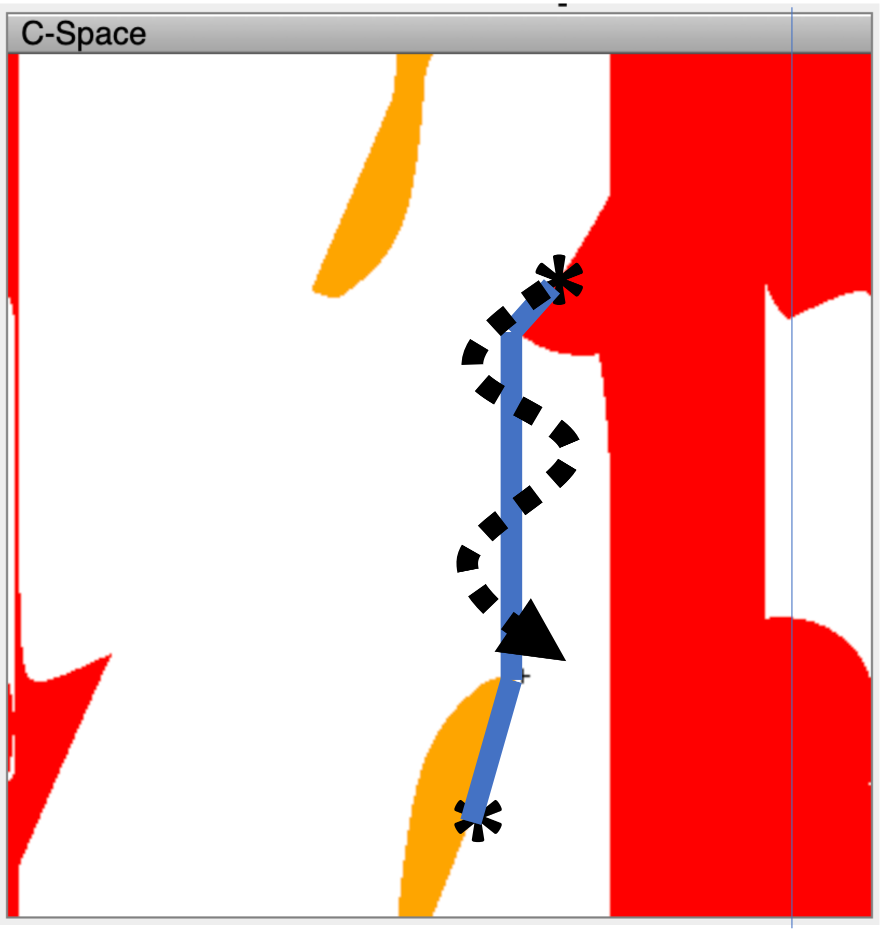
A proportional controller (P-controller) adds some extra torque in proportion to the error:

$$\frac{d^2}{dt^2}\begin{bmatrix}\theta_1 \\ \theta_2\end{bmatrix} = K(q(t) - r(t))$$



P-controller Problems

A P-controller tends to result in oscillating overshoot.

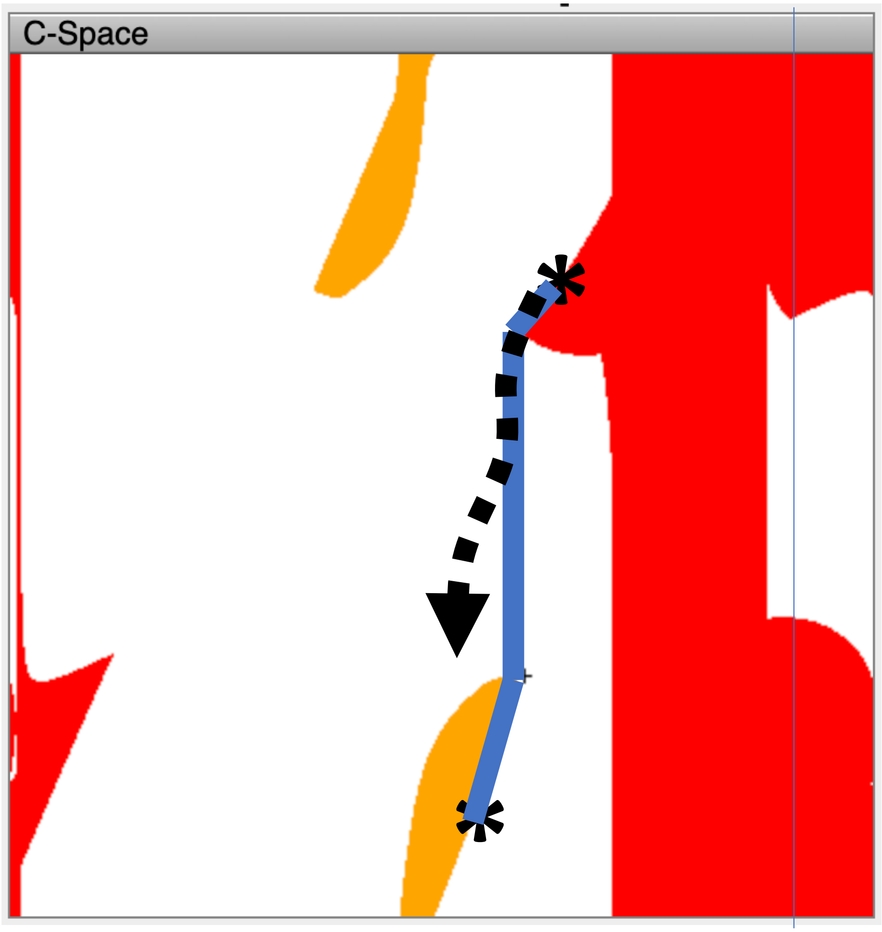


PD-controller

A proportional-derivative controller (PD-controller) adds some extra torque in proportion to the error of the derivative:

$$\frac{d^2}{dt^2}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = K_P(q(t) - r(t)) + K_D(\dot{q}(t) - \dot{r}(t))$$

Doing this can smooth out the trajectory, but can leave some long-term error



PID-controller

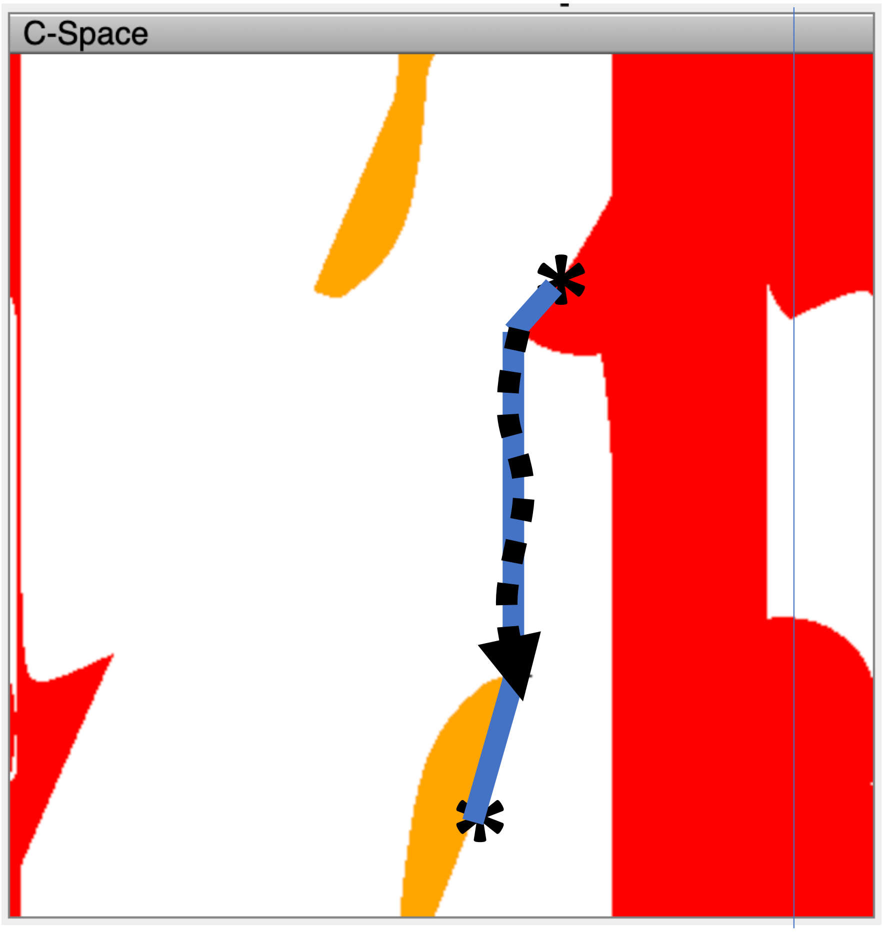
A proportional-integral-derivative controller (PID-controller) adds some extra torque in proportion to the error of the integral:

$$\frac{d^2}{dt^2}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = K_P(q(t) - r(t))$$
$$+K_I\int_0^t (q(\tau) - r(\tau))d\tau$$
$$+K_D(\dot{q}(t) - \dot{r}(t))$$

The P term fixes short-term errors.

The I term fixes long-term errors.

The D term smooths out oscillations.



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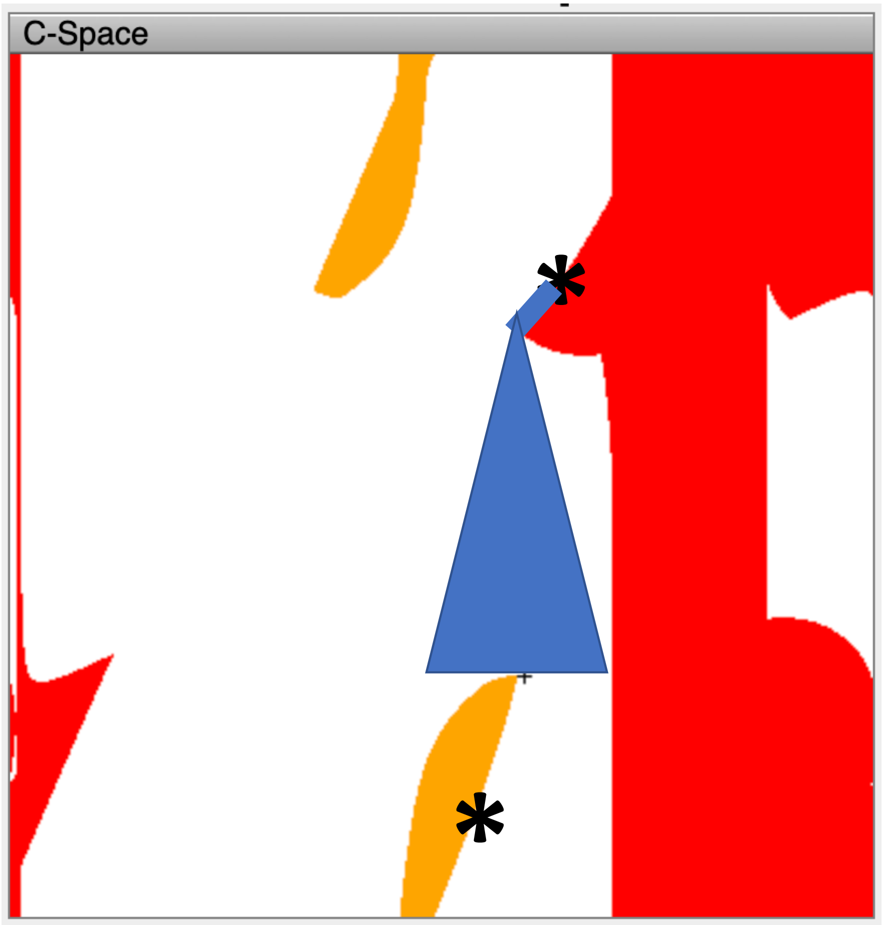
What if your motors behave randomly?

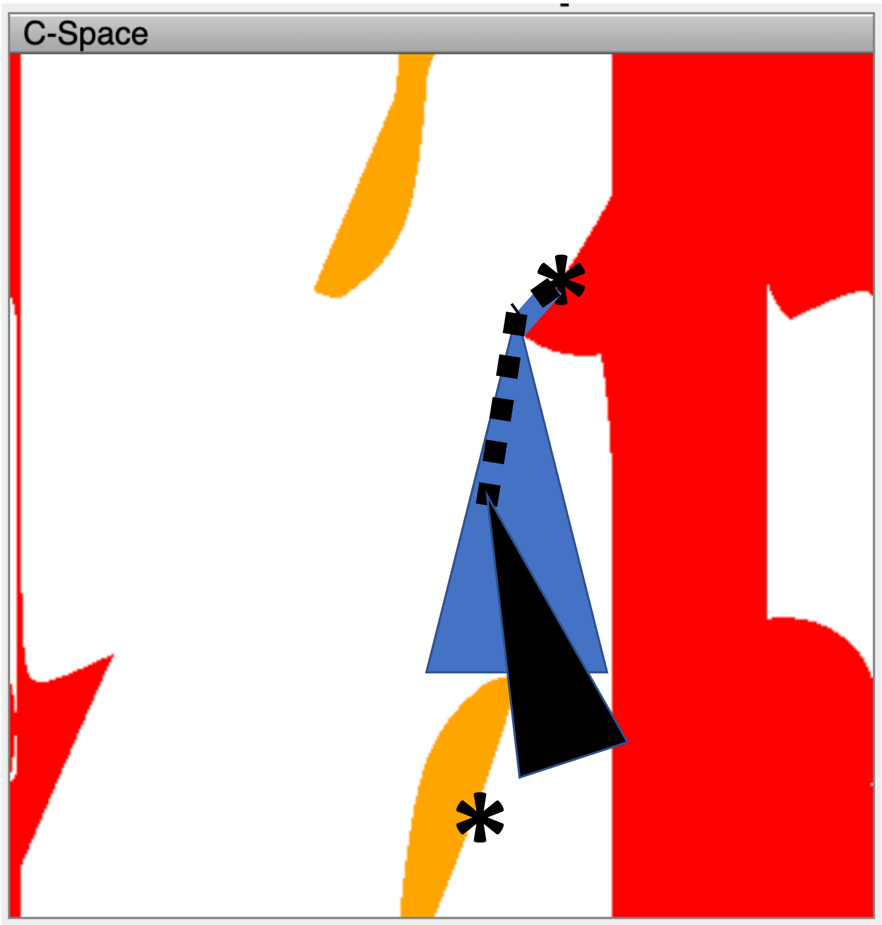
- What if your motors have some randomness?
- Then you might not be able to plan an exact trajectory.
- The best you can do is plan a trajectory that goes in the right general direction.



Model predictive control

… means the following strategy.

1. Plan an optimum trajectory
2. Go partway
3. Observe where you are
4. Recalculate the optimal trajectory
5. Repeat



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What if your transition probabilities are unknown?

- So far, we assume that you have some idea what your motors will do.
- What if you have no idea?
- Reinforcement learning!

What if your transition probabilities are unknown?

- Model-based reinforcement learning:
  - Try some stuff, observe what happens
  - Update a model of your motors and your workspace
  - Update your policy
- Model-free reinforcement learning
  - Try some stuff, observe what happens
  - Update a Q-table that tells you what actions to perform

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